Constraints on space-time dimensionality in the classical approximation.

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Abstract

We note the existence, in Newtonian gravity, of two simple arguments to constraint space time dimensionality. They can be used to give space time dimension a values of D=4, in the first argument, or D<5, in the second case.

Any theory of gravity will include Newtonian gravity as a limit. Thus it is worth to look there for arguments that in the full theory can be used to restrict the dimensionality of space time, or alternatively to signal a preferred number of non-compactified dimensions. Lets use this sheet to annotate two arguments of this kind. The calculations are so fast that we will dispense the reader from them.

Theories having discrete units of area and time will meet Kepler's second law in the following way: Consider a test particle orbiting circularly around a body of mass M. Ask for which radius will the particle to sweep a units of Planck area in b units of Planck time. We find that

- Only for space-time dimension D=4 will Newton' constant G cancel out from the calculation.
- In this case, the radius sweeping one unit of Planck area in one unit of Planck time is R=h/Mc, the Compton radius of the particle creating the gravitational potential.

Those considering quantities such as some density of bound states in a theory will meet Kepler's third law in a peculiar way. Consider two different circular orbits of radius R_1, R_2 and ask which orbit will a test particle sweep more area for, in the same interval of time. The area being proportional to the square of the radius, the third law tell us that periodes, radiuses and total areas are as $T^2 \sim R^{D-1} \sim A^{(D-1)/2}$, so that for D=5 total area is linear with the period of the orbit. Associated to this, we have the following dependence for swept area:

-When D<5, it increases when radius increases

-When D=5 the area swept by the test particle does not depend of the radius of the orbit, and

-When D>5, it decreases when the radius of the bound orbit increases

Of course we should expect that any theory beyond Newtonian gravity will destabilize the criticality of D=5, tipping the balance towards one of the two other alternatives.