Distance to the drip lines

Alejandro Rivero *

July 30, 2004

Abstract

It can be found that with the adequate measure, the beta stability line is equidistant from neutron and proton drip lines. We explore this fact and its predictive potentiality in the simplest case, the classic liquid drop formula.

For a nucleus of atomic number $A = N + Z$ in the beta stability line, we can consider the corresponding nuclei $(Z - k, N)$ in the proton drip line and $(Z, N - k')$ in the neutron drip line, with respective masses A_p, A_n. The main mass models in the market (eg, from [2]) predict a very small difference $A_p - A_n$, which even becomes zero in isolated points under the action of microscopical corrections.

We have studied this difference for the classical Weizsäcker formula[1],

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A}$$

An analytical -even if very large- expression can be given if instead of taking A as the independent variable, we fix the mass A_0 in the drip lines. Then solving the second degree equation in the proton drip line

$$M[Z, A_0] - M[Z-1, A_0 - 1] - m_z = 0$$

and the third degree one in the stability line (we take $m_p - m_n \sim 0$ but it is not necessary)

$$Z = \frac{2a_4 A}{a_3 A^{2/3} + 4a_4}$$

we can get the corresponding mass A and proton and neutron numbers $Z, N(= A - Z)$ of the stable nucleus. We compare this neutron number with the one got from the neutron drip line equation

$$M[A_0 - N_0, A_0] - M[A - N_0, A_0 - 1] - m_n = 0.$$

The difference $d(A_0) = N - N_0$ results a very convenient function to input in a numerical-analytical program such as, for instance, *Mathematica*, because we

*EUPT (Universidad de Zaragoza), 44003 Teruel, Spain arivero@unizar.es

can plot dependences with any of the four free parameters of the model, as well as mixed plots $d(A_0, a_i)$ or $d(a_i, a_j)$. It is specially relevant to check the dependence in a_3, because it has a natural minimum for the zero value, but it is not uniformly increasing; there is a second minimum in the ~ 1 MeV area, but this one has also a dependence on A_0 so we can no expect it to coincide exactly with the usual value $a_3 = 0.711$ MeV. Still, this minimum can be interpreted as the cause of our equidistancy.

For the usual values

$$a_1 = 15.75\text{MeV}, a_2 = 17.8\text{MeV}, a_3 = 0.711\text{MeV}, a_4 = 23.0\text{MeV},$$

it can be seen that $d(A_0)$ gets the higher value for $A_0 \sim 300$; it is only -0.815 when proton and neutron masses are equal, and this maximum discrepancy moves by only about two units when proton and neutron masses are given different value, so for simplicity one can find convenient to keep with $m_p - m_n \sim 0$ as we do here.

The function is not linear, so for mid-range masses the difference is appreciably smaller. Generically we can say that the equidistance property $k = k'$ with different proton and neutrons masses holds within a 2%.

As a numerical experiment, and to test how strong a rule is the equidistance to the driplines, we measured the square discrepancy at four different values, using an averaging function

$$\text{mean} = d(50)^2 + d(100)^2 + d(150)^2 + d(200)^2,$$

and we took the bold bet of searching for minima[1] in the allowed parameter space of $a_1 ... a_4$.

After discarding unphysical or non-convergent zones, we explored a detailed area for $a_1 = 6..18, a_2 = 0..40, a_3 = 0.3..1.2, a_4 = 5..45$. We seed the algorithm with values in this area and then we ask it to descend numerically into a local minimum. Note that smaller values of a_3 cause the algorithm to descent into the trivial $a_3 = 0$ case. The results are seem in the table. The stability line depends only of a_3/a_4, so it is a good indication to get both increasing, even if below the empirical value. It can be said that we are always out, in the four parameters, by a sensible percentage, but if one takes into account that no empirical input is used in the calculation[2], the exploration is successful enough. Besides, we have chosen the averaging function in a very arbitrary way. This choosing is surely more important than the seed. For the usual values, $d(A_0)$ has three zeros besides the trivial, about $A = 25, 600, 2000$. This is very generic, and it means that up to three discrete values can be adjusted in an unnatural manner. The averaging formula must evaluate a number of points enough to be sure that no zeros happen in the range of validity of the mass formula, say $A = 30...300$. From the shape of $d(A_0)$ in this range it seems that a weighting proportional to the binding energy of A_0 (say, in the stability line) could be adequate, but it

[1] Mathematica 5.x users beware: the default method in `FindMinimum` notices the sum of squares and it fails; you must use any of the older methods.

[2] except, if you want, the value of the atomic mass unit

is still an ad-hoc suggestion, as any function with the same shape would do the same role.

Experiments can be done with variations in only three, two or one parameters, but then the extant ones must be fixed with other methods, beyond the purpose of this brief note. Contrary to the others, the parameter a_4 seems not be very able to vary, as it stops about two or three units from the seed. But fixing it to 23.0 does not improve the match.

As we have said, the equidistance property can be noticed in most models of nuclear masses, and our function $d(A_0)$, or alternatively any measure of the discrepancy between k and k', is an interesting parameter to consider when studying the properties of a mass model.

An explanation of this property should be that really some important mass dependent effect is enhanced at the drip lines, so this effect forces mass formulae to adjust themselves to fit. The effect does not need to happen uniformly in all the mass range, it is enough to force the coincidence in isolated points, perhaps the ones having strong microscopical corrections, as noted above.

Perhaps it is related to the extra nuclear stability coming from magicity, because an additional phenomena appears when we consider magic numbers at the drip lines: we can use nuclear mass to pair Z and N numbers, in such way that a nucleus in the proton drip line given a magic (or semimagic) Z number correspond to another nucleus of the same mass in the proton neutron drip line, having again a magic (or semimagic) N number:

At neutron dripline, N=	28-30	50	(64)	82	126	184
At proton dripline, Z=	28	(40)	50	(58)	82	114

mean sqr discrepancy	a_1	a_2	a_3	a_4
0.00329185	11.1382	18.2911	0.521987	18.9961
0.00450291	11.0895	19.869	0.556106	17.1047
0.0173419	13.9102	20.5569	1.62424	13.6503
0.0158552	14.8943	19.1126	2.07992	14.2993
0.0144648	15.4414	18.2103	2.34985	14.7491
0.00267294	12.2724	18.3904	0.568557	22.0129
0.0021288	13.5535	18.1768	0.606681	26.1328
0.00175986	15.0299	17.7684	0.668602	30.3896
0.00149186	16.6956	17.0859	0.748912	34.9357
0.00108511	18.1636	12.6318	0.812369	41.3469
0.00110731	18.2587	12.8536	0.823127	41.2093
0.00113735	18.1823	13.6355	0.811909	40.8982
0.00212587...(*)	14.2744	16.8097	0.70215	26.5332
0.00214133...(*)	13.7113	18.2199	0.628928	25.981

Table 1: Some typical predictions from Mathematica' `FindMinimum` explorations of the allowed parameter range for the liquid drop model. To be compared with the usual $a_1 = 15.75, a_2 = 17.8, a_3 = 0.711, a_4 = 23.0$. Lines marked with (*) took 15.75 and 17.8 as seed

By the way: the author was too worried about other phenomena and he thought the fact here reported were well known. So I must thank M. Asorey by asking why the drip lines I kept drawing were always equidistant of the stability, and A. Zuker by suggesting to look first in the simplest model, Weizsaecker's liquid drop.

References

[1] H. A. Bethe and R. F. Bacher *Nuclear Physics A. Stationary States of Nuclei* Rev. Mod. Phys. 8, 82-229 (1936)

`http://link.aps.org/abstract/RMP/v8/p82`

[2] M. Uno, *Current status of nuclear mass formulae and their predictability* RIKEN review, 26 (2000), 38-44

`http://www.riken.go.jp/lab-www/library/publication/review/conts/conts26.html`