Lecture 6: Non-SUSY Duality Extensions

The Dualised Standard Model — Part III Lecture Notes

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Contents

1 What We Lose Without SUSY **2**

1.1 The four pillars of SUSY duality 2

1.2 What is lost 3

1.3 What survives 3

2 The Sannino–Schechter Toy Model (1997) **4**

2.1 The key insight 4

2.2 What this suggests—and what it does not prove 5

2.3 The Cheng–Shadmi tension 5

3 The Non-SUSY QCD Dual (Sannino, 2009) **5**

3.1 The electric theory 5

3.2 The conjectured magnetic dual 6

3.3 A non-trivial solution 6

3.4 Consistency checks 6

4 The Standard Model as a Magnetic Dual (Sannino, 2011) **7**

4.1 The SUSY precursor: Maekawa–Takahashi 7

4.2 The Pati–Salam embedding 7

4.3 Three conjectural consequences 7

5 Emergent SUSY at Fixed Points (Antipin–Mojaza–Pica–Sannino, 2011) **8**

5.1 The setup 8

5.2 The result: emergent SUSY 9

5.3 Limitations 9

5.4 Complementary large-N arguments 9

6 The Central Vulnerability **9**

6.1 The vulnerability stated 10

6.2 Then why study it? 10

6.3 Recent developments 11

7 Summary **12**

Conventions for Lecture 6

All conventions from Lectures 1–5 remain in force. No new conventions are introduced in this lecture.

Quantity	**Convention**	**Comment**
Units	$\hbar = c = 1$ (natural)	
Metric	$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$	mostly minus
Dynkin index	$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$	fundamental of $\mathrm{SU}(N_c)$
Cubic anomaly	$A(\square) = 1,\ A(\overline{\square}) = -1$	per fundamental Weyl
Beta function	$b_0 = (11N_c - 2N_f)/3$	N_f Dirac flavours
R-charge	$R[Q] = (N_f - N_c)/N_f$	SUSY reference point
N_f counting	Dirac flavour pairs	each = 2 Weyl fermions

Epistemic convention for this lecture. Every statement asserting or assuming non-supersymmetric duality carries an explicit conjectural qualifier: "if the duality holds," "assuming the conjectured duality," "conjectural," or equivalent. The anomaly matching *equations* presented below are exact; the *duality interpretation* of those equations is conjectural.

1 What We Lose Without SUSY

In Lectures 2–4 we constructed Seiberg duality for $\mathcal{N} = 1$ SQCD as a *proven* equivalence between two gauge theories. That proof rested on four independent pillars working in concert. Before extending duality beyond supersymmetry, we must be explicit about which of these pillars survive and which do not.

1.1 The four pillars of SUSY duality

(i) **Holomorphy of the superpotential** (Lecture 2). The superpotential W is a holomorphic function of the chiral superfields and is not renormalised beyond one loop. This non-renormalisation theorem [1] constrains the effective superpotential exactly, leaving the dynamical scale Λ as the only non-perturbative input.

(ii) **NSVZ exact beta function** (Lecture 2). The exact relation between the beta function, the anomalous dimensions, and the gauge coupling [2] determines the boundaries of the conformal window *exactly*: $\frac{3}{2}N_c < N_f < 3N_c$ for $\mathrm{SU}(N_c)$ with N_f fundamental flavours.

(iii) **Moduli space matching** (Lecture 3). The classical moduli space of flat directions is not lifted quantum mechanically (in the conformal window), and the electric and magnetic moduli spaces can be mapped one-to-one via the meson operator $M^i_j = Q^i \widetilde{Q}_j/\Lambda$. This provides an infinite family of consistency checks—one for each point on the moduli space.

(iv) **'t Hooft anomaly matching** (Lecture 1, verified in Lecture 3). All six independent anomaly coefficients—$\mathrm{SU}(N_f)^3_L$, $\mathrm{SU}(N_f)^3_R$, $\mathrm{SU}(N_f)^2_L\,\mathrm{U}(1)_B$, $\mathrm{SU}(N_f)^2_R\,\mathrm{U}(1)_B$, $\mathrm{U}(1)^3_B$, $\mathrm{U}(1)_B$-gravitational—match exactly between the electric and magnetic descriptions as polynomial identities in N_c and N_f (validated by `verify_anomaly_matching.py`, 7/7 symbolic checks).

1.2 What is lost

Without supersymmetry, three of the four pillars collapse:

(a) **Holomorphy:** There is no superpotential in a non-SUSY theory. The powerful non-renormalisation theorems that fix the form of W have no analogue. All couplings run and mix under renormalisation without holomorphic constraints.

(b) **Exact beta function:** The NSVZ formula relies on the superfield structure. Without SUSY, only the perturbative coefficients b_0, b_1, ... are available. The boundaries of the conformal window become *estimates* rather than exact results.

(c) **Moduli space:** Flat directions require cancellation of the scalar potential, which in SUSY theories is guaranteed by the F-term and D-term conditions. Without SUSY, quantum corrections generically lift all flat directions. There is no moduli space to match.

1.3 What survives

The sole surviving tool

Of the four pillars, **only 't Hooft anomaly matching survives** without SUSY. Anomaly matching is a non-perturbative, exact constraint that relates the UV and IR spectra of *any* gauge theory. It was derived by 't Hooft [3] in 1980 without any reference to supersymmetry and requires only:

(i) a UV gauge theory with a global symmetry group G_{global},

(ii) massless fermions charged under G_{global} in both the UV and IR descriptions.

This is exactly the tool we developed in Lecture 1 and applied in Lecture 3 to verify Seiberg duality. It remains available—and equally powerful—in the absence of SUSY.

SUSY vs. non-SUSY evidence for duality

Evidence type	**SUSY duality**	**Non-SUSY duality**
Holomorphy	✓ (exact)	×
Exact beta function (NSVZ)	✓ (exact)	×
Moduli space matching	✓ (infinite checks)	×
't Hooft anomaly matching	✓ (exact)	✓ (exact)
Superconformal index	✓ (independent check)	×
Total independent pillars	**5**	**1**

Summary: Non-SUSY duality proposals rest on anomaly matching alone. Anomaly matching is *necessary* but *not sufficient* for duality: multiple distinct IR spectra can satisfy the same UV anomaly conditions. Every non-SUSY duality statement in this lecture is therefore **conjectural**.

2 The Sannino–Schechter Toy Model (1997)

The first concrete evidence that SUSY duality structures might survive the breaking of supersymmetry came from a toy model by Sannino and Schechter [4].

2.1 The key insight

Sannino and Schechter started from the effective Lagrangian for $\mathcal{N} = 1$ super Yang–Mills theory (the Veneziano–Yankielowicz Lagrangian [5]) and asked: what happens if we add supersymmetry-breaking terms and integrate out the heavy superpartners (gluinos and squarks)?

Their key observation was that the ordinary QCD degrees of freedom—mesons and glueballs—*emerge from the auxiliary fields* of the supersymmetric effective Lagrangian. Specifically:

- The scalar and pseudoscalar glueball fields of QCD are contained in the auxiliary F-field of the chiral superfield S (see Eq. (2.4) of [4]):

$$F = \frac{g^2}{32\pi^2}\left[-\tfrac{1}{2}F_a^{mn}F_{mn,a} - \tfrac{i}{4}\epsilon_{mnrs}F_a^{mn}F_a^{rs} + \cdots\right]. \tag{1}$$

- The meson fields of QCD are hidden in the auxiliary field $(F_T)_{ij} \sim \psi_i\widetilde{\psi}_j$ of the meson superfield.

When SUSY is broken at a scale m and the superpartners are integrated out, these auxiliary fields become the *dynamical* degrees of freedom of the low-energy theory. Sannino and Schechter showed that the resulting effective Lagrangian, under suitable assumptions about the SUSY-breaking terms, reproduces features of the ordinary QCD effective Lagrangian.

2.2 What this suggests—and what it does not prove

The Sannino–Schechter model *suggests* that the duality structure encoded in the SUSY effective Lagrangian may persist, in some form, after SUSY breaking. The QCD-like degrees of freedom that emerge are not ad hoc additions but arise organically from the SUSY structure. Sannino and Schechter themselves described this as an "amusing feature" of their model [4].

However, the model does *not* prove that non-SUSY duality exists. The SUSY-breaking procedure involves arbitrary choices (the form of the breaking terms, the assumption that the dominant part of the effective Lagrangian retains a "tree-level holomorphicity"), and the resulting QCD Lagrangian depends on these choices. The model is a proof of concept, not a derivation.

2.3 The Cheng–Shadmi tension

Shortly after the Sannino–Schechter proposal, Cheng and Shadmi [6] performed a systematic analysis of Seiberg duality in the presence of soft SUSY-breaking terms. Their approach was to couple both the electric and magnetic theories to a common dynamical SUSY-breaking (DSB) sector and track the induced soft masses through the duality map.

Cheng–Shadmi tension (unresolved)

Cheng and Shadmi found that in the limit of **large SUSY breaking**—precisely the limit relevant for obtaining non-SUSY QCD from broken SQCD—the magnetic scalars generically acquire **negative mass-squared**. This destabilises the magnetic vacuum: the magnetic theory does not have a stable ground state in the regime where the electric theory reduces to ordinary QCD.

Specifically, assuming a canonical Kähler potential for the magnetic theory (the simplest and most natural choice), they showed that the soft scalar masses satisfy $m^2_{\text{scalar}} < 0$ for many cases in the conformal window [6]. The resulting symmetry breaking pattern in the magnetic theory is **incompatible** with the expected chiral symmetry breaking pattern of QCD.

This tension is unresolved. Sannino's subsequent programme (Sections 3–5) *sidesteps* rather than *resolves* the Cheng–Shadmi problem by conjecturing non-SUSY duality directly, without deriving it from broken SUSY. The Cheng–Shadmi result remains a cautionary example of what can go wrong when SUSY-derived structures are extrapolated to the non-SUSY regime.

3 The Non-SUSY QCD Dual (Sannino, 2009)

Rather than deriving non-SUSY duality from broken SUSY—an approach plagued by the Cheng–Shadmi tension—Sannino [7] proposed in 2009 to *conjecture* non-SUSY duality directly and check it for consistency via anomaly matching.

3.1 The electric theory

The starting point is ordinary QCD-like physics: an SU(N_c) gauge theory (specialised to $N_c = 3$ for QCD) with N_f Dirac flavours of fundamental quarks. The quantum global symmetry group is:

$$G_{\text{global}} = \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \,, \tag{2}$$

where the classical axial $\mathrm{U}(1)_A$ is broken by the Adler–Bell–Jackiw anomaly (as discussed in Lecture 1).

3.2 The conjectured magnetic dual

Sannino sought a *magnetic* gauge theory with gauge group $\mathrm{SU}(X)$—where X is to be determined by anomaly matching—and a matter content of magnetic quarks q, $\widetilde{q}$ plus gauge-singlet fermions identifiable as baryons built from the electric quarks. The key constraint is that the magnetic spectrum must reproduce *all* independent 't Hooft anomaly coefficients of the electric theory:

$$\mathrm{SU}(N_f)^3_{L/R}\,, \qquad \mathrm{SU}(N_f)^2_{L/R}\,\mathrm{U}(1)_V\,. \tag{3}$$

(For a vector-like theory, these are the independent anomaly types; *cf.* Lecture 1.)

3.3 A non-trivial solution

Sannino found a novel solution to the anomaly matching conditions with [7]:

$$\boxed{X \;=\; 2N_f - 5N\,,} \tag{4}$$

where $N = N_c = 3$ for QCD, giving a dual gauge group $\mathrm{SU}(2N_f - 15)$. (For $N_f = 9$ and $N = 3$, the magnetic gauge group is SU(3)—the same as the electric theory, though with different matter content and interactions.)

The solution includes magnetic quarks in the fundamental of $\mathrm{SU}(X)$ plus gauge-singlet Weyl fermions in specific representations of the global symmetry group, with multiplicities and $\mathrm{U}(1)_V$ charges determined by the anomaly matching equations. The full spectrum is tabulated in Table III of [7].

3.4 Consistency checks

If the conjectured duality holds, Sannino's solution passes several non-trivial consistency checks:

(a) **Anomaly matching:** All independent 't Hooft anomaly coefficients match between the electric and magnetic descriptions. The anomaly matching equations are polynomials in N_f and N, and the solution satisfies them as identities.

(b) **Conformal window prediction:** Requiring asymptotic freedom of the dual theory gives a lower bound on N_f:

$$N_f \;\geq\; \frac{11}{4}\,N\,, \tag{5}$$

which for $N = 3$ gives $N_f \geq 8.25$. Remarkably, this *coincides* with the lower boundary of the conformal window predicted by the all-orders conjectured beta function of Ryttov and Sannino [8] when the anomalous dimension of the mass operator takes the value $\gamma = 2$.

(c) **Involution:** Dualising the magnetic theory recovers the electric gauge group structure—the duality is its own inverse [7].

Anomaly matching is necessary but not sufficient

The existence of a consistent anomaly matching solution is **evidence for** duality, not **proof of** it. The anomaly matching conditions are a *finite* set of polynomial equations in the multiplicities and quantum numbers of the magnetic spectrum. Multiple distinct solutions can satisfy the same conditions [7].

Sannino's solution is distinguished by additional physics requirements (involution, flavour decoupling, consistency with the conjectured beta function), but these are *consistency checks*, not derivations. The duality itself remains a conjecture.

4 The Standard Model as a Magnetic Dual (Sannino, 2011)

In 2011, Sannino [9] extended the QCD dual framework to the full Standard Model by embedding it in a Pati–Salam-like structure. *If the non-SUSY duality holds*, the Standard Model can be viewed as the *magnetic dual* of a purely fermionic electric theory.

4.1 The SUSY precursor: Maekawa–Takahashi

The direct SUSY precursor of this construction was provided by Maekawa and Takahashi [10], who constructed the Seiberg dual of a supersymmetric theory with Pati–Salam gauge group $\mathrm{SU}(4)_{\mathrm{PS}}$. In the SUSY context, their construction naturally produces:

- A top quark Yukawa coupling of order unity (from the magnetic superpotential $W_{\mathrm{mag}} \sim M\, q\, \widetilde{q}$), explaining the large top mass without fine-tuning.
- A natural μ-parameter for the Higgs sector.

Sannino's 2011 proposal extends the Maekawa–Takahashi construction to non-SUSY by replacing the SUSY consistency arguments with anomaly matching.

4.2 The Pati–Salam embedding

The Standard Model fermion content—quarks and leptons—is naturally embedded in a Pati–Salam SU(4) framework by treating the lepton number as a fourth colour [11]. In Sannino's construction, the **magnetic** (SM-side) description includes [9]:

- Quarks and leptons: p and $\widetilde{p}$ in the fundamental and antifundamental of $\mathrm{SU}(4)_{\mathrm{PS}}$, carrying flavour indices under the global symmetry group.
- The Higgs field H: transforming as a bifundamental of the flavour group—*not* as an elementary scalar of the UV theory, but as a **composite** built from electric fermions.
- An adjoint Weyl fermion λ_m of $\mathrm{SU}(4)_{\mathrm{PS}}$.
- A meson field M: a scalar transforming as a bifundamental of the flavour group, mediating interactions between the magnetic quarks and the gauge-singlet fermions.

4.3 Three conjectural consequences

If the SM is the magnetic dual of a Pati–Salam-like electric theory, three remarkable consequences follow:

(a) **No elementary scalars in the UV = no hierarchy problem.** The electric theory contains *only fermions and gauge bosons*—no fundamental scalars. The Higgs doublets of the SM are composite "mesons" of the electric theory ($H \sim Q\lambda\lambda\widetilde{Q}$ schematically), analogous to the meson M of Seiberg duality. Since the electric theory has no scalars, there are no quadratic divergences in the fundamental description, and the hierarchy problem is, if the duality holds, an artefact of the magnetic (composite) description.

(b) $N_{\mathrm{gen}} \geq 3$ **from the dual gauge group.** The dual *electric* gauge group is $\mathrm{SU}(2n_g - 4) = \mathrm{SU}(N_f - 4)$, where n_g is the number of SM generations and $N_f = 2n_g$ [9]. For this group to be non-abelian (a necessary condition for a non-trivial gauge theory), we need:

$$2n_g - 4 \ \geq\ 2 \qquad \Longrightarrow \qquad \boxed{n_g \ \geq\ 3\,.} \tag{6}$$

If the duality holds, the existence of at least three generations of quarks and leptons is not an accident but a *mathematical requirement* for the electric dual to be a non-trivial gauge theory. Requiring asymptotic freedom of the magnetic theory further constrains $n_g \leq 6$ [9].

(c) **Common Yukawa origin.** The Yukawa couplings of the SM arise from the magnetic interaction Lagrangian, which has the schematic form $\mathcal{L}_{\mathrm{mag}} \supset Y\, q\, M\, \widetilde{q}$—the non-SUSY analogue of the SUSY magnetic superpotential $W_{\mathrm{mag}} = M\, q\, \widetilde{q}/\mu$. *If the duality holds*, all Yukawa couplings share a common dynamical origin rather than being independent free parameters.

Remark 4.1 (Why "if the duality holds" matters). Each of the three consequences above is conditional on the *conjectured* non-SUSY duality being correct. Without a proof of duality, they are **predictions of the framework**, not established results. They are compelling because they address genuine puzzles of the SM (hierarchy problem, number of generations, Yukawa structure), but their validity depends entirely on whether the underlying duality conjecture is correct.

5 Emergent SUSY at Fixed Points (Antipin–Mojaza–Pica–Sannino, 2011)

The most striking piece of perturbative evidence for non-SUSY duality was provided by Antipin, Mojaza, Pica, and Sannino [12], who showed that *supersymmetric fixed points can emerge from non-supersymmetric RG flow* in the magnetic dual theory.

5.1 The setup

The magnetic dual theory of Section 3, augmented by a Weyl adjoint fermion λ_m (as in the SM-as-magnetic construction of Section 4), has the quantum global symmetry:

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{AF}\,, \tag{7}$$

where $\mathrm{U}(1)_{AF}$ is an anomaly-free axial-like symmetry under which the adjoint fermion is charged.

The theory has multiple couplings: the gauge coupling α_g, Yukawa-type couplings α_y mediating interactions between the magnetic quarks and the scalar mesons, and quartic scalar couplings. In the Veneziano limit ($N_c, N_f \to \infty$ with N_f/N_c fixed), the perturbative beta functions for all these couplings can be computed systematically.

5.2 The result: emergent SUSY

Antipin *et al.* classified all perturbatively accessible fixed points of the coupled beta function system and discovered that among a large number of non-supersymmetric fixed points, there exist fixed points where [12]:

(i) the gauge coupling, Yukawa couplings, and quartic scalar couplings all take *precisely the values predicted by supersymmetry*;

(ii) these SUSY fixed points are reached by the RG flow starting from *generic non-supersymmetric* initial conditions—no fine-tuning of bare couplings is required;

(iii) the SUSY fixed points have finite basins of attraction in the full coupling-constant space.

The perturbative analysis *suggests* that supersymmetry can emerge dynamically as an IR phenomenon: a non-SUSY UV Lagrangian flows to a SUSY fixed point without any SUSY being imposed at the fundamental level. This supports the "quasi-supersymmetric" interpretation of the magnetic spectrum [9]: even without imposing SUSY, the IR dynamics organises fields into SUSY-like multiplets at the fixed point.

5.3 Limitations

Perturbative result in the Veneziano limit

The emergent SUSY result of Antipin *et al.* is a **perturbative** result valid in the **Veneziano limit**: $N_c, N_f \to \infty$ with N_f/N_c held fixed. The perturbative analysis is reliable because, in this limit, the fixed point couplings are parametrically small (controlled by $\epsilon = N_f^{\mathrm{AF}} - N_f \ll N_f^{\mathrm{AF}}$).

Whether emergent SUSY survives at **finite** N_c and N_f —and in particular at the SM values $N_c = 3$, $N_f = 6$ (three generations)—is an **open question**. Finite-N corrections to the beta functions are uncontrolled and could qualitatively change the fixed-point structure.

The claim that SUSY emerges at the SM point extrapolates a perturbative large-N result to a regime where it is not controlled. This does not invalidate the result—it means that the result is *suggestive*, not *conclusive*.

5.4 Complementary large-N arguments

A complementary approach to non-SUSY duality at large N was provided by Armoni, Shifman, and Unsal [13], who used *orientifold planar equivalence* to argue that certain non-SUSY gauge theories share the same planar limit as their SUSY counterparts. This provides another piece of evidence for non-SUSY duality, but again limited to the large-N regime. Whether the planar equivalence extends to finite N is unknown.

6 The Central Vulnerability

We now state, as the concluding point of this lecture, the **central vulnerability** of the non-SUSY duality programme. This is not a footnote or caveat—it is the most important intellectual content of the lecture.

6.1 The vulnerability stated

Central vulnerability of non-SUSY duality

Non-SUSY duality has no independent cross-validation beyond 't Hooft anomaly matching.

In SUSY, duality is established by **four independent pillars** working together (§1): anomaly matching, holomorphy, moduli space matching, and the NSVZ exact beta function (plus the superconformal index as a fifth check). The convergence of multiple independent lines of evidence is what makes SUSY duality *proven*.

In non-SUSY, **only anomaly matching survives**. The construction *postulates* the duality and checks consistency via anomaly matching. It does not *prove* the duality.

The Sannino programme is the simplest *consistent* framework that:

(i) satisfies all 't Hooft anomaly conditions,

(ii) passes the involution and flavour decoupling tests,

(iii) reproduces the correct SM gauge group and matter content,

(iv) predicts $n_g \geq 3$ and offers a mechanism for the hierarchy problem.

But **consistency is not proof**. Anomaly matching is necessary but not sufficient: multiple IR spectra can satisfy the same UV anomaly conditions.

6.2 Then why study it?

A student who has followed the rigorous development of SUSY duality in Lectures 2–4 may reasonably ask: "If non-SUSY duality is unproven, why are we studying it?"

The answer involves four considerations:

(a) **Anomaly matching is highly constraining.** While not sufficient by itself, anomaly matching is an extremely powerful consistency condition. The number of solutions that simultaneously satisfy all anomaly conditions, pass the involution test, and allow flavour decoupling is small. Finding even one consistent solution is non-trivial.

(b) **The SUSY evidence is suggestive.** Seiberg duality for SQCD is proven. The physical picture of tumbling plus complementarity (Lecture 5) provides a mechanism that does not intrinsically require SUSY. The Sannino–Schechter toy model (§2) shows that QCD-like degrees of freedom emerge naturally from the SUSY structure. The emergent SUSY result (§5) suggests that the fixed-point structure survives beyond SUSY. None of this is proof, but collectively it makes the conjecture *well-motivated*.

(c) **The framework produces non-trivial predictions.** *If the duality holds*, it explains the hierarchy problem (no fundamental scalars), predicts $n_g \geq 3$ (non-abelian electric gauge group), and provides a common origin for Yukawa couplings. These are testable consequences of the framework, not tautologies.

(d) **Belief is not proof.** We study the framework *because* it is the most economical and well-motivated conjecture consistent with all available evidence—not because it is established. Independent validation is needed:

- **Lattice:** Direct numerical verification of the non-SUSY conformal window structure and the duality map.
- **New formal tools:** Non-perturbative methods beyond anomaly matching that could provide additional cross-checks.
- **Experimental signatures:** Predictions of the dualised SM (composite Higgs, extra matter at the Pati–Salam scale) that could be tested at future colliders.

6.3 Recent developments

The non-SUSY duality programme remains an active area of research. Bi and Terning [14] have explored non-SUSY duality candidates for chiral gauge theories with Spin(8) gauge group, verifying 21 independent anomaly matching conditions. Their work demonstrates that the programme extends beyond QCD-like (vector-like) theories to chiral gauge theories—a broader and more challenging setting.

The backbone paper of Cacciapaglia and Sannino [15] provides the most comprehensive treatment of the dualised Standard Model, including the quasi-supersymmetric spectrum and quantitative IR matching, which we will develop in Lectures 7–8.

Preview: Lectures 7–8

In Lectures 7–8, we will *assume* the non-SUSY duality conjecture and construct the dualised Standard Model following Cacciapaglia and Sannino [15]. We will:

- Specify the electric theory (Pati–Salam gauge group, purely fermionic matter content).
- Construct the magnetic dual and identify the SM fields as composites.
- Derive the quasi-supersymmetric spectrum and check anomaly matching for the full $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge group.
- Compute the fermion mass relations and compare with observation.

Every prediction derived in Lectures 7–8 is conditional on the conjectured non-SUSY duality being correct.

7 Summary

Lecture 6: Summary

1. **Without SUSY, three of four duality pillars are lost:** holomorphy, the exact beta function, and moduli space matching. Only 't Hooft anomaly matching survives—exact, non-perturbative, and requiring no SUSY (§1).

2. **Sannino–Schechter (1997):** QCD degrees of freedom emerge from the auxiliary fields of the SUSY effective Lagrangian, *suggesting* that duality structures may survive SUSY breaking (§2).

3. **Cheng–Shadmi tension (1998):** Seiberg duality with large SUSY breaking produces tachyonic magnetic scalars—an **unresolved** tension. Sannino's programme sidesteps this by conjecturing non-SUSY duality directly (§2.3).

4. **QCD dual (Sannino, 2009):** Non-trivial anomaly matching solutions exist for non-SUSY QCD with a dual gauge group $\mathrm{SU}(2N_f - 5N)$. The conformal window prediction *coincides* with the all-orders conjectured beta function (§3). *If the duality holds*, this provides a dual description of QCD.

5. **SM-as-magnetic (Sannino, 2011):** *If the non-SUSY duality holds*, the SM is the magnetic dual of a purely fermionic electric theory with Pati–Salam structure. Consequences: no hierarchy problem, $n_g \geq 3$, common Yukawa origin. Maekawa–Takahashi [10] provided the direct SUSY precursor (§4).

6. **Emergent SUSY (Antipin–Mojaza–Pica–Sannino, 2011):** SUSY fixed points emerge from non-SUSY RG flow in the Veneziano limit. This perturbative result *suggests* that the quasi-SUSY spectrum is dynamically natural, but finite-N validity is uncontrolled (§5).

7. **Central vulnerability:** Non-SUSY duality has **no independent cross-validation beyond anomaly matching**. The construction postulates the duality and checks consistency. Consistency is not proof (§6).

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