Standard Model Masses and Models of Nuclei

Alejandro Rivero*

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Abstract

We note an intriguing coincidence in nuclear levels, that the subshells responsible for doubly magic numbers happen to bracket nuclei at the energies of the Standard Model bosons. This could show that these bosons actually contribute to the effective mesons of nuclear models.

Dear Grubendol,

I want to report you and the fellows of your society about some strange coincidences I have found in nuclei tables. They are related to the availability of virtual J=0 particles from the Standard Model.

Let me to organise the letter in small sections, so I will first comment on global issues, then to look into the coincidences in detail, and finally to do some remarks about their possible significance.

1 Introduction

It is known that magic numbers in nuclei are generated by separating levels via spin-orbit coupling. Numbers 50, 82 and 126 are got because the respective subshells 5g9/2, 6h11/2 and 7i13/2 get large gaps and fall into the lower shell.

Traditionally this was done by a purely phenomenological Hamiltonian, consisting of an empirical spin-orbit coupling and a Nilsson term $+K_n J^2$ depending of the shell. Modernly a relativistic approach based in meson exchange lets one to derive the spin orbit term from first principles. Still, the models are restricted to low nucleon numbers, because of computational effort. Some high N,Z models have been essayed, but they fail to reproduce the phenomena of double magicity.

Here we report on some empirical coincidences which seem to imply a need to correct meson exchange by adding the whole set of standard model interactions.

2 Global view

There are four highly massive particles in the standard model: the vector bosons W,Z, the scalar Higgs boson, and the Top quark. This one can not appear free but in composites, the simplest being the bosons $(t\bar{u}), (t\bar{d})$, etc, all of them having a mass near to the Top mass. Thus we will refer to all these particles as "the standard model bosons".

In figure 1 we present the traditional plot of energy per nucleon for all the nuclei in the Audi-Wapstra experimental tables. Taking note that 1 uma = 0.9xx GeV, we have drawn over the plot four lines indicating the respective masses of W,Z,H (conjectured by LEP-2) and Top.

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*Zaragoza University at Teruel. 44003 Teruel (Spain). `arivero@unizar.es`

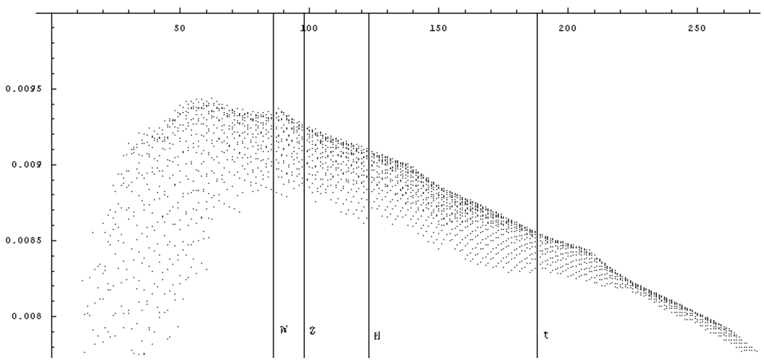

Figure 1: Energy per nucleon as usual. Lines are the SM masses

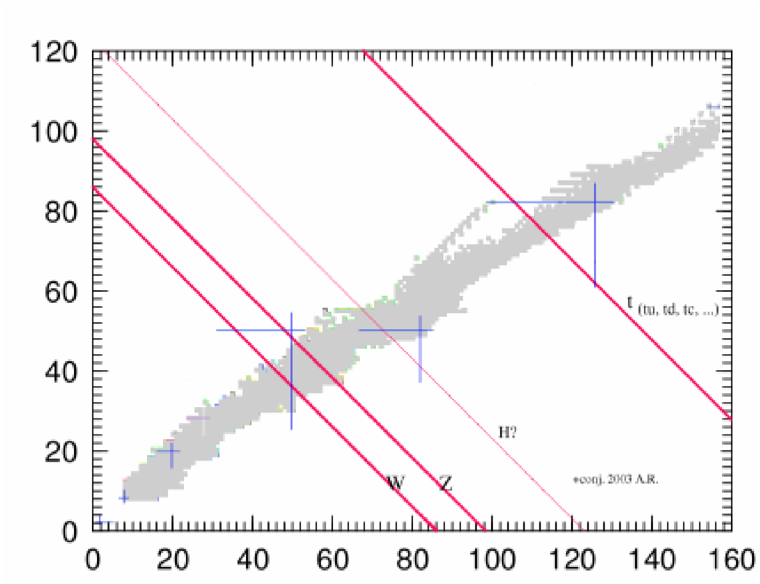

Figure 2: Blue marks the magic numbers. Red isobars correspond to the masses of W,Z, H (conjectured [3]) and top)

We note that boson masses happen slightly before of the peaks. Now, the peaks in this figure are mostly due to the neutron magic numbers, but if one remembers that inside each neutron line 50, 82, 126 there is a doubly magic number, then it is more sensible to examine the proximity of the SM masses to the doubly magic numbers. This is done in figure 2, by using the traditional N-P plot of nuclides. Here we plot diagonal isobaric lines for each massive particle, and we see that effectively the lines happen near the crosses corresponding to doubly magic numbers.

In order to determine how near is "near", we need to add some information to the plot. Namely, we will consider the subshell structure before each double magic number.

We take 1 atomic mass unit (amu) = 0.931494 GeV. Errors in masses can be checked for instance in [9]. They are negligible for the main arguments here.

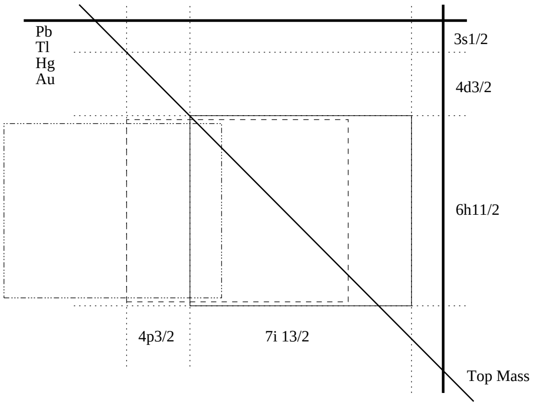

Figure 3: Diagonal isobar signals the mass of the top quark. Levels and solid rectangle according scheme from [6]. Other possibilities are drawn as dashed rectangles

3 P=82,N=126

The mass of the top quark was measured by the Tevatron in 1994, and it amounts, by direct observation, to 174.3 GeV, about 187.1 amu.

The proton magic number 82 is due to the 6h11/2 subshell, which actually is under two other levels, the 3s1/2 and the 4d3/2. The neutron magic number 126 is due to subshell 7i13/2. If we use Klinkenberg 1952 filling scheme [6] to draw the rectangle of nuclei with partial fillings of (6h11/2,7i13/2), the diagonal approaches closely to the t quark isobaric line. The situation is shown in figure 3.

We should alert the reader that the filling scheme is not well defined because the energy difference between odd and even number of nucleons is enough to alter the energy levels. Thus if we get the shell scheme for 208Pb from Bohr-Mottelson 1969[1], then the levels 5f5/2 and 4p3/2 seem to raise above the 7i13/2, and then the isobar line just cuts a corner of the rectangle. This kind of complications are usual and we will find the same problem in the N=82 range.

One should consider also that a 5 GeV range is available starting from the top mass, if we want to consider here all the "mesons" from $t\bar{d}$ to $t\bar{b}$. Of course, the highly unstable $t\bar{t}$ "meson" has a mass far away from the nuclear data.

4 P=50,N=50

Here we still have the same subshell for protons and neutrons, namely the 5g9/2. Thus the relevant nuclei are in a square.

The masses of W and Z were measured by CERN collaborations in 1982-3. They amount to 80.423 and 91.1876 GeV, resp. 86.3 nd 97.9 amu.

While both masses are inside the square bracketed by the subshells, some models could also be interested in its average, 92.1 uma, which roughly closes the diagonal of the square.

If one draws the mass lines over the plot of errors in the microscopic-macroscopic mass formula FRDM-1992[8], it can be noticed that outside from the square the lines coincide with an huge error area in the model. In figure 4 we have included this data.

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[1] indirectly quoted in [4] this year 2003, so it should be still considered state-of-the-art!

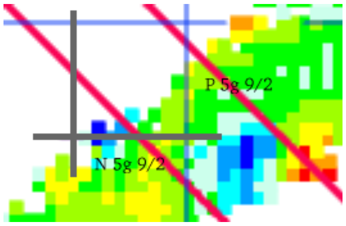


Figure 4: Blue lines mark magic shell closures at N=50, P=50. Black lines mark start of the respective shell. The background shows FDRM(1992) error [8] in this area of the nuclide table

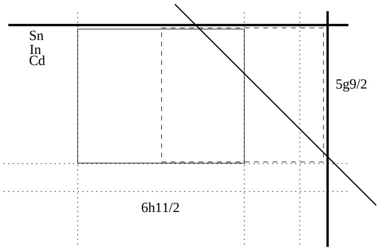


Figure 5: Diagonal isobar marks the conjectured mass of the SM Higgs scalar. solid rectangle takes subshell filling order from [6], dashed rectangle takes subshell ordering from HFB models,

5 P=50,N=82

Even without the higgs, the two previous coincidences should be enough to consider an extension of nuclear models. Now, a Higgs-like event was reported by LEP-2 collaborations in the year 200x , with a mass of 115 GeV, roughly 123.46 uma. We can plot it and see how near it is of the extant doubly magic number.

The magic number P=50 is due to the subshell 5g9/2 as in the previous case. The magic number N=82 is due to subshell 6h11/2. This should do a 10×12 rectangle.

An unexpected complication is the overlapping of neutron subshells 6h11/2, 3s1/2 and 4d3/2. Single nucleons prefer one subshell, while paired nucleons prefer another. It is difficult to establish the right order, if it exists. Even when the 6h11/2 subshell is supposed to be under the other two (or three!), the first excited level is always provided by it.

If we ignore the overlap and we we accept the old criteria from Klinkenberg, putting the 6h subshell as the lower one, then the LEP-2 event simply cuts a corner of the rectangle. The same discrepancy, at a smaller scale, happens if we take Bohr-Mattelson. And if we draw a whole elongated rectangle accounting for all the possible occupations, then it is not so clear where should we draw the diagonal. In any case, we have reflected all these possibilities in figure 5.

On other hand, we should point out that all the modern models based on HFB mass formulae fail to reproduce the narrow overlap and the priority of 4d3/2, and they assign the highest energy level to the 6h11/2 subshell, which then closes the whole shell (check the plots and data in [4]). This should be the best approach, as then the 10×12 rectangle is directly attached to the doubly magic corner.

6 Analysis and Remarks

While it is possible to believe that this effect is just a random numerological coincidence, it covers all the cases where the naive shell+spin-orbit does not suffice to justify the observed magicity, and specially it suggest that a motivation can exist for the effect of double magicity. It could be argued that the 28 subshell is not included in the list, but this shell needs only a mid spin-orbit splitting, at the reach of classical models.

Our empirical approach is partly mudded because of the proximity between different subshell splittings. We refer the reader again to [6] to get an idea of the complexity of these determinations. On the other hand, it could be that all the levels are actually trying to compete to fit in the isobaric lines.

The enhancement happens in the depressed subshells, which agrees with the phenomenological approach that enhances the spin-orbit separation by incorporating a dependence on angular momenta.

We can not, at this moment, provide an explanation of the effect. The simultaneous fit of both proton and neutron subshells suggest that the effect includes an isospin symmetry.

If the LEP-2 event is confirmed to be the Higgs scalar particle, then we should look for couplings to J=0 top-mesons and for similar combinations of the W,Z bosons.

It should be remembered that $J = 0$ coupling is the trademark of the scalar σ-meson in relativistic models of the nucleus.

The status of the σ meson is experimentally troubled. It is usually assimilated to a $f_0(600)$ boson (see [9, pg 450]) by the HEP community, but nuclear authors prefer to remark that "there is no experimental evidence for a free σ meson, although the σ field is a crucial ingredient of relativistic mean-field models" [1]. The particles from the Standard Model could interfere with the effect of the σ. It could happen either as an small modification of the $1/m_\sigma$ mass term, or as a direct interaction with the nucleon density.

It seems that the meson exchange interaction is enhanced when the mass of the whole ensemble $N + Z$ of nucleons equals the mass of some available SM boson. This could be more explicit in the second case.

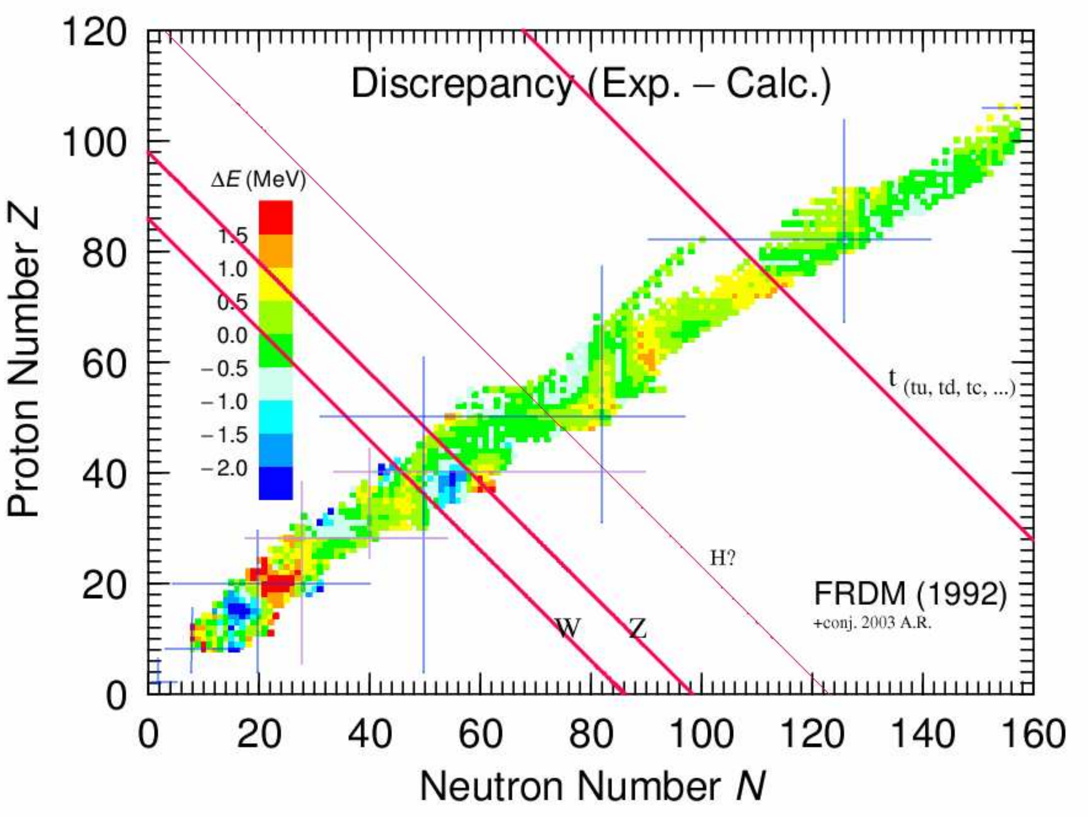
Also a effect via three-body interactions [5] could be searched.

Well, it has been long time since Fermi remarked to Maria if "Is there any indication of spin-orbit coupling". Perhaps now we have a definitive clue to the particles originating it.

Yours,
Alejandro Rivero
Teruel, November 2003.

7 Appendix

The following figure is an ampliation of figure 2 with the whole droplet discrepancies over-imposed. It is taken from [8] and it is provided here for editorial reference only, not for publication.



References

[1] M. Bender, P-H. Heenen, P-G. Reinhard *Self-consistent mean-field models for nuclear structure* Rev. Mod. Phys. 75, 121 (2003) http://link.aps.org/abstract/RMP/v75/p121

[2] B. L. Cohen *Observations on the Energies of Single-Particle Neutron States* Phys. Rev. 130, 227-233 (1963) http://link.aps.org/abstract/PR/v130/p227

[3] J. R. Ellis *The 115 GeV Higgs Odyssey* http://arxiv.org/abs/hep-ex/0011086

[4] S. Goriely et al., *Further explorations...*, Phys. Rev. C 68, 054325 (2003).

[5] N. Kaiser *Three-body spin-orbit forces from chiral two-pion exchange* Phys. Rev. C 68, 054001 (2003) http://link.aps.org/abstract/PRC/v68/e054001

[6] P. F. A. Klinkenberg *Tables of Nuclear Shell Structure* Rev. Mod. Phys. 24, 63-73 (1952) http://link.aps.org/abstract/RMP/v24/p63

[7] D. Lunney, J. M. Pearson, C. Thibault *Recent trends in the determination of nuclear masses* Rev. Mod. Phys. 75, 1021 (2003) http://link.aps.org/abstract/RMP/v75/p1021

[8] *P. Mller, J. R. Nix, W. D. Myers, W. J. Swiatecki* Nuclear Ground-State Masses and Deformations Atom.Data Nucl.Data Tabl. 59, 185-381 (1995) http://arxiv.org/abs/nucl-th/9308022

[9] Particle Data Group, *Review of Particle Physics*, Phys. Rev. D 66 (1-I), p. 1 (2002) http://pdg.lbl.org/