Exercise Set 2: The Conformal Window

The Dualised Standard Model — Part III Exercises

Lent Term 2027

Conventions for these exercises

Quantity	**Convention**
Beta function	$\mu\, d\alpha_s/d\mu = -(b_0/2\pi)\,\alpha_s^2 - (b_1/4\pi^2)\,\alpha_s^3 + \cdots$
One-loop coefficient	$b_0 = (11N_c - 2N_f)/3$ for N_f Dirac flavours
Dynkin index	$T(\square) = \frac{1}{2}$, $T(\text{adj}) = N_c$
R-charge	$R[Q] = (N_f - N_c)/N_f$; $R[W] = 2$
SUSY conformal window	$3N_c/2 < N_f < 3N_c$ (exact, Seiberg)

References: Lecture 1, §??.

Problem 1: Beta function coefficient survey ⋆ (15 min)

Background. The one-loop beta function coefficient b_0 determines whether a gauge theory is asymptotically free ($b_0 > 0$) and sets the scale for the conformal window.

Questions.

(a) Compute $b_0 = (11N_c - 2N_f)/3$ for SU(3) gauge theory with the following numbers of Dirac flavours:

$$\begin{array}{lll} N_f = 0 & \text{(pure Yang–Mills)} & b_0 = ? \\ N_f = 2 & \text{(2-flavour QCD)} & b_0 = ? \\ N_f = 6 & \text{(6-flavour QCD)} & b_0 = ? \\ N_f = 16 & \text{(near AF boundary)} & b_0 = ? \\ N_f = 17 & \text{(above AF boundary)} & b_0 = ? \end{array}$$

(b) For each case, state whether the theory is asymptotically free.

(c) **Convention check.** Verify that the formula

$$b_0 = \frac{11}{3}\,T(\text{adj}) - \frac{4}{3}\,N_f\,T(\square) \tag{1}$$

with $T(\square) = 1/2$ and $T(\text{adj}) = N_c$ gives the same result as $b_0 = (11N_c - 2N_f)/3$.

Solution to Problem 1. **(a)** Using $b_0 = (11N_c - 2N_f)/3$ with $N_c = 3$:

N_f	Description	$b_0 = (33 - 2N_f)/3$	AF?
0	Pure Yang–Mills	$(33-0)/3 = 11$	Yes
2	2-flavour QCD	$(33-4)/3 = 29/3$	Yes
6	6-flavour QCD	$(33-12)/3 = 7$	Yes
16	Near AF boundary	$(33-32)/3 = 1/3$	Yes
17	Above AF boundary	$(33-34)/3 = -1/3$	No

(b) The theory is asymptotically free when $b_0 > 0$, *i.e.* when $N_f < 11N_c/2 = 33/2 = 16.5$. So $N_f \leq 16$ gives AF; $N_f \geq 17$ does not.

(c) Convention check.
$\frac{11}{3}\,T(\text{adj}) - \frac{4}{3}\,N_f\,T(\square) = \frac{11}{3}\,N_c - \frac{4}{3}\,N_f \times \frac{1}{2} = \frac{11N_c}{3} - \frac{2N_f}{3} = \frac{11N_c - 2N_f}{3}$.
This is identical to $b_0 = (11N_c - 2N_f)/3$. ✓

Important: The factor of 4/3 in $-\frac{4}{3}\,N_f\,T(\square)$ comes from the fermion loop contribution, where the 4 counts the Dirac degrees of freedom (2 spin × 2 particle/antiparticle, or equivalently 2 Weyl fermions per Dirac pair times the factor 2 from the loop integral). With $T(\square) = 1/2$, the overall contribution per Dirac fermion is $\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$. □

Problem 2: The Banks–Zaks fixed point ⋆⋆ (30 min)

Background. When N_f is just below the asymptotic freedom boundary $11N_c/2$, the one-loop coefficient b_0 is small and positive. If the two-loop coefficient b_1 is negative, the two-loop beta function has a non-trivial zero at a small coupling—the Banks–Zaks fixed point [**?**].

Questions.

(a) Write the two-loop beta function in the convention

$$\mu\frac{d\alpha_s}{d\mu} = -\frac{b_0}{2\pi}\,\alpha_s^2 - \frac{b_1}{4\pi^2}\,\alpha_s^3, \tag{2}$$

where

$$b_1 = \frac{1}{3}\Big[34N_c^2 - N_f\Big(13N_c - \frac{3}{N_c}\Big)\Big]. \tag{3}$$

(b) Show that setting $\beta(\alpha_s^*) = 0$ (with $\alpha_s^* \neq 0$) gives

$$\alpha_s^* = -\frac{2\pi\, b_0}{b_1}. \tag{4}$$

State the conditions on b_0 and b_1 for α_s^* to be physical (positive).

(c) For SU(3) with $N_f = 16$:

(i) Compute b_0.
(ii) Compute b_1.
(iii) Compute α_s^*.
(iv) Is the perturbative expansion reliable?

(d) For SU(3) with $N_f = 15$, repeat the computation. How does the reliability of the perturbative result change?

(e) Explain why $\alpha_s^* \to 0$ as $N_f \to 11N_c/2$ from below.

Solution to Problem 2. **(a)** The two-loop beta function is

$$\mu\frac{d\alpha_s}{d\mu} = -\frac{b_0}{2\pi}\,\alpha_s^2 - \frac{b_1}{4\pi^2}\,\alpha_s^3 + \mathcal{O}(\alpha_s^4),$$

with $b_0 = (11N_c - 2N_f)/3$ and $b_1 = \frac{1}{3}[34N_c^2 - N_f(13N_c - 3/N_c)]$.
This is equations (**??**) and (**??**) of Lecture 1.

(b) Setting $\beta(\alpha_s^*) = 0$ and dividing by α_s^{*2} (since $\alpha_s^* \neq 0$):

$$0 = -\frac{b_0}{2\pi} - \frac{b_1}{4\pi^2}\,\alpha_s^*, \tag{5}$$

$$\frac{b_1}{4\pi^2}\,\alpha_s^* = -\frac{b_0}{2\pi}, \tag{6}$$

$$\alpha_s^* = -\frac{b_0}{2\pi} \times \frac{4\pi^2}{b_1} = -\frac{2\pi\, b_0}{b_1}. \tag{7}$$

For α_s^* to be physical (*i.e.* positive), we need:

- $b_0 > 0$ (asymptotic freedom), and
- $b_1 < 0$ (so the ratio $-b_0/b_1 > 0$).

(c) SU(3)**,** $N_f = 16$**:**

(i) $b_0 = (33 - 32)/3 = 1/3$.

(ii) $b_1 = \frac{1}{3}[34 \times 9 - 16(13 \times 3 - 3/3)]$:

$$34N_c^2 = 34 \times 9 = 306, \tag{8}$$

$$13N_c - \frac{3}{N_c} = 13 \times 3 - \frac{3}{3} = 39 - 1 = 38, \tag{9}$$

$$N_f \times 38 = 16 \times 38 = 608, \tag{10}$$

$$b_1 = \frac{306 - 608}{3} = \frac{-302}{3}. \tag{11}$$

So $b_1 = -302/3 < 0$. ✓

(iii)

$$\alpha_s^* = -\frac{2\pi \times (1/3)}{(-302/3)} = -\frac{2\pi/3}{-302/3} = \frac{2\pi}{302} \approx 0.0208. \tag{12}$$

(iv) Since $\alpha_s^* \approx 0.021 \ll 1$, the perturbative expansion is *reliable*. Higher-loop corrections are suppressed by powers of $\alpha_s^*/(4\pi) \sim 0.002$ and are negligible. The Banks–Zaks fixed point is under perturbative control.

(d) SU(3)**,** $N_f = 15$**:**

$b_0 = (33 - 30)/3 = 1$.

b_1: $13N_c - 3/N_c = 38$ (same as before). $N_f \times 38 = 15 \times 38 = 570$.

$$b_1 = \frac{306 - 570}{3} = \frac{-264}{3} = -88. \tag{13}$$

$$\alpha_s^* = -\frac{2\pi \times 1}{-88} = \frac{2\pi}{88} = \frac{\pi}{44} \approx 0.071. \tag{14}$$

This is larger than the $N_f = 16$ case but still reasonably small ($\alpha_s^*/(4\pi) \approx 0.006$). The perturbative result is *marginally reliable*—higher-loop corrections may contribute at the 10–20% level, but the qualitative conclusion (an IR fixed point exists) is robust.

Comparison:

N_f	b_0	b_1	α_s^*	Reliability
16	1/3	$-302/3$	$2\pi/302 \approx 0.021$	High
15	1	-88	$\pi/44 \approx 0.071$	Marginal

As N_f decreases further, b_0 increases and α_s^* grows larger. Eventually the perturbative two-loop approximation breaks down entirely.

(e) As $N_f \to 11N_c/2 = 16.5$ from below, $b_0 \to 0^+$ while b_1 remains finite and negative. Therefore $\alpha_s^* = -2\pi b_0/b_1 \to 0$. The fixed point approaches the free-field point, and the perturbative expansion becomes arbitrarily reliable. This is the key insight of Banks and Zaks [**?**]: by tuning N_f close to the AF boundary, one obtains a *perturbatively accessible* interacting conformal field theory. □

Problem 3: The SUSY conformal window ⋆⋆⋆ (30 min)

Background. In $\mathcal{N} = 1$ supersymmetric QCD (SQCD) with gauge group $SU(N_c)$ and N_f flavours of chiral superfield pairs $(Q_i, \widetilde{Q}^i)$, the conformal window boundaries can be determined *exactly*, in striking contrast to the non-SUSY case.

Questions.

(a) State (without proof) that the exact NSVZ beta function for $SU(N_c)$ SQCD gives the one-loop SUSY coefficient $b_0^{\mathrm{SUSY}} = 3N_c - N_f$. What is the condition for asymptotic freedom?

(b) **Upper boundary.** At $N_f = 3N_c$, the theory becomes free (no interacting fixed point). Explain this using the NSVZ beta function: the numerator is proportional to $3N_c - N_f(1 - \gamma_Q)$, and at $N_f = 3N_c$ with $\gamma_Q = 0$ (free field), $\beta = 0$.

(c) **Lower boundary.** At the superconformal fixed point, the scaling dimension of a chiral operator $\mathcal{O}$ is $\Delta(\mathcal{O}) = \frac{3}{2}R[\mathcal{O}]$. The unitarity bound requires $\Delta \geq 1$ for a scalar operator (equivalently $R[\mathcal{O}] \geq 2/3$).

  (i) Using $R[Q] = (N_f - N_c)/N_f$, compute $R[M]$ for the meson $M = Q\widetilde{Q}$ (the R-charges are additive).

  (ii) The unitarity bound $R[M] \geq 2/3$ gives the lower boundary of the conformal window. Derive the bound on N_f.

(d) **Tabulate** the SUSY conformal window ($3N_c/2 < N_f < 3N_c$) for:

  (i) SU(2)

  (ii) SU(3)

  (iii) SU(5)

  For each, list the integer values of N_f inside the window.

(e) Compare the SUSY conformal window with the non-SUSY conformal window for SU(3). Why is the SUSY result more powerful?

Solution to Problem 3. **(a)** The one-loop SUSY beta function coefficient for $SU(N_c)$ SQCD with N_f chiral pairs is:

$$b_0^{\mathrm{SUSY}} = 3N_c - N_f. \tag{15}$$

This follows from the NSVZ (Novikov–Shifman–Vainshtein–Zakharov) exact beta function (to be derived in Lecture 2). The factor of $3N_c$ (instead of $11N_c/3$) reflects the different field content of SQCD: each gauge boson is paired with a gaugino (adjoint Weyl fermion), and each matter field is a chiral superfield containing both a scalar and a Weyl fermion.

Asymptotic freedom requires $b_0^{\mathrm{SUSY}} > 0$, *i.e.*:

$$\boxed{N_f < 3N_c} \qquad \text{(condition for AF in SQCD)}. \tag{16}$$

For SU(3): $N_f < 9$.

(b) Upper boundary.

The exact NSVZ beta function has the form:

$$\beta(g) \propto \frac{3N_c - N_f(1-\gamma_Q(g))}{1 - C_2(\text{adj})\, g^2/(8\pi^2)}, \tag{17}$$

where $\gamma_Q(g)$ is the anomalous dimension of Q. The beta function vanishes when the numerator vanishes:

$$3N_c - N_f(1-\gamma_Q^*) = 0, \qquad \Longrightarrow \qquad \gamma_Q^* = 1 - \frac{3N_c}{N_f}. \tag{18}$$

At $N_f = 3N_c$: $\gamma_Q^* = 1 - 1 = 0$. A vanishing anomalous dimension means the quarks are *free fields* at the fixed point—the theory is non-interacting. Therefore $N_f = 3N_c$ is the boundary between the conformal window (where $\gamma_Q^* \neq 0$, interacting CFT) and the free phase.

For $N_f > 3N_c$: $b_0^{\text{SUSY}} < 0$, so the theory is not asymptotically free and there is no interacting UV or IR fixed point at weak coupling.

(c) Lower boundary.

(i) The non-anomalous R-charge of Q is $R[Q] = (N_f - N_c)/N_f$. Since R-charges are additive:

$$R[M] = R[Q] + R[\widetilde{Q}] = 2\,\frac{N_f - N_c}{N_f} = \frac{2(N_f - N_c)}{N_f}. \tag{19}$$

(We use $R[\widetilde{Q}] = R[Q]$ by the $Q \leftrightarrow \widetilde{Q}$ symmetry of the R-charge assignment.)

(ii) The unitarity bound for a scalar chiral primary is $\Delta \geq 1$, equivalently $R[M] \geq 2/3$. Imposing this:

$$\frac{2(N_f - N_c)}{N_f} \geq \frac{2}{3}, \tag{20}$$

$$3(N_f - N_c) \geq N_f, \tag{21}$$

$$3N_f - 3N_c \geq N_f, \tag{22}$$

$$2N_f \geq 3N_c, \tag{23}$$

$$N_f \geq \frac{3N_c}{2}. \tag{24}$$

Therefore the conformal window has a lower boundary at $N_f = 3N_c/2$. Below this value, the meson M violates the unitarity bound, signalling that the electric conformal description breaks down and the IR is better described by the magnetic (confined/free-magnetic) variables.

Check at the boundary: $N_f = 3N_c/2$:

$$R[M] = \frac{2(3N_c/2 - N_c)}{3N_c/2} = \frac{2 \times N_c/2}{3N_c/2} = \frac{N_c}{3N_c/2} = \frac{2}{3}. \quad \checkmark \tag{25}$$

The unitarity bound is exactly saturated.

Combining with the upper boundary from part (b):

$$\boxed{\frac{3N_c}{2} < N_f < 3N_c} \qquad \text{(SUSY conformal window)}. \tag{26}$$

(d) Tabulation.

Gauge group	$3N_c/2$	$3N_c$	Integer N_f in window
SU(2)	3	6	$N_f = 4,\, 5$
SU(3)	4.5	9	$N_f = 5,\, 6,\, 7,\, 8$
SU(5)	7.5	15	$N_f = 8,\, 9,\, 10,\, 11,\, 12,\, 13,\, 14$

Checks:

- SU(3), $N_f = 5$: $R[Q] = (5-3)/5 = 2/5 > 1/3$ and $R[M] = 4/5 > 2/3$. Inside window. ✓
- SU(3), $N_f = 9$: $R[Q] = 6/9 = 2/3$ and $R[M] = 4/3 > 2/3$, but this is the upper boundary ($\gamma_Q = 0$, free theory). $N_f = 9$ is *not* in the window. ✓
- SU(3), $N_f = 4$: $R[Q] = 1/4$ and $R[M] = 1/2 < 2/3$. *Below* the window (confining). ✓

(e) Comparison with non-SUSY.

For non-SUSY SU(3) gauge theory with N_f Dirac fundamentals:

- **Upper boundary:** $N_f = 11 \times 3/2 = 16.5$ (exact, one-loop).
- **Lower boundary:** Debated. Different methods give:
  - Ladder approximation (Appelquist–Terning–Wijewardhana): $N_f^* \approx 4N_c = 12$ for SU(3).
  - Lattice simulations: conflicting results for $N_f = 8$–12. For SU(3) with $N_f = 12$, some lattice studies find evidence for conformality and others find confinement.
  - Functional RG: $N_f^* \approx 8$–10 for SU(3).

The SUSY result is more powerful because:

1. **Both boundaries are exact.** The lower boundary follows from the unitarity bound (a rigorous consequence of the superconformal algebra), not a perturbative approximation.
2. **No lattice input needed.** The non-SUSY lower boundary requires non-perturbative methods (lattice, functional RG) that are computationally demanding and sometimes give conflicting results.
3. **Seiberg duality provides a dual description.** Below the conformal window, Seiberg duality maps the strongly-coupled electric theory to a weakly-coupled magnetic theory, giving analytical control over the confined phase as well.

□

Problem 4: Phases of gauge theories ⋆⋆⋆⋆ (45 min)

Background. The phase structure of SU(N_c) gauge theory with N_f fundamental flavours is rich and directly relevant to the dualised Standard Model programme.

Questions.

(a) For SU(3) with N_f Dirac fundamental flavours, describe the IR physics in each of the following regimes:

(i) $N_f = 0$ (pure Yang–Mills)

(ii) $N_f = 1, \ldots, \sim 4$ (few flavours)

(iii) $N_f \approx 5$–8 (SUSY conformal window)

(iv) $N_f \approx 9$–16 (Banks–Zaks regime)

(v) $N_f \geq 17$ (loss of asymptotic freedom)

(b) For each regime, describe the IR physics (confined, conformal, or free) and state what anomaly matching tells us.

(c) Explain why the conformal window is important for the dualised SM programme.

(d) **(Challenge.)** For SU(3) with $N_f = 12$, near the lower boundary of the non-SUSY conformal window: discuss why lattice simulations are needed and what the current state of evidence is.

Solution to Problem 4. **(a) and (b) Phase diagram of** SU(3) **with** N_f **fundamental Dirac flavours.**

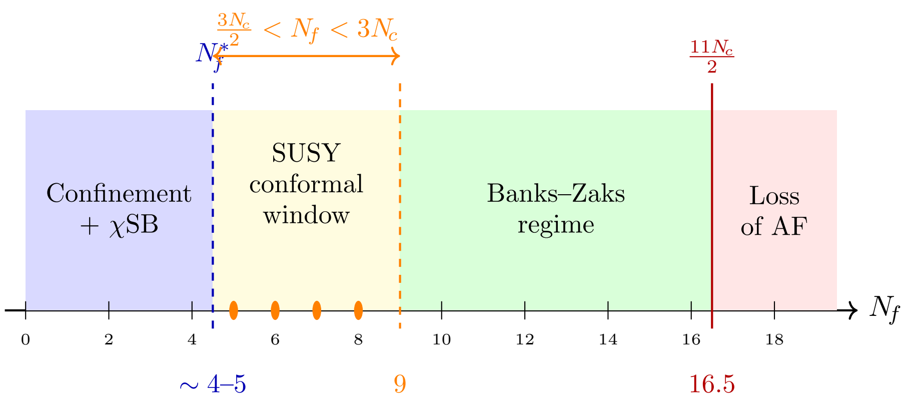

Figure 1: Phase diagram of SU(3) gauge theory as a function of N_f (Dirac flavours). Orange dots mark the SUSY conformal window integers $N_f = 5, 6, 7, 8$. The non-SUSY lower boundary N_f^* is model-dependent.

Important caveat. The SUSY conformal window (shaded orange, exact: $3N_c/2 < N_f < 3N_c$) is shown alongside the non-SUSY estimates (blue/green regions, approximate). *These are distinct theories with different dynamics.* The horizontal axis counts Dirac flavours in both cases, but the SUSY result applies to $\mathcal{N} = 1$ SQCD (with superpartners), while the non-SUSY boundaries (N_f^* and the Banks–Zaks regime) refer to ordinary QCD-like theories. The SUSY window is overlaid to show where exact results are available; one should *not* read it as a subregion of the non-SUSY phase diagram.

(i) $N_f = 0$: Pure Yang–Mills.

The theory has no matter fields. $b_0 = 11 > 0$: strongly asymptotically free. The IR is confining with a mass gap $\sim \Lambda_{\rm QCD}$. There are no global flavour symmetries, so anomaly matching is vacuous. The spectrum consists of massive glueballs.

(ii) $N_f = 1, \ldots, \sim 4$: Confinement with chiral symmetry breaking.

$b_0 > 0$ (AF), the coupling grows in the IR, and the theory confines. The classical global symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_B$ is spontaneously broken:

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V,$$

producing $N_f^2 - 1$ massless Goldstone bosons (pions).

't Hooft anomaly matching constrains the IR spectrum: the anomaly coefficients (computed in Problem 2 for $N_f = 2$ and Problem 3 for general N_f) must be matched by the massless fermions in the confined phase. For $N_f = 2$, the nucleon doublet provides the matching.

(iii) $N_f \approx 5$–8 (SUSY conformal window for $\mathrm{SU}(3)$):

In the SUSY theory, the exact result is that $N_f = 5, 6, 7, 8$ lie in the conformal window $3N_c/2 < N_f < 3N_c = 9$. The theory flows to an interacting superconformal fixed point in the IR. There is no confinement, no chiral symmetry breaking, and no mass gap.

Anomaly matching is automatically satisfied at the conformal fixed point: both the UV (free) and IR (interacting CFT) descriptions have the same massless particle content (the theory is scale-invariant, not confining).

For the *non-SUSY* theory, the situation is uncertain: the lower boundary N_f^* of the non-SUSY conformal window is model-dependent, with estimates ranging from ~ 8 to ~ 12 for SU(3).

(iv) $N_f \approx 9$–16: Banks–Zaks regime.

$b_0 > 0$ (AF) and the two-loop coefficient $b_1 < 0$ for N_f sufficiently large. The theory flows to a Banks–Zaks fixed point at $\alpha_s^* = -2\pi b_0/b_1$. Near $N_f = 16$, the fixed point coupling is small and perturbatively reliable (Problem 2). As N_f decreases toward N_f^*, the coupling at the fixed point grows and eventually the perturbative description breaks down.

(v) $N_f \geq 17$: Loss of asymptotic freedom.

$b_0 \leq 0$: the coupling is irrelevant in the UV and the theory is IR free (trivial). There is no interacting fixed point. The theory is not confining and has no interesting strong-coupling dynamics. It behaves like QED at low energies.

(c) Relevance to the dualised SM programme.

The conformal window is central to the dualised Standard Model because:

1. **Seiberg duality operates in the conformal window.** In SQCD, theories with $3N_c/2 < N_f < 3N_c$ have both an "electric" description (gauge group $\mathrm{SU}(N_c)$, strongly coupled in the IR) and a "magnetic" description (gauge group $\mathrm{SU}(N_f - N_c)$, weakly coupled in the IR). These are dual descriptions of the *same* physics.

2. **The SM could be the magnetic dual.** If the SM gauge sectors arise as the magnetic (weakly coupled) dual of a strongly-coupled electric theory, then the matter content we observe (quarks, leptons, Higgs) would be the magnetic "quarks" and "mesons" of the dual theory. This requires the electric theory to lie in or near the conformal window.

3. **Analytical control.** Within the conformal window, both sides of the duality are under analytical control. Anomaly matching provides exact constraints on the duality map. This is far more powerful than attempting to study the non-SUSY conformal window, where no exact results are available.

(d) Challenge: SU(3) **with** $N_f = 12$.

The theory with $N_f = 12$ is at the heart of the debate about the non-SUSY conformal window. At two loops:

$$b_0 = (33 - 24)/3 = 3, \tag{27}$$

$$b_1 = (306 - 12 \times 38)/3 = (306 - 456)/3 = -50, \tag{28}$$

$$\alpha_s^* = -2\pi \times 3/(-50) = 6\pi/50 = 3\pi/25 \approx 0.38. \tag{29}$$

This value $\alpha_s^* \approx 0.38$ is *not* small—the perturbative two-loop result is unreliable. We are deep in the non-perturbative regime, and the two-loop beta function cannot be trusted to determine whether the theory confines or flows to a fixed point.

Why lattice simulations are needed:

1. No perturbative or analytical method can determine whether $\alpha_s^* \approx 0.38$ corresponds to a true fixed point or is an artifact of the two-loop truncation.
2. The SUSY exact result does not apply ($N_f = 12$ is above the SUSY window $N_f < 9$ for SU(3), and this is a non-SUSY theory).
3. Only a non-perturbative method (lattice gauge theory) can definitively determine the IR behaviour.

Current lattice evidence (summary):

- **Evidence for conformality:** Some lattice studies (notably the LatKMI collaboration) find that the mass spectrum scales consistently with a conformal IR fixed point, with a slowly running coupling consistent with approximate scale invariance.
- **Evidence for confinement:** Other studies find a non-zero chiral condensate and a non-zero string tension, suggesting confinement with chiral symmetry breaking.
- **The difficulty:** Near the conformal window boundary, the coupling runs very slowly ("walking"). This makes it hard to distinguish between true conformality and very slow running toward confinement. Finite-volume and finite-lattice-spacing effects are significant and hard to control.

The bottom line: The question of whether SU(3) with $N_f = 12$ is inside or outside the conformal window remains open as of 2027. This uncertainty is one motivation for the dualised SM programme: by working within the *SUSY* conformal window (where the boundaries are exact), one avoids this ambiguity entirely. □

End of Exercise Set 2. The conformal window and anomaly matching from Exercise Sets 1–2 provide the foundation for Seiberg duality (Lecture 3) and the dualised Standard Model construction (Lectures 6–7).

References

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