# The $\sin\theta_W$ found in a 1924 timecapsule

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We comment on the derivation of a quantity similar, at experimental 0.13 $\sigma$ level, to the measured mass-shell Weinberg's angle.

De Broglie's relativistic quantum orbit rule [1]

$$\frac{m_0\beta^2c^2}{\sqrt{1-\beta^2}}T_r = nh \tag{1}$$

was proposed about the same time that Landé-Pauli substitution rule for 3D angular momentum[2, 3],

$$\frac{1}{j^2} \to -\frac{d}{dj}(\frac{1}{j}) \to \frac{1}{j} - \frac{1}{j+1} \to \frac{1}{j(j+1)} \tag{2}$$

but the fast pace of the events in the mid-twenties did not allow for a fusion of both ideas; almost immediately (2) was rigorised in the Heisenberg-Born matrix mechanics – even allowing for half-integer j –, while De Broglie's suggestions for wave mechanics were absorbed into Schrödinger's analytic methodology.

In November of 2004, eighty years later, during an empirical study of gyromagnetic ratios [8], Hans de Vries suggested to combine (2) and (1) with the extra requirement

$$T_r = \frac{h}{m_0c^2} \tag{3}$$

on the orbital period, so that rest mass and Planck constant are canceled out and we are left with a relationship between relativistic speed and angular momentum:

$$\frac{\beta^2}{\sqrt{1-\beta^2}} = \sqrt{j(j+1)} \tag{4}$$

Solving $\beta$ for the $j = 1/2$, $j = 1$, and via the ratio of speeds, de Vries produced the following adimensional quantity

$$s^2_{dV} \equiv 1 - \left(\frac{\beta_{1/2}}{\beta_1}\right)^2 = 0.22310132... \tag{5}$$

which remembers closely to the mass-based experimental Weinberg's sine.

At the time of calculation the data on $W^+$ mass and the global fits to standard model parameters were putting de Vries' sine at more than $1\sigma$ deviation from the measured value. So the result was put aside as one-line footnote in the preprint report. But the new data released from LEP II during 2005 and the fits from the particle data group have moved the experimental value to be [6, 7]

$$s^2_W = 0.22306 \pm 0.00033 \tag{6}$$

so that $s^2_{HdV}$ is now inside the experimental error, centered at $0.13\sigma$. If you prefer, lets say that the quotient $s^2_{W,exp}/s^2_{dV}$ between experimental and theoretical quantities is now $0.9998 \pm .0015$.

While the experimental error is still too big, the centrality of the calculated result seems to grant that the agreement will continue under further experimental improvements. In any case lets keep in mind that this theoretical number comes from plain relativistic quantum mechanics, thus from the point the view of QFT it is a tree level statement and we should do not expect to push it beyond 0.1% level; in fact it should be surprising if the experimental error decreases but the central value keeps fixed, because in such case a 0.01% agreement level would be reached.

De Vries reasonment started from orbital radius instead of orbital period. Indeed one can use the condition (3) to get an orbital radius

$$r = \beta\ c\ T_r = \beta\frac{h}{mc} = \frac{h}{c}\frac{\beta}{m} \tag{7}$$

proportional to Compton length and thus inverse proportional to the orbiting mass.

Thus we can do the additional remark that if a particle of mass $\propto M_{W^\pm}$ orbits according (3) producing j=1/2 according (2)(1), then a particle of mass $\propto M_{Z^0}$ orbiting at the same radius under the same conditions will produce j=1. This can be formulated producing an extra mass scale $M$ from the fixed radius, getting

$$M_s^2 = \frac{1}{2}(-M^2S^2 + \sqrt{(M^2S^2)^2 + 4M^2(M^2S^2)}) \tag{8}$$

(here we have used operator notation so that the two Casimir invariants of Poincare group are almost explicit)

Independently of this remark, we think that model builders can find useful this result. The electroweak scale can be defined as the point at which the renormalised Weinberg's angle, running down from its GUT-theoretical value, reaches the value of de Vries's angle. Besides, de Vries number comes from a pair of well calculated adimensional numbers,

$$\beta_{1/2} = \sqrt{\tfrac{3}{8}(\sqrt{19/3}-1)} = 0.7541414352817\ldots \tag{9}$$

$$\beta_1 = \sqrt{\sqrt{3}-1} = 0.855599677167\ldots \tag{10}$$

so it contains slightly more information. It could be used for instance to pinpoint mass values at $\beta_{1/2}M_Z \propto$

$\beta_1 M_W \propto 68.76\ GeV$ or $M_W/\beta_{1/2} \propto M_Z/\beta_1 \propto 106.5\ GeV$. Also, the attempt of providing physical meaning to the quotient of speeds (or, via an arbitrary potential, of binding radius) seems to underline composite, top-condensation like, models of the Higgs sector, but we do not put forward a definitive statement on this.

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[1] L. de Broglie, *Ondes et Quanta*, Note presentee par M. Jean Perrin. Seance du 10 Septembre 1923, Academie des Sciencies, pages 507 to 510.

[2] W. Pauli, *Zur Frage der Zuordnung...* Z Phys 20 p 371. See also Z. Phys 31 p 765. See also [4, ch VIII, near p. 193]

[3] A. Lande, Z. Phys 15 p 189-205 and Z. Phys 19 p 112-123. See also [4]

[4] O. Darrigol, *From c-Numbers to q-Numbers*, University of California Press, 1992. http://ark.cdlib.org/ark:/13030/ft4t1nb2gv/

[5] H. de Vries and A. Rivero, preprint hep-ph/0503104

[6] Particle Data Group, *Review of Particle Properties 2006*, to be published

[7] J. Erler, *Precision Electroweak Physics*, preprint hep-ph/0604035

[8] See [5]; from observations on $(g_e - 2)$ and $(g_\mu - g_e)$