Mass terms from Casimir Invariants

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Abstract. A general strategy to obtain mass terms for (super)symmetry breaking is presented, and applied to get mass relationships in the electroweak sector.

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Under Poincare symmetry, suppose we have a family of particles (m_i, s_i) labeled using the two Casimirs of the group, $\mathcal{C}_1, \mathcal{C}_2$ with respective eigenvalues $c_1 = m^2$, $c_2 = -m^2 s(s+1)$.

We ask for constructions of operators M_s^2 with dimension $[mass]^2$ built exclusively from combinations of this casimirs (excluding inversion) and with the additional asymptotic condition

$$\lim_{s\to\infty} m_s^2 = m \tag{1}$$

of recovering the original mass eigenvalue in the high spin limit. This condition allows for preservation of the string tension (from the asymptotic Regge trajectory) if for instance our spectrum of particles comes from a string theory.

The simplest combination $\alpha\mathcal{C}_1 + \beta\mathcal{C}_2$ of the Casimirs has the adequate dimensions but fails to meet the asymptotic condition. The next simplest try, and the simplest one fulfilling our condition, is got from square roots of the quartic combination. This is, from the solution of the equation

$$M_s^4 - M_s^2\mathcal{C}_2 + \mathcal{C}_1\mathcal{C}_2 = 0 \tag{2}$$

And if we want to dispose of square roots we must rewrite it in terms of Pauli Matrices

$$M_s^2 = \sigma^+ \otimes \mathcal{C}_1\mathcal{C}_2 + \sigma^- \otimes \mathbf{I} + \frac{\mathbf{I} - \sigma_z}{2} \otimes \mathcal{C}_2 \tag{3}$$

Note that this operator can be also got from conditions different to (1). An interesting alternative could be to ask

$$\mathrm{Tr}\, M_s^2 = \mathrm{Tr}\,\mathcal{C}_2 \tag{4}$$

The goal of this note to point out that our method seems to have a role in electroweak breaking. Meeting with the same equation in a relativistic mechanics context, Hans de Vries discovered [1] that the positive eigenvalues of this operator for s=1/2 and s=1 let one to build the quantity

$$s_{dV}^2 \equiv 1 - \frac{m_{s=1/2,+}^2}{m_{s=1,+}^2} = 0.22310132... \tag{5}$$

unexpectedly near of the mass shell Weinberg angle [2, 3]

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2} = 0.22306 \pm 0.00033 \tag{6}$$

In fact the quotient between de Vries and Weinberg angles is $s_{dV}^2/s_W^2 = 0.9998 \pm .0015$ even too good for a tree level prediction, and we should expect it to survive to further experimental updates.

With this ansatz, we can insert the measured $M_Z^2 = (91.1874 GeV)^2$ as input for the eigenvalue $m_{1,+}^2$ and get the other three eigenvalues:

$$m_{s=1/2,+}^2 = (80.3717 GeV)^2 \tag{7}$$

$$m_{s=1,-}^2 = -(176.154 GeV)^2 \tag{8}$$

$$m_{s=1/2,-}^2 = -(122.384 GeV)^2 \tag{9}$$

This last negative value is not used in electroweak models, but we find that the negative eigenvalue $m_{1,-}^2$ is actually in the expected range for the negative mass square operator we use to break the electroweak symmetry. Remember that

$$\frac{< v >}{\sqrt{2}} = \sqrt{\frac{-m_h^2}{\lambda_h}} = 174.1042\ (\pm 0.00075)\ GeV \tag{10}$$

The experimental value coming from Fermi constant [2]. So, we are compatible with $\lambda_h \approx 1$. In fact we could fix it equal to 1 and pivot on the standard model to get a tree level estimate of the fine structure constant, getting $\alpha^{-1} = 135.28\ldots$

It is mysterious why so easily two predictions are got. If we add the actual measurement[4] of the top yukawa coupling, $\lambda_t = 0.991 \pm 0.013$ to our basket and we take it as hint for a technicolor/topcolor mechanism, then one could suspect that techniforces has also stringy properties –not surprisingly– and that its associated string carries somehow a supersymmetry –surprisingly, but a good excuse for M_W to come packed in a $s = 1/2$ object.

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[3] Erler J "Precision Electroweak Physics" *Preprint* hep-ph/0604035
[4] vv. aa. "Combination of CDF and D0 Results on the Mass of the Top Quark" *Preprint* hep-ex/0603039