

It is the ambiguity. (But only three generations)

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Abstract

It is suggested that generations are linked to the need of calculating curvature of space via a deformed or discrete calculus. Quantization would limit the deformation, building three generations, and not four, as other interpretation could imply.

1 Introduction

It is known [8, 11] that the usual ambiguity in the definition of (partial) derivatives of a function becomes a source of problems when we go to quantum theory. The simplest example is Feynman quantum mechanics (or 0+1 dimensional field theory), where different elections of discretisation for the derivative drive to different ordering rules in the quantized theory; Weyl corresponds to the symmetric one, Born-Jordan to the forward derivative, and so on, and even more exotic effects could be got by gauging the ambiguity.

My paper [17] has been understood in [2] as if it were giving theoretical support to the existence of a fourth generation. Indeed, as it is pointed in [17] and also previously in [16], I believe that the ambiguities of a discrete curvature only point to three generations, the extant ambiguity being absorbed in a scale parameter which relates to Plank constant (and, to the deformation parameter of the calculus). It is my fault that this process is just half cooked, in sparse references at the end of [15] and, as appendix, in [14]. I apologize I can not help with a detailed example yet, so in order to try to clear the confusion, this paper can only to expand on the ideas of our previous [17].

So, please consider this note as a traditional conference poster, trying to put illustrations to the previous work. The examples of the previous papers were formulated in the context of non commutative geometry. Here we will keep ourselves in the conceptual cloud.

2 Pictorial image

The main observation, figure (1), is that to define a vector tangent at a point x_0 we need to build two series of points approaching x_0 , so the limit $(x_2 - x_1)/\epsilon$ will give the tangent vector. It seems that there are two ambiguities, to choose

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x_2 and to choose x_1 , but one of them can be absorbed into the scale parameter. The extant ambiguity is the one we proposed to consider a mass.

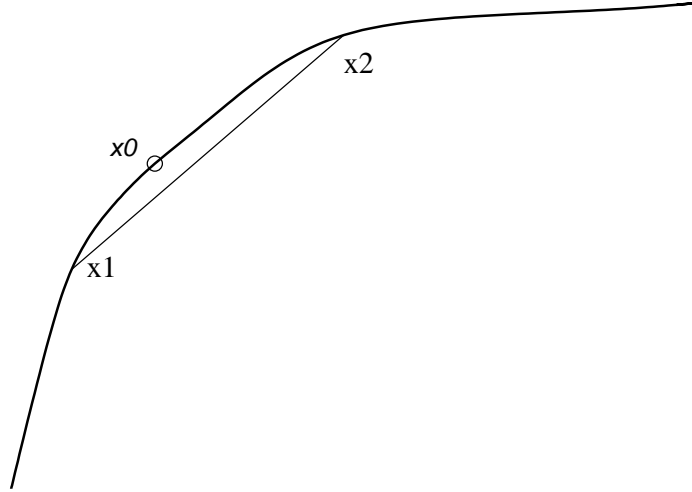


Figure 1: q-tangent vector to the point x_0

Consider now, as in figure (2), a curve of which we want to know the curvature at a point. This implies to give four points, so the orthogonal to two q-tangent vectors will cross marking the position of the radius of curvature, and giving us the inverse of the curvature when the continuous limit is approached.

Again the scale can absorb one of the ambiguities, and we have three free parameters, that we can identify with three masses.

When going to higher dimensions, the play is not more ambiguous, but it is more complicated (Figure 3). The curvature tensor is build from the curvatures of the geodesic surface tangent to each 2-plane. Every direction must be considered, and dependences between the ambiguities of orthogonal directions are not clear.

There are ways to simplify the task, for instance asking for additional restrictions (isotropy, homogeneity) to the space-time manifold. In any case, we should expect now to multiply our degeneration times the number of dimension of space time, to be able to cope with every curvature. Thus we will get four particles (from space-time dimension) and three generations (from ambiguity).

Last, consider how the scale parameter could come into play. Every tangent vector is defined via tales theorem, figure (4), relating the time (or parameter of the curve) with the distance. Classically each triangle is well defined at each point of the curve. Now, doubts can be raised when we make an integration (figure 5) in the "deformed" way, before any limit. It seems that we should introduce an scale parameter ϵ , to be able to add the quantities of each triangle, and that the continuous limit should come when $\epsilon \rightarrow 0$.

The process as I see it is a little more involved, as first one needs to use scale invariance to go from a bare ϵ to a renormalized h , and the classical limit is $h \rightarrow 0$. This method is needed because the groupoid of paths given by Connes does not add $(x, y, t)(y, z, t)$ to $(x, z, 2t)$ but to (x, z, t) .

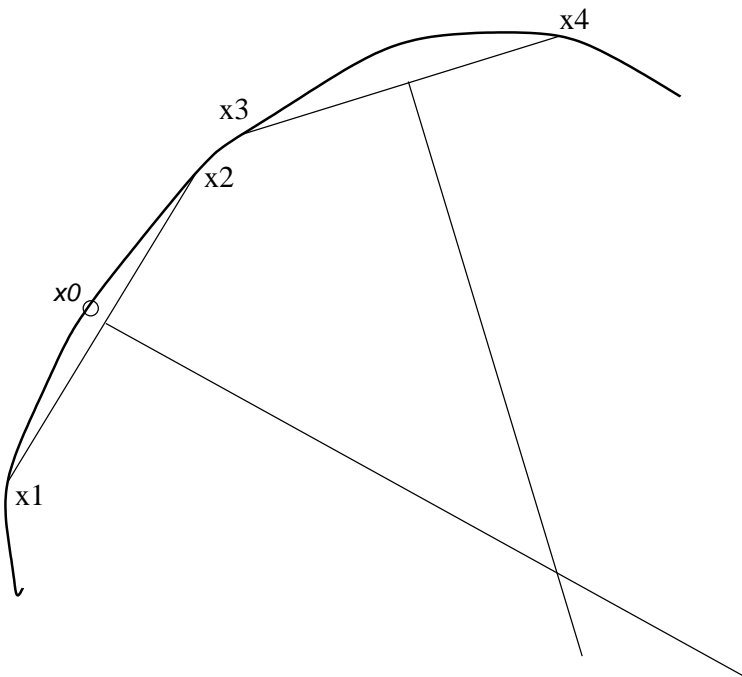


Figure 2: Curvature of a curve through x_0

3 Interdisciplinary work

The most popular of all the regularisation problems is the one of fermion doubling, which happens when we try to quantize fermions in lattice quantum field theory. Here, if we use only the symmetric definition of derivative we finish with a set of 2^d fermions for each initial fermion in the theory. Wilson cures this by incorporating the ambiguity to the theory, building each derivative as a combination of symmetric and antisymmetric part, and then giving a high mass to the antisymmetric combination, which controls the unwanted degrees of freedom. Recently [12, 9] Wilson' approach has been resuscitated and the Ginsparg-Wilson condition is looking for a place in non commutative geometry. Lüscher' approach is perhaps closest to Dimakis or Majid ones, but Balachandran [1] is already looking for a role for it near to the axiomatic of NCG manifolds.

From the point of view of non commutative geometry, it seems [10] that naive lattice field theory does not qualify as a differential geometry, as even when multiple copies of elements are taken (which seems needed to cope with NCG first order axiom), it fails to fulfill Poincare duality. To get out of this trap, the suggestion should be to act with the Dirac operator in different points of the space.

We will do this by introducing small finite differences between the fields attached to each fermion, and relating this difference to the inverse of the mass reasoned in the pictorial show, so that in the very low energy limit the implemented difference becomes equal to the derivative it discretizes. Our goal is to expand NCG Lagrangian to contain information about quantization ambiguity,

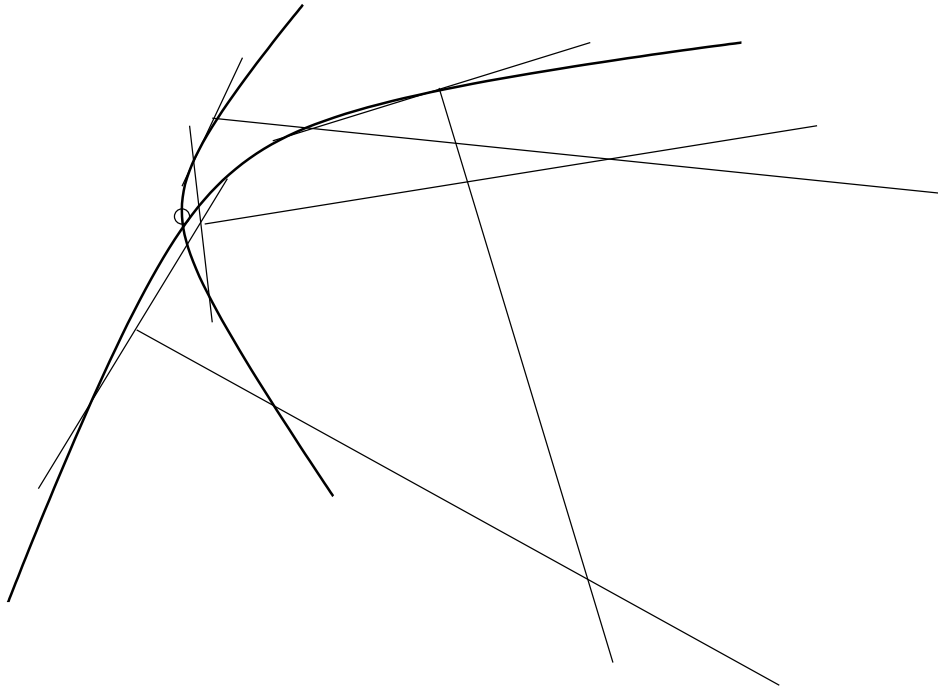


Figure 3: Two curvatures of a surface

but with this technique we also get to introduce the spectrum of masses.

Time ago in Barcelona, Alain, in a dual session with Asthekar, suggested that the newer versions [5, 6] of the Connes-Lot approach should be seen as an low energy approximation to a completely non commutative space only visible at high energy. In some sense, here the methodology is reversed, betting first of a "very non commutative" model and trying to guess a method to go down to low energy. At very low energy, only gravity should be seen, while at intermediate energy, Connes-Chamseddine or Connes-Lot should be suitable approximations.

Just to visualize such approximation we want to keep ourselves using the Dirac operator formalism. On the other hand, from the Tangent Groupoid construction, we know that the set of functions over the tangent space TM of a manifold can be pasted, via Weyl quantization [3, 4], to the set of kernels $k(x, y)$ of operators in a Hilbert space, and this formalism is very near to the finite difference scheme proposed above. It should be nice if our candidates for differential forms had some duality with this space of operators. Also because the tangent groupoid seems closer to q-deformations as made by Majid and others, and to the non-commutative formalism used by lattice theoretists in the above referred works.

4 Mass

Basic mass, as we have seen, should be fixed by the position of the vector of figure 1 respective of the point which "differentiation" is assigned to.

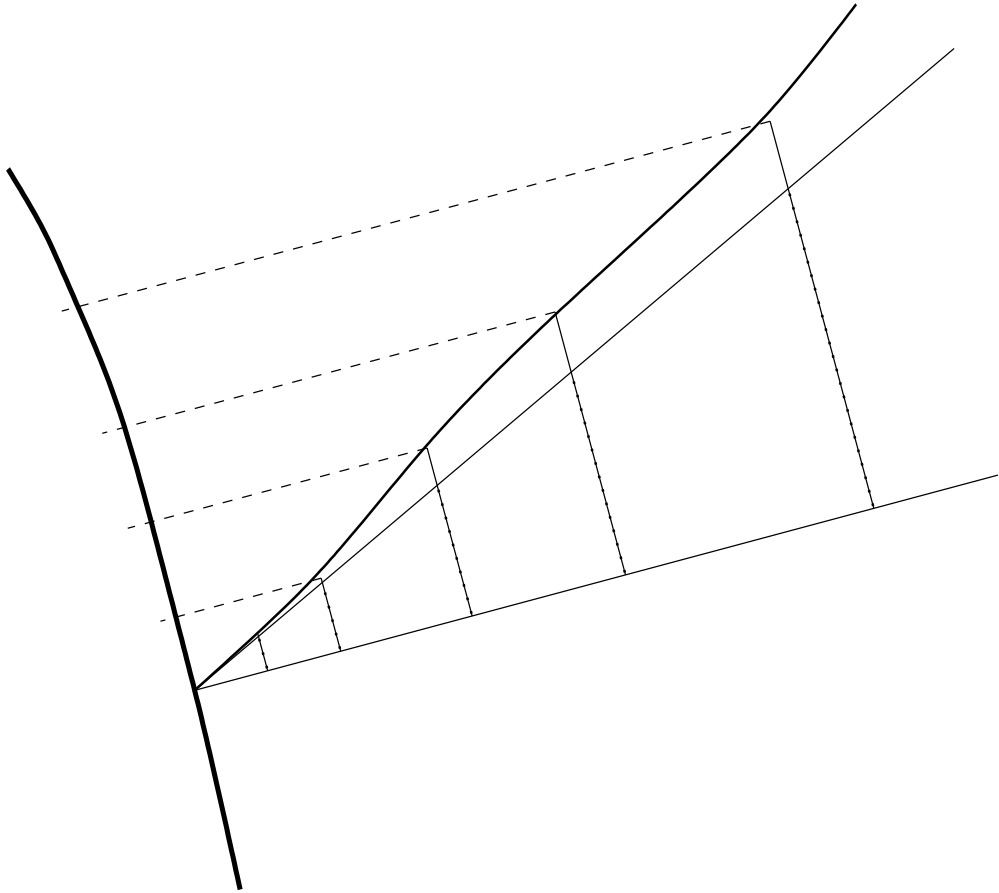


Figure 4: Tangent vector as a limit or "instantaneous velocity"

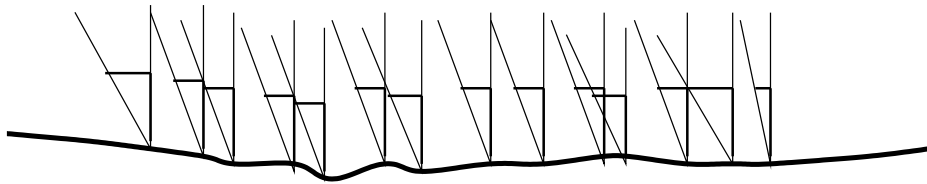


Figure 5: Summation (integration) across a section of the tangent bundle. Should a length scale be defined at every point?

For each particle, a mass relationship can be imposed asking to the second derivatives to give the same result as a iteration of the first derivatives. So the two vectors of a curvature would be given by the positions of the extreme points of the vector giving the first derivative.

Such relationship could be not needed if some geometrical consistency conditions where imposed to the mass matrix. For instance, it is know that Poincare Duality forces the Connes-Lott model generations to be degenerated in mass. More restringent conditions could appear.

Finally, the mass relation between different particles is the touchiest point. My bet is to link it to a preferred kind of metrics, with a preferred set of coordinate systems. Very much as it happens in a spherical set of coordinates: a variation across δr has no additional weight, variations across $\delta\theta, \delta\phi$ carry an additional weight r , and variation across $\delta\phi$ carries an additional term respect to $\delta\theta$. In [16], this was stated with an obscure comparison between quarks and angles.

5 Acknowledgements

To gauge the ambiguity is a suggestion of E. Follana, yet to explore. Cheerfully acknowledged, as well as a lot of support to discuss other ideas. Momentum space was seen as configuration space with a field of forces in some coffee talks with J. I. Martinez, according my old notebooks. Again, yet to explore. And, as noted elsewhere, the main thesis of this paper surfaced while a bedroom talk with J. Guerrero at Vietri, where I was driven to think in analogies between the four fundamental fermions and a four dimensional volume form. More about this can be forthcoming in [18], where we wonder about the relation between junk removal and the Pauli antisymmetrization of N fermions.

The relaxing ambiance provided by the folks of *La Latina*, the Madrid inner neighborhood where I am doing a half sabbatical, should also be acknowledged.

References

- [1] A. P. Balachandran et al., *The Fermion Doubling Problem and Noncommutative Geometry*, hep-th/9911087
- [2] Chao-Shang Huang et al., *The $B \rightarrow X_s l^+ l^-$ and $B \rightarrow X_s \gamma$ decays with the fourth generation*, hep-ph/9911203
- [3] JF Cariñena et al., *Connes' Tangent Groupoid and Deformation Quantization*, J. of Geom. and Phys, v 32 (1999). math/9802102
- [4] A. Connes, *Non Commutative Geometry*, Academic Press 1994
- [5] A. Connes, *Non Commutative Geometry and Reality*
- [6] A. Chamseddine, A. Connes, *A Universal Action Formula*, preprint hep-th/9606056
- [7] A. Connes, *Gravity coupled with matter and the foundation of non commutative geometry*, preprint hep-th/9603053

- [8] J.S. Dowker, Path Integrals and Ordering Rules, *J. Math. Phys.* **17** (1976)
- [9] , T. Fujikawa et al., *Non-commutative Differential Calculus and the Axial Anomaly in Abelian Lattice Gauge Theories*, hep-lat/9906015
- [10] M. Gökeler and T. Shüker, *Does noncommutative geometry encompass lattice gauge theory*, hep-th/9805077
- [11] Roy R. Gould, *Am. J. Phys.*, **63**, n. 2 (1995)
- [12] M. Lüscher, *Chiral gauge theories on the lattice with exact gauge invariance*, hep-lat/9909150
- [13] S. Majid, *Advances in Quantum and Braided Geometry*, q-alg/9610003 v2
- [14] A. Rivero, *Introduction to the tangent grupoid*, dg-ga/9710026.
- [15] A. Rivero, *A short derivation of Feynman formula*, quant-ph/9803035
- [16] A. Rivero, *Some conjectures looking of a NCG theory*, hep-th/9804169
- [17] A. Rivero, *On generations*, hep-th/9905021
- [18] A. Rivero, *Junk: the fermionic ansatz*, work in course.
- [19] M. Santander, *Interpretacion geometrica de la gravitacion*, DFTUZ/93/11