Evidence for radiative generation of lepton masses

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Abstract

We present a fit to the experimental charged lepton masses as coming from radiative corrections in QED.

1 Introduction

In November of the last year, one of us (HdV) noticed that the values of $a_e \equiv (g_e - 2)/2$, the anomalous moment of electron, and of the difference $a_\mu - a_e$ with the one of the muon, were amazingly close to the mass quotients m_μ / m_Z and m_e / m_W. This happened during an on-going internet quest for accurate empirical relationships between fundamental constants, but we felt that the accuracy of this particular case deserved further investigation:

0.00115869 = muon / Z mass ratio
0.00115965 = electron magnetic anomaly
0.00000635 = electron / W mass ratio
0.00000626 = difference of muon and electron magnetic anomaly

table 1.

Of course the calculation of $(g-2)/2$ involves the very well known series on the electromagnetic couping α. A coincidence with simple combinations of lepton masses can be explained if such masses come themselves from expressions containing α. Then it strongly suggests that such masses are generated radiatively in such way that at low order both perturbative series can be related. It has been observed from time ago [1] that lepton masses have quotients of order α, and a whole industry of model-making starts from trying to fit it [1, 4], getting the masses as radiative series on α. But until now, no new evidence had been observed for this kind of schemes

2 Self Energy and Vacuum Polarization

As we expect only a parallel between structures, we can do the ansatz of comparing the first quotient exclusively to self-energy graphs, and to ascribe all

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of the vacuum polarisation (v.p.) contribution to the second. This ansatz was guided by Hans' observation of the similarity between the quotient m_W/m_Z and the ratio of semiclassical[1] velocities $\beta_1/\beta_{\frac{1}{2}}$. In any case, it amounts to excluding the electron vacuum polarisation loop in the α^2 order and, because precision requires it, the v.p. and light by light diagrams in the third order. These are, respectively [6]

$$a_e^{vp} = (\frac{119}{36} - \frac{1}{3}\pi^2)(\frac{\alpha}{\pi})^2 - 0.099(\frac{\alpha}{\pi})^3 + 0.37(\frac{\alpha}{\pi})^3 = 88.0\ 10^{-9} \tag{1}$$

Our table becomes

0.001 158 692 3 = muon / Z mass ratio
0.001 159 564 2 = exp. a_e electron magnetic anomaly - a_e^{vp}
0.000 000 871 9 = difference

0.001 165 046 0 = muon / Z mass ratio + electron / W mass ratio
0.001 165 920 8 = exp muon magnetic anomaly a_μ
0.000 000 874 8 = difference

0.000 006 353 7 = electron / W mass ratio
0.000 006 356 7 = exp. $a_\mu -$ exp. $a_e + a_e^{vp})$
0.000 000 003 0 = difference

table 2.

The uncertainty due to W mass is $2.99 \cdot 10^{-9}$. Actually, the third loop perturbation, above incorporated, has a positive contribution $3.4 \cdot 10^{-9}$ against a perfect match. In any case, it is empirically very satisfying to find oneself inside the experimental error with only an ansatz on the diagrams. Still, the μ/Z ratio is accurate only up to $O(\frac{\alpha}{\pi})$, and it seems to ask for an additional $O((\frac{\alpha}{\pi})^2)$ term.

3 First QED approximation

So, lets try this ansatz in a pure calculational setup, without recurring to the experimental data, and lets see if -or how- the coincidence can be related to a parallell of mathematical structures. The QED calculation of a_e excluding vacuum polarisation is [6],

$$a_e^{QED-v.p} = \frac{1}{2}\frac{\alpha}{\pi} - 0.3441668(\frac{\alpha}{\pi})^2 + 0.943(\frac{\alpha}{\pi})^3 = 0.001\ 159\ 564\ 60 \tag{2}$$

while the whole QED result for a_μ is[3]

$$a_\mu^{QED} = 0.5\frac{\alpha}{\pi} + 0.765857388(\frac{\alpha}{\pi})^2 + 24.0505(\frac{\alpha}{\pi})^3 + 126.04(\frac{\alpha}{\pi})^4 = 0.001\ 165\ 847\ 00 \tag{3}$$

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[1] β_s is the velocity of a mass rotating on a orbit with angular momentum $\sqrt{s(s+1)}\hbar$ and a frequency corresponding to its rest mass. The quotient $\beta_{\frac{1}{2}}/\beta_1$ is about 0.8814

The difference being $a_\mu - a_e = 0.00000628240$ The coincidences are thus initially of 99.92% and 98.88% and by themselves they should constitute at least collateral evidence of radiative terms for most part of the m_e, m_μ. Note that by betting for a mathematical structure with leptons only, we have lost the hadronic contribution, of order $67 \cdot 10^{-9}$, so now we are too in need of a corrective term for the missing 01.12% if we want to increase the order of accuracy.

4 Additional Terms

Our first research must be how the fit to m_μ/m_Z can be improved by using additional terms. There is no very much playroom using only electroweak mass data, but a bit surprisingly, there are possibilities of improvement. Keeping with simple quotients of Z, it is possible to enter into the one-sigma experimental precision of Z, $26.68 \cdot 10^{-9}$, by using $(1/2\pi) m_e/m_Z$. If we are willing to admit more higher powers of mass quotients, a term $m_\mu^2/2m_W^2$ drives the estimate up to almost full coincidence with the central values. And if we do not like extra coefficients, we can instead use m_μ^2/m_X^2 for an undiscovered mass X of 114.5 GeV. Let us compare these possibilities:

$$a_e^{QED-v.p} = 0.00115956460 \tag{4}$$

$$\frac{m_\mu}{m_Z} + \frac{1}{2\pi}\frac{m_e}{m_Z} = 0.00115958417 \quad : -0.000\,000\,019\,57 \tag{5}$$

$$\frac{m_\mu}{m_Z} + \frac{m_\mu^2}{2m_W^2} = 0.00115955526 \quad : 0.000\,000\,009\,34 \tag{6}$$

$$\frac{m_\mu}{m_Z} + \frac{m_\mu^2}{m_X^2} = .00115954381 \quad : 0.000\,000\,020\,79 \tag{7}$$

$$\text{Z error} \quad : 0.000\,000\,026\,68 \tag{8}$$

The last column shows differences, to be compared with the uncertainness induced from the experimental measurement of Z_0.

The fit at X is appealing because Z is a neutral particle, and the experimental hint of CERN at this value was for the neutral scalar. While waiting for news in the experimental front, we can happily admit the correction of (6).

Another motivation to prefer quadratic correction terms is that we can use also a term in $m_e m_\tau$ to recover almost completely the precision we lost for the second quotient when we decided to do not include the hadronic (quark loop) contributions. We have

$$a_\mu^{QED} - a_e^{QED-v.p} = 0.00000628240 \tag{9}$$

$$\frac{m_e}{m_W} - \frac{m_e m_\tau}{2m_W^2} = 0.00000628354 \quad : 0.000\,000\,001\,14 \tag{10}$$

$$\frac{m_e}{m_W} - \frac{m_e m_\tau}{m_X^2} = 0.00000628447 \quad : 0.000\,000\,002\,07 \tag{11}$$

$$\text{W error} \quad : 0.000\,000\,002\,99 \tag{12}$$

Again, the last column shows differences, to be compared with the uncertainness induced from the experimental measurement of W^+. And besides the already mentioned hadronic contribution, $67 \cdot 10^{-9}$, we could consider also the pure electroweak contribution, $1.51 \cdot 10^{-9}$, to be added to a_μ. We mention it separately to show that we can not decide if we are comparing against the structure of a pure QED kind of series or against an electroweak series.

5 Remarks

Remark 1. It can be asked if there is a role for the tau anomalous moment in this scheme. It is a touchy issue, because while the tau lives at order α of the electroweak vacuum[2], it is also at the mass scale typical of SU(3) colour, while the next lepton, the muon, lives at the mass scale of the chiral breaking (whose goldstone boson is in some sense the pion). We can suspect things are not very clear cut in its radiative process, and in fact one could prefer to admit quarks in the calculation instead of using the correction of formula (10) above, and then to adjust the a_e term with formula (4).

As for the a_τ correction it refers, it is tempting to try to guess if a simple expression does it exist. This value is not known experimentally, but from Samuel et al [8], we know its calculated QED value, 0.0011732. If we ask for a simple quotient, we would again to use the electron mass over some particle X^+, which we could expect (but not necessarily) to be a charged one, to imitate the use of W. The total expression

$$\frac{m_\mu}{m_Z} + \frac{m_e}{m_W} + \frac{m_e}{m_{X^+}} + \frac{m_\mu^2 - m_e m_\tau}{m_X^2} \tag{13}$$

actually matches a_τ for a mass of X^+ about 68 GeV [3].

Remark 2. In principle, if all the three formulae above are taken seriously, a matching order-by-order in α could be done to estimate the corresponding coefficients of the radiative series for each lepton mass. But without further understanding of the role of the electroweak bosons, or of the full electroweak scale and the role of τ, such matching becomes merely a mathematical exercise.

Remark 2.5 Another consequence of taking seriously the quadratic formulae is that their simultaneous use gives an hyperbolic relationship between electroweak masses. It should be interesting if some family of GUT models were able to generate this kind of relationship:

$$\frac{m_\tau}{m_Z} + \frac{m_\mu}{m_W} = \frac{m_\tau}{m_\mu} a_\mu^{s.e.} + \frac{m_\mu}{m_e} a_\mu^{v.p.} \tag{14}$$

Note that $a_\mu^{v.p.}$, containing the vacuum polarisation (and light by light) terms, has also an internal dependence on the quotients m_e/m_μ, m_τ/m_μ.

Remark 3 As the pure self-energy contributions do not depend (in QED) of lepton mass, it is indifferent if we extract then from a_e or a_μ. Along this note we have kept with a_e due to historical reasons, but it results more symmetric to refer to $a_\mu^{QED,s.e.}$ and $a_\mu^{QED,v.p.}$, as we have done in formula (14) above.

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[2] As Jay R Yablon reminded us recently

[3] It is perhaps worth to note here that the existence of a charged scalar at this value was pursued [7] in the LEP, while the final evaluation reduced the value of the events down to a two sigma deviation. So in some sense this scalar has presently the same experimental status that the events at 115 GeV assigned to a neutral scalar.

References

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