Research Statement

A. Rivero, November 2006

1) Effective supersymmetry

A not usually advertised detail about the standard model is that the strong coupled sector is able to hold the number of degrees of freedom needed for supersymmetry almost exactly. Plainly, if we produce bosons by pairing the strongly interacting fermions that are able to form pairs (this is, excepting the top quark) we get

Charge	***Colour***	***# degr of fr.***	***List of "bosons"***	***"associated fermions"***
+4/3	anti	3	uu cc uc	?
+1	no	6	u$\underline{d}$ u$\underline{s}$ u$\underline{b}$ c$\underline{d}$ c$\underline{s}$ s$\underline{b}$	e+ mu+ tau+
+2/3	yes	6	$\underline{dd}$ $\underline{ds}$ $\underline{db}$ $\underline{ss}$ $\underline{sb}$ $\underline{bb}$	u c t
+1/3	anti	6	ud us ub cd cs cb	$\underline{d}$ $\underline{s}$ $\underline{b}$
0	no	13	u$\underline{u}$ c$\underline{c}$ u$\underline{c}$ c$\underline{u}$; d$\underline{d}$ s$\underline{s}$ b$\underline{b}$ d$\underline{s}$ s$\underline{d}$ d$\underline{b}$ b$\underline{d}$ s$\underline{b}$ b$\underline{s}$	Neutrinos (+ ?)
-1/3	yes	6	$\underline{ud}$ $\underline{us}$ $\underline{ub}$ $\underline{cd}$ $\underline{cs}$ $\underline{cb}$	d s b
-2/3	anti	6	dd ds db ss sb bb	$\underline{u}$ $\underline{c}$ $\underline{t}$
-1	no	6	$\underline{u}$d $\underline{u}$s $\underline{u}$b $\underline{c}$d $\underline{c}$s $\underline{c}$b	e- mu- tau-
-4/3	yes	3	u$\underline{u}$ c$\underline{c}$ u$\underline{c}$	?

We have one extra degree of freedom in the neutrino sector and six extra degrees of freedom in a coloured diquark sector with charge 4/3. They probably can be discarded on the basis of group or representation-theoretical requirements. Barring them, the number of degrees of freedom of this bosonic sector coincides, charge-by-charge, with the number of degrees of freedom of the elementary fermionic sector. A coincidence that is the hallmark of supersymmetry.
Of course this is a very special low energy limit: it only works for three generations, at a energy low enough for the SU(3) coupling to be strong, but at the same time not enough to bind the top quark. But amazingly the Standard Model seems to live in this region, and furthermore the symmetry does not seem to be very badly broken, the mass of the pion being very near of the mass of the muon.

Connes's formulation of the standard model, with a spectral triple composed of a product of a continuous space times a discrete one, stresses the separation between spatial and internal properties of the particles, with the added peculiarity that the Yukawa couplings, while being responsible of the masses, live in a Dirac operator of the discrete part.
I see worthwhile to try to extend his formulation to allow bosons to live in the discrete part in equal footing that fermions do. A such extension would allow us: a) to build the above bosons from a tensor product of the discrete spectral triple, and b) to model confinement by the simple resource of not tensoring the continuous part, which should be still the spatial part of a single particle, not of a coupled pair. Note that (b) really asks for a bosonic version of Connes's product triple, then we will need to go deep into the role of bosons and fermions in Connes's model, in a way far more general than this standard-model-inspired example.

2) Lattices

A feature already present in the old model was the use of a topological dimension *d mod 8* getting its periodicity from Bott periodicity in K-theory. It was thought to be equal to the spectral dimension coming from the Dirac operator. Now such equality has been removed, and it becomes of interest to find out more meaning in the topological dimension of the standard model. For the new model, this dimension is 6 mod 8, or 2 mod 8 if you include the continuous part.

Of course 2 mod 8 invites to think about gravity or strings. Furthermore, strings have a kind of mod 8 periodicity already incorporated, that singles out 10 mod 8 = 26 mod8. This periodicity can be interpreted as coming down from the one of selfdual unimodular lattices. Dimension 26 is known to be related to the existence of the Leech lattice in 24 dimensions and its descendant in 25+1.

The periodicity in lattices is derived from a Gauss sum formula for elements in the quotient lattice L'/L built from a lattice L and its dual L'. In the self-dual case, the formula implies a mod 8 periodicity.

Now, it is interesting that one of the ways to build non commutative spaces is to use crossed products to quotient a space under the action of a group. When the commutative tools give a trivial space, non commutative geometry is able to keep not trivial aspects of the quotient structure. So in the case of a selfdual lattice, where the quotient space is a trivial point, we should be interested on see what can be learn from doing this quotient with the non-commutative tools. Particularly interesting is to see how the two 8-periodicities, the one of lattices and the one of K-theory, are related. This enquiry is mathematically interesting by itself, but also it could allow for better understanding of the discrete spectral triple related to the standard model.

3) Generic aspects of Connes Non Commutative Geometry Models

Now that the papers hep-th/0610241 and hep-th/0608221 have revived the phenomenology of Connes's models for elementary particles, it is worthwhile to review their minor aspects. I am interested on searching more about the role of Kasparov K-theory and homology, and how it defines a manifold via Poincare Duality. It is specially attractive that the commutative case involves a pairing between the algebra of complex functions and the Clifford bundle of the manifold.
Another issue is the relationship between the Spectral Trace and Dixmier Trace. The former has been only worked for the 4-dimensional case, while the later is generic. When applied to the Dirac operator, Dixmier trace can recover the euclidean Einstein-Hilbert action up to a dimension-dependent constant (in Gracia-Varilly-Figueroa normalization, this constant is (d-2)/24, amusingly). The Minkowskian versions of these traces and Dirac operators must still be produced.
In the phenomenology, there is the question of asking if the new version of the model still is restrictive about the number of Higgs doublets and other particles, or if on the contrary additional Higgses can be introduced, which could be useful to model neutrino angles. Besides, to be able to vary the particle content could be of interest to adjust the meeting of the running coupling constants.