Lecture 5: Tumbling, the MAC Criterion, and Complementarity

The Dualised Standard Model — Part III Lecture Notes

Lent Term 2027

Contents

Conventions for Lecture 5

All conventions from Lectures 1–4 remain in force. Additional conventions specific to non-perturbative strong dynamics:

Quantity	Convention	Comment
Units	$\hbar = c = 1$ (natural)	
Metric	$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$	mostly minus
Dynkin index	$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$	fundamental of $\mathrm{SU}(N_c)$
Cubic anomaly	$A(\square) = 1,\ A(\overline{\square}) = -1$	per fundamental Weyl
Quadratic Casimir	$C_2(\square) = \frac{N_c^2-1}{2N_c}$	fundamental
	$C_2(\mathrm{adj}) = N_c$	adjoint
	$C_2(\wedge^2) = \dfrac{(N_c-2)(N_c+1)}{N_c}$	antisymmetric rank 2
	$C_2(\mathrm{S}_2) = \dfrac{(N_c+2)(N_c-1)}{N_c}$	symmetric rank 2
MAC attraction	$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_c)$	Raby–Dimopoulos–Susskind

1 Chiral Gauge Theories and Strong Dynamics

In Lectures 1–4 we studied *vector-like* gauge theories: theories in which the left-handed and right-handed fermions transform in representations that are related by complex conjugation. QCD is the canonical example: each quark flavour comes with a Dirac partner, and the gauge representation $\square \oplus \overline{\square}$ is real (in the sense of being self-conjugate as a combined representation). In such theories, the standard expectation is chiral symmetry breaking through a $\langle \bar{\psi}\psi \rangle$ condensate that preserves the gauge symmetry.

In this lecture, we turn to a richer class of theories: **chiral gauge theories**, in which the fermion content does *not* form a real representation of the gauge group. The Georgi–Glashow SU(5) model with fermions in the $\mathbf{10} \oplus \overline{\mathbf{5}}$ representation is a classic example: the $\mathbf{10}$ (antisymmetric rank-2) is not the conjugate of the $\overline{\mathbf{5}}$, and the combined representation is irreducibly complex.

In chiral gauge theories, a $\langle \bar{\psi}\psi \rangle$ condensate of the QCD type is not available—such a condensate would break the gauge symmetry itself. Instead, the strong dynamics can lead to **dynamical gauge symmetry breaking** through condensation of gauge-invariant fermion bilinears in specific channels. Which channel condenses first? This is the question addressed by the Most Attractive Channel (MAC) criterion.

Plan for this lecture. We develop three tools for analysing strong dynamics in gauge theories:

1. The **MAC criterion** (§2): a heuristic for identifying which fermion bilinear condenses first.
2. **Tumbling** (§3): the mechanism of sequential condensation and gauge group breaking, illustrated by the SU(5) Georgi–Glashow model.
3. **Complementarity** (§4): the Fradkin–Shenker theorem relating the Higgs and confinement phases.

In §5 we explain how these tools connect to the electric–magnetic duality framework from Lectures 3–4, setting up the non-SUSY extensions in Lecture 6.

2 The Most Attractive Channel (MAC) Criterion

2.1 The one-gluon exchange argument

Consider a gauge theory with gauge group $\mathrm{SU}(N_c)$ and fermions in representations R_1 and R_2. At one-loop order in perturbation theory, the exchange of a single gluon between two fermions produces an attractive potential in the colour channel R_c that appears in the tensor product $R_1 \otimes R_2$. The strength of this attraction is proportional to the **quadratic Casimir difference**:

$$\boxed{\Delta C_2 \;=\; C_2(R_1) + C_2(R_2) - C_2(R_c)\,.} \tag{1}$$

The channel R_c with the **largest** ΔC_2 experiences the strongest attraction. The MAC hypothesis [1] asserts that this most attractive channel is the one in which a fermion bilinear condensate forms first as the gauge coupling runs to strong coupling.

2.2 Physical picture

The argument has a simple physical origin. In the non-relativistic limit, the one-gluon exchange potential between two colour sources in representations R_1 and R_2, forming a composite in channel R_c, is:

$$V(r) \;\sim\; -\frac{\alpha_s}{r}\,\big[C_2(R_1) + C_2(R_2) - C_2(R_c)\big] \;=\; -\frac{\alpha_s}{r}\,\Delta C_2\,. \tag{2}$$

A larger ΔC_2 means a deeper attractive potential well, which favours the formation of a bound state (condensate) in that channel. The condensate with the strongest attraction forms first as the coupling increases.

Remark 2.1 (Why the smallest $C_2(R_c)$?). Since $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_c)$ and $C_2(R_1)$, $C_2(R_2)$ are fixed by the fermion content, ΔC_2 is maximised when $C_2(R_c)$ is *minimised.* In a tensor product $R_1 \otimes R_2 = \bigoplus_c R_c$, the smallest $C_2(R_c)$ typically corresponds to the smallest (lowest-dimensional) representation in the decomposition. For $\square \otimes \square$, this is the singlet (if available) or the antisymmetric representation.

2.3 Quadratic Casimir values

For reference, we collect the quadratic Casimirs for the common representations of $\mathrm{SU}(N)$, using the conventions locked in Lecture 1 (with $T(\square) = \frac{1}{2}$):

Representation	**Dimension**	$C_2(R)$
Fundamental $\square$	N	$\dfrac{N^2-1}{2N}$
Antifundamental $\overline{\square}$	N	$\dfrac{N^2-1}{2N}$
Adjoint adj	N^2-1	N
Antisymmetric $\wedge^2$	$\dfrac{N(N-1)}{2}$	$\dfrac{(N-2)(N+1)}{N}$
Symmetric S_2	$\dfrac{N(N+1)}{2}$	$\dfrac{(N+2)(N-1)}{N}$

(3)

Verification for SU(3)**:** $C_2(\square) = (9-1)/6 = 4/3$. $C_2(\text{adj}) = 3$. $C_2(\wedge^2) = (1)(4)/3 = 4/3$. (This is correct: for SU(3), the antisymmetric rank-2 representation is the $\overline{\mathbf{3}}$, which is the antifundamental, so $C_2(\wedge^2) = C_2(\overline{\square})$.) ✓

2.4 Limitations of the MAC criterion

MAC is a heuristic, not a theorem

The MAC criterion has significant limitations that must be stated honestly:

(a) **MAC is a conjecture, not a theorem.** There is no rigorous proof that the most attractive channel (as identified by one-gluon exchange) is the one that actually condenses in the full non-perturbative theory.

(b) **Perturbative reasoning at strong coupling.** The one-gluon exchange argument (2) is a leading-order perturbative calculation. However, condensation occurs in the strong-coupling regime ($\alpha_s \sim 1$), where higher-order corrections are not small and the perturbative picture is unreliable.

(c) **Known counterexamples.** When two or more channels have comparable ΔC_2 values, the MAC prediction can fail. Kosower [4] demonstrated that for certain SU(N) theories, the competition between channels is not resolved by the leading-order criterion alone.

(d) **Channel, not scale.** Even when the MAC correctly identifies the condensation channel, it does not predict the condensation *scale*. The scale is set by the full non-perturbative dynamics, which the one-gluon exchange does not capture.

Usage: The MAC criterion *suggests* the condensation channel. We do not say "the condensate forms in the MAC." When the dominant channel is clearly separated from competitors (as in our SU(5) example below), the criterion is most reliable. Within SUSY, the exact results of Seiberg duality (*cf.* Lecture 3) supersede the MAC approximation.

3 Tumbling: The SU(5) Worked Example

We now demonstrate tumbling in the classic Georgi–Glashow SU(5) model [1], with fermion content:

$$\psi^{ab} \in \mathbf{10}\ (\wedge^2 \text{ of SU}(5)), \qquad \chi_a \in \overline{\mathbf{5}}\ (\overline{\square} \text{ of SU}(5)), \tag{4}$$

where $a, b = 1, \ldots, 5$ are SU(5) fundamental indices, and $\psi^{ab} = -\psi^{ba}$. Both ψ and χ are left-handed Weyl fermions.

Remark 3.1 (Anomaly cancellation)*.* This fermion content is anomaly-free. The cubic SU$(5)^3$ anomaly is: $A(\mathbf{10}) + A(\overline{\mathbf{5}}) = A(\wedge^2) + A(\overline{\square}) = (5-4) + (-1) = 1 - 1 = 0$. ✓ (Recall from Lecture 1 that $A(\wedge^2) = N - 4$ and $A(\overline{\square}) = -1$.)

3.1 Step 1: Enumerate condensation channels

We consider bilinears formed from $\psi^{ab} \in \mathbf{10}$. The tensor product of two antisymmetric rank-2 representations of SU(5) decomposes as:

$$\mathbf{10} \otimes \mathbf{10} = \overline{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \mathbf{50}. \tag{5}$$

The $\overline{\mathbf{5}}$ channel corresponds to the contraction $\langle \psi^{ab}\, \psi^{cd} \rangle \sim \epsilon^{abcde}$, which produces a composite transforming as $\overline{\mathbf{5}}$.

We also consider the mixed bilinear $\mathbf{10} \otimes \overline{\mathbf{5}}$:

$$\mathbf{10} \otimes \overline{\mathbf{5}} \;=\; \mathbf{5} \;\oplus\; \mathbf{45}\,. \tag{6}$$

3.2 Step 2: Compute ΔC_2 for each channel

Using the Casimir values from (3) with $N = 5$:

$$C_2(\mathbf{10}) = C_2(\wedge^2 \text{ of SU}(5)) = \frac{(5-2)(5+1)}{5} = \frac{18}{5} = 3.6\,, \tag{7}$$

$$C_2(\overline{\mathbf{5}}) = C_2(\square \text{ of SU}(5)) = \frac{25-1}{10} = \frac{12}{5} = 2.4\,. \tag{8}$$

Channel comparison:

Bilinear	**Channel** R_c	$\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_c)$	**Value**
$\mathbf{10} \times \mathbf{10}$	$\overline{\mathbf{5}}$	$\frac{18}{5} + \frac{18}{5} - \frac{12}{5}$	$\frac{24}{5} = 4.8$
$\mathbf{10} \times \overline{\mathbf{5}}$	$\mathbf{5}$	$\frac{18}{5} + \frac{12}{5} - \frac{12}{5}$	$\frac{18}{5} = 3.6$
$\mathbf{10} \times \mathbf{10}$	$\overline{\mathbf{45}}$	$\frac{18}{5} + \frac{18}{5} - C_2(\overline{\mathbf{45}})$	< 4.8

(9)

The $\overline{\mathbf{45}}$ channel has $C_2 > C_2(\overline{\mathbf{5}})$, so its ΔC_2 is smaller. The most attractive channel is:

$$\boxed{\text{MAC:} \quad \mathbf{10} \times \mathbf{10} \;\to\; \overline{\mathbf{5}}\,, \qquad \Delta C_2 = \frac{24}{5}\,.} \tag{10}$$

The MAC suggests that the SU(5) theory first forms the condensate:

$$\langle \psi^{ab}\, \psi^{cd} \rangle \;\sim\; \epsilon^{abcde}\, v_e\,, \tag{11}$$

where v_e is a vector in the fundamental representation space of SU(5) that specifies the direction of condensation.

3.3 Step 3: Gauge symmetry breaking

The condensate (11) transforms as a $\overline{\mathbf{5}}$ under SU(5). A non-zero expectation value for this composite breaks the gauge symmetry. Choosing (by gauge rotation) $v_e = v\,\delta_{e5}$, the unbroken subgroup is:

$$\text{SU}(5) \xrightarrow{\langle\psi\psi\rangle\neq 0} \text{SU}(4)\,. \tag{12}$$

The condensate picks out the fifth direction, breaking SU(5) to the SU(4) that rotates the remaining four indices $a = 1, 2, 3, 4$.

This is tumbling: the gauge group has broken to a smaller group. The residual SU(4) theory now contains the *surviving* massless fermions—those components of the original ψ and χ that did not acquire masses from the condensate—and the process repeats. The cascade of sequential condensation steps is the **tumbling mechanism** [1].

3.4 Fermion decomposition under SU(4)

Under SU(5) → SU(4), the representations decompose as:

$$\mathbf{10}\,(\psi^{ab}) \;\to\; \mathbf{6}\,(\psi^{ij}) \;\oplus\; \mathbf{4}\,(\psi^{i5})\,, \tag{13}$$

$$\overline{\mathbf{5}}\,(\chi_a) \;\to\; \overline{\mathbf{4}}\,(\chi_i) \;\oplus\; \mathbf{1}\,(\chi_5)\,, \tag{14}$$

where $i, j = 1, \ldots, 4$ are SU(4) fundamental indices. The $\mathbf{6}$ is the antisymmetric rank-2 of SU(4), and the $\mathbf{4}$ comes from the components ψ^{i5} with one index in the broken direction.

The condensate $\langle \psi^{ab}\,\psi^{cd}\rangle \sim \epsilon^{abcd5}\, v$ involves the components ψ^{ij} (forming the SU(4)-invariant combination $\epsilon^{ijkl}\,\psi_{ij}\,\psi_{kl}$). The condensation gives mass to a linear combination of fermions, while the orthogonal combinations remain massless.

The surviving massless fermions in the SU(4) theory are:

$$\mathrm{SU}(4): \qquad \psi^{i5} \in \mathbf{4}\,, \qquad \chi_i \in \overline{\mathbf{4}}\,, \qquad \chi_5 \in \mathbf{1}\,. \tag{15}$$

(The $\mathbf{6}$ of SU(4) from ψ^{ij} condenses and the corresponding fermions acquire mass from the gap equation.)

3.5 Step 4: Anomaly matching at the tumbling step

We now verify that 't Hooft anomaly matching—the exact constraint from Lecture 1, §3—is satisfied at the first tumbling step. This provides a non-trivial consistency check on the fermion identification above.

The original SU(5) theory has a global U(1) symmetry (generalised baryon number) under which the fermions carry charges:

$$B[\psi] = +1\,, \qquad B[\chi] = -3\,. \tag{16}$$

(The relative charge is fixed by requiring $B[\langle\psi\psi\rangle] \neq 0$ and by the anomaly-free condition for the gauge group $\mathrm{SU}(5)^2\,\mathrm{U}(1)_B$: $T(\mathbf{10})\cdot(+1) + T(\overline{\mathbf{5}})\cdot(-3) = \frac{3}{2}\cdot 1 + \frac{1}{2}\cdot(-3) = 0$.)

The relevant anomaly coefficient for $\mathrm{U}(1)_B$ is the mixed gauge–global anomaly $\mathrm{SU}(5)^2\,\mathrm{U}(1)_B$, which vanishes by construction. However, we can check the *purely global* anomaly $\mathrm{U}(1)_B^3$ as a non-trivial test.

UV (before condensation):

$$\mathcal{A}[\mathrm{U}(1)_B^3]_{\mathrm{UV}} = \dim(\mathbf{10})\cdot B[\psi]^3 + \dim(\overline{\mathbf{5}})\cdot B[\chi]^3 = 10\cdot 1 + 5\cdot(-27) = 10 - 135 = -125\,. \tag{17}$$

IR (after condensation): The surviving fermions in (15) carry the same $\mathrm{U}(1)_B$ charges as the original fields from which they descend:

$$\mathcal{A}[\mathrm{U}(1)_B^3]_{\mathrm{IR}} = \dim(\mathbf{4})\cdot(1)^3 + \dim(\overline{\mathbf{4}})\cdot(-3)^3 + 1\cdot(-3)^3 = 4 - 108 - 27 = -131\,. \tag{18}$$

The mismatch (-125 vs -131) indicates that this simple decomposition does not yet account for all the massless fermion content at the SU(4) stage—additional composite massless fermions ('t Hooft's "persistent mass condition") must appear to satisfy anomaly matching, or some of the fermion components from the condensed representation remain massless as Goldstone partners. This is precisely the subtlety that makes tumbling analysis non-trivial: anomaly matching *constrains* the surviving spectrum but does not uniquely determine it from the condensation channel alone.

Remark 3.2 (The role of anomaly matching in tumbling). The point of this exercise is not that anomaly matching fails—it is an exact theorem and *must* be satisfied—but that *identifying the* *correct massless spectrum* at each tumbling step requires careful analysis beyond MAC alone. The MAC tells us *which channel condenses*; anomaly matching constrains *which fermions remain massless*. Together, they provide the tools for analysing the tumbling cascade. We refer to the original analysis of Raby, Dimopoulos, and Susskind [1] and the pedagogical treatment in Peskin [7] for the complete determination of the massless spectrum at each step of the SU(5) cascade.

The key lesson from Lecture 1 applies here: **'t Hooft anomaly matching does not determine the dynamics, but it powerfully constrains the possible low-energy spectra.** Any proposed tumbling pattern that violates anomaly matching is ruled out.

4 Complementarity: The Fradkin–Shenker Theorem

We now turn to a different but complementary perspective on the relationship between the Higgs mechanism and confinement.

4.1 Statement of the theorem

Fradkin and Shenker [2] (see also Osterwalder and Seiler [3]) proved a remarkable result on the lattice:

Theorem 4.1 (Fradkin–Shenker, 1979)**.** *For a gauge theory with a scalar field in the* ***fundamental representation*** *of the gauge group, the Higgs phase and the confinement phase are* ***analytically connected****: there is no phase transition separating them.*

Physical meaning. In a gauge theory with a fundamental Higgs field ϕ (and no other order parameter distinguishing the phases), one can continuously deform the theory from the "Higgs regime" (large scalar VEV, perturbative gauge boson masses) to the "confinement regime" (strong coupling, flux tube formation) without encountering a phase transition. The two regimes are *not distinct phases*—they are different descriptions of the same physics.

4.2 The particle identification

The physical consequence of complementarity is a **one-to-one map** between the particle spectra in the two descriptions:

Higgs description	**Confining description**
Massive gauge bosons (W, Z)	Bound states of $\phi^\dagger D_\mu \phi$
Higgs boson(s)	Bound states $\phi^\dagger\phi$
Matter fermions ψ	Confined composites $\phi^\dagger\psi$

The confined composites on the right are gauge-invariant operators built from the scalar ϕ and the other fields. Because there is no phase transition, these gauge-invariant composites must match the gauge-fixed particle descriptions on the left—their quantum numbers (spin, flavour charges, baryon number, etc.) agree exactly. This identification is sometimes called the **complementarity principle** for gauge theories [5].

4.3 Conditions and limitations

Complementarity is not universal

The Fradkin–Shenker theorem holds under two specific conditions:

(a) **The scalar must be in the fundamental representation** of the gauge group.

(b) **The gauge group must be completely broken** by the scalar VEV (no residual gauge symmetry).

When either condition is violated, phase transitions *can* separate the Higgs and confinement phases:

- For **adjoint Higgs** (or higher-representation scalars), the Higgs and confinement regimes can be separated by a genuine phase transition. A discrete unbroken gauge symmetry (such as the centre $\mathbb{Z}_N$ of $\mathrm{SU}(N)$) provides a local order parameter that distinguishes the phases.
- For **partial gauge breaking**—where the scalar VEV leaves a residual gauge group—additional arguments are needed. The Standard Model electroweak sector has $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$, which is partial breaking. While complementarity-type arguments have been advanced for this case [5], the rigorous Fradkin–Shenker theorem does not directly apply.
- Shifman and Yung [6] showed that in the Seiberg–Witten framework (Lecture 4), the quark confinement phase is **not** analytically connected to the Higgs phase. This provides a concrete counterexample to naïve universal complementarity when the conditions of the Fradkin–Shenker theorem are not met.

Remark 4.2 (Complementarity and the Standard Model). The SM Higgs doublet is in the fundamental ($\mathbf{2}$) of $\mathrm{SU}(2)_L$, satisfying condition (a). However, the electroweak breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$ is *partial*: condition (b) is not satisfied because $\mathrm{U}(1)_{\mathrm{EM}}$ survives. In the context of the dualised Standard Model (Lecture 6 and beyond), the Pati–Salam embedding provides a more natural setting where a larger gauge group is *completely* broken, and complementarity applies more directly.

5 Connection to Duality

5.1 Tumbling + complementarity = a physical picture for duality

We can now assemble the tools from this lecture into a **physical picture** that motivates electric–magnetic duality:

(i) **Electric theory (confining description).** Start with a strongly-coupled gauge theory with fermions. The MAC criterion suggests which fermion bilinear condenses first; the tumbling mechanism produces a cascade of condensation steps that break the gauge group sequentially. At each step, massless composite fermions survive, constrained by anomaly matching.

(ii) **Transition to the Higgs picture.** At the end of the tumbling cascade, the strongly-coupled theory has produced condensates that break the gauge symmetry. By complementarity, this confining description is continuously connected to a *Higgs description* of the same physics: the condensates play the role of elementary Higgs fields, and the composites of the electric theory become the elementary particles of the Higgs phase.

(iii) Magnetic theory (Higgs description). The Higgs-phase description is a weakly-coupled theory with elementary scalars (the "mesons" of the electric theory) and gauge bosons—precisely the structure of the magnetic dual from Lecture 3. The confined composites of the electric theory are identified with the elementary fields of the magnetic theory via their shared quantum numbers.

Remark 5.1 (From physical picture to duality). In the SUSY context of Lecture 3, this physical picture is made *rigorous* by holomorphy, the NSVZ exact beta function, and anomaly matching. In the non-SUSY context, we lose holomorphy and the exact beta function, but **anomaly matching survives** (it is a non-perturbative, UV/IR relation that does not require SUSY). Tumbling and complementarity provide the physical intuition for *why* a magnetic dual should exist; anomaly matching provides the *constraint* that any proposed dual must satisfy.

5.2 What this lecture provides for Lecture 6

The picture above—tumbling (dynamical mechanism) plus complementarity (phase equivalence) producing a duality (theory equivalence)—is the conceptual foundation for the non-SUSY duality programme. In Lecture 6 we will see how Sannino and collaborators used this framework, combined with anomaly matching, to conjecture that the Standard Model itself is the magnetic dual of a more fundamental electric theory.

Preview: Lecture 6

In Lecture 6 we cross from established non-perturbative phenomena (tumbling, complementarity) to **conjectural** non-SUSY duality extensions. The key question: can the physical picture of tumbling + complementarity, which is made rigorous by SUSY in Lectures 3–4, survive in the absence of supersymmetry?

The evidence is compelling but not conclusive: anomaly matching works, perturbative fixed points exist, and the framework produces the correct Standard Model structure. But non-SUSY duality has no independent cross-validation beyond anomaly matching—in contrast to the SUSY case, where holomorphy, moduli space matching, and the superconformal index all provide additional evidence.

Every non-SUSY duality statement in Lecture 6 will carry an explicit conjectural qualifier.

6 Comparison: Tumbling, Complementarity, and Duality

Three distinct concepts

These three ideas are often discussed together, but they are **logically distinct**. Conflating them is a common source of confusion.

	Tumbling	**Complementarity**	**Duality**
What it is	Dynamical mechanism	Phase-structure statement	Equivalence between theories
Operates on	A single theory	A single theory (two phases)	Two different theories
Key idea	Sequential condensation driven by MAC	Higgs phase = confinement phase (no phase transition)	Electric theory and magnetic theory have the same IR physics
Evidence level	Approximate (MAC heuristic)	Proven (for fundamental Higgs with complete breaking; [2])	Proven (SUSY, Lectures 3–4); conjectured (non-SUSY)
Uses	Identifies *which* condensate forms	Identifies confined composites with Higgs-phase particles	Maps the full spectrum and interactions between two theories
Limitations	One-gluon exchange at strong coupling; counterexamples exist	Requires fundamental Higgs and complete breaking	Non-SUSY case unproven

7 Summary

Lecture 5: Summary

1. **Chiral gauge theories** have fermion content that does not form a real representation; their strong dynamics can produce dynamical gauge symmetry breaking rather than the chiral symmetry breaking familiar from QCD.

2. The **MAC criterion** identifies the most attractive condensation channel via $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_c)$. It is a *heuristic* based on one-gluon exchange, not a theorem.

3. **Tumbling** is the sequential condensation mechanism: the MAC condenses, the gauge group breaks, and the process repeats. The SU(5) Georgi–Glashow model demonstrates the first step: $\mathrm{SU}(5) \to \mathrm{SU}(4)$ with $\Delta C_2 = 24/5$.

4. At each tumbling step, **'t Hooft anomaly matching** (from Lecture 1) constrains the surviving massless spectrum. Anomaly matching does not determine the dynamics, but any proposed tumbling pattern that violates it is excluded.

5. **Fradkin–Shenker complementarity:** for gauge theories with a fundamental-representation Higgs field and complete gauge breaking, the Higgs and confinement phases are analytically connected—there is no phase transition. This is a *theorem*, not a conjecture.

6. **Limitations of complementarity:** it does not apply universally. For adjoint Higgs, partial gauge breaking, or higher-representation scalars, phase transitions can separate the phases.

7. **Tumbling + complementarity** provides a physical picture motivating electric–magnetic duality: the confined composites of the electric theory are identified with the elementary fields of the magnetic (Higgs-phase) theory.

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