Lecture 7: The Dualised Standard Model

The Dualised Standard Model — Part III Lecture Notes

Lent Term 2027

Contents

Conventions for Lecture 7

All conventions from Lectures 1–6 remain in force. New conventions for the Pati–Salam embedding and hypercharge identification are introduced below.

Quantity	**Convention**	**Comment**
Units	$\hbar = c = 1$ (natural)	
Metric	$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$	mostly minus
Dynkin index	$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$	fundamental of $\mathrm{SU}(N_c)$
Cubic anomaly	$A(\Box) = 1,\ A(\overline{\Box}) = -1$	per fundamental Weyl
Beta function	$b_0 = (11N_c - 2N_f)/3$	N_f Dirac flavours
N_f counting	Dirac flavour pairs	each $= 2$ Weyl fermions
New in this lecture:		
Hypercharge	$Y = T^3_R + \frac{1}{6}\,Q_V$	Eq. (6) of [1]
Duality map	$X = N_f - N$	magnetic gauge group $\mathrm{SU}(X)$
Epistemic	"if the duality holds"	all non-SUSY claims (P1)

Epistemic convention. Every statement asserting or assuming non-supersymmetric duality carries an explicit conjectural qualifier. The field content *tables* and anomaly matching *equations* are exact group-theoretic results; the *duality interpretation* is conjectural.

1 The General Duality Framework

In Lecture 6, we introduced the conjectured non-SUSY duality programme and stated its central vulnerability: anomaly matching is the sole surviving consistency check. We now *apply* this framework to construct the dualised Standard Model, following Cacciapaglia and Sannino [1].

1.1 The electric theory

The starting point is an $\mathrm{SU}(N)$ gauge theory with the following matter content [1]:

- N_f Dirac fermions Q^i, $\widetilde{Q}_i$ ($i = 1, \ldots, N_f$) in the fundamental and antifundamental representations of $\mathrm{SU}(N)$;
- One Weyl fermion λ in the adjoint representation of $\mathrm{SU}(N)$.

This is the *non-SUSY cousin* of $\mathcal{N} = 1$ SQCD with an adjoint: the same fermion content as the SUSY theory, but without squarks or sleptons as fundamental fields. The quantum global symmetry group is:

$$G_{\text{global}} \;=\; \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{AF}\,, \tag{1}$$

where $\mathrm{U}(1)_V$ is the vector (baryon-number) symmetry acting as $Q \to e^{i\alpha}Q$, $\widetilde{Q} \to e^{-i\alpha}\widetilde{Q}$, and $\mathrm{U}(1)_{AF}$ is an anomaly-free axial symmetry under which the adjoint fermion λ is charged.

Electric theory (UV)					
Fields	SU(N)	SU(N_f)$_L$	SU(N_f)$_R$	U(1)$_V$	U(1)$_{AF}$
λ	Adj	$\mathbf{1}$	$\mathbf{1}$	0	1
Q	F	F	$\mathbf{1}$	1	$-N/N_f$
$\widetilde{Q}$	$\bar{F}$	$\mathbf{1}$	$\bar{F}$	-1	$-N/N_f$

Table 1: Electric theory field content (Table I, top block, of [1]). F denotes the fundamental representation, $\bar{F}$ the antifundamental, and Adj the adjoint.

Key feature: The electric theory contains **no elementary scalars**. All gauge fields and matter fields are fermions (and gauge bosons). *If the duality holds*, all scalars in the low-energy description are composites.

1.2 The conjectured magnetic dual

If the conjectured non-SUSY duality of Lecture 6 holds, the magnetic (IR) description is an SU(X) gauge theory with [1]

$$\boxed{X \;=\; N_f - N\,,} \tag{2}$$

and the following quasi-supersymmetric spectrum (Table I, bottom block, of [1]). The term "quasi-supersymmetric" reflects the organisation of the magnetic degrees of freedom into multiplet-like pairs reminiscent of SUSY, even though *no* supersymmetry is imposed [5]:

$$(G_\mu,\,\lambda_m)\,,\qquad (q,\,\phi)\,,\qquad (\widetilde{q},\,\widetilde{\phi})\,,\qquad (M,\,\Phi_H)\,. \tag{3}$$

Each pair consists of a fermion and a scalar partner. The complete quantum numbers are:

Magnetic theory (IR)					
Fields	SU(X)	SU(N_f)$_L$	SU(N_f)$_R$	U(1)$_V$	U(1)$_{AF}$
λ_m (gaugino)	Adj	$\mathbf{1}$	$\mathbf{1}$	0	1
q (mag. quark)	F	$\bar{F}$	$\mathbf{1}$	N/X	$-X/N_f$
$\widetilde{q}$ (mag. antiquark)	$\bar{F}$	$\mathbf{1}$	F	$-N/X$	$-X/N_f$
M (mesino)	$\mathbf{1}$	F	$\bar{F}$	0	$-1+2X/N_f$
ϕ (squark)	F	$\bar{F}$	$\mathbf{1}$	N/X	$1-X/N_f$
$\widetilde{\phi}$ (anti-squark)	$\bar{F}$	$\mathbf{1}$	F	$-N/X$	$1-X/N_f$
Φ_H (mes-Higgs)	$\mathbf{1}$	F	$\bar{F}$	0	$2X/N_f$

Table 2: Magnetic theory field content (Table I, bottom block, of [1]). **Fermions:** λ_m, q, $\widetilde{q}$, M. **Scalars:** ϕ, $\widetilde{\phi}$, Φ_H. The naming reflects the quasi-SUSY pairing (3).

The *magnetic interactions*, *if the duality holds*, take the form [1]:

$$\mathcal{L}_m \;\supset\; y\,q\,\widetilde{q}\,\Phi_H \,+\, y'\,q\,\widetilde{\phi}\,M \,+\, \widetilde{y}'\,\widetilde{q}\,\phi\,M \,+\, \xi_L\,\lambda_m\,q\,\phi^\dagger \,+\, \xi_R\,\lambda_m\,\widetilde{q}\,\widetilde{\phi}^\dagger \,+\, \text{h.c.} \,+\, \text{scalar quartics}\,, \tag{4}$$

where the first three terms emerge from a superpotential coupling if $y = y' = \widetilde{y}'$, and the last two from gauge interactions if $\xi_L = \xi_R = g_m$ (Eq. (4) of [1]). The **universal Yukawa coupling** y is a central prediction of the framework.

1.3 Composite fields as electric bilinears

The scalar fields in the magnetic theory are naturally interpreted as composite states of the electric theory [1]:

$$M \;=\; \langle \widetilde{Q}\,\lambda\,Q\rangle\,, \qquad \Phi_H \;=\; \langle \widetilde{Q}\,\lambda\,\lambda\,Q\rangle\,. \tag{5}$$

The mesino M is a three-fermion composite, while the mes-Higgs Φ_H is a four-fermion composite. **The Higgs field is not an elementary scalar**—it is a deeply composite state of the electric theory. This is the non-SUSY analogue of the Seiberg duality meson $M \sim Q\widetilde{Q}$ from Lecture 3, but with the adjoint fermion λ providing additional "glue."

Remark 1.1 (Connection to Seiberg duality). Compare with Lecture 3: in SUSY Seiberg duality, the meson $M^i_j = Q^i\widetilde{Q}_j/\Lambda$ is a two-fermion composite. The non-SUSY version requires the adjoint fermion λ to "saturate" the additional quantum numbers. The quasi-SUSY multiplet structure (3) mirrors the SUSY structure (vector multiplet, chiral multiplets, meson multiplet) even though no supersymmetry is imposed.

2 Pati–Salam Embedding and Hypercharge

The general duality framework of Section 1 applies to any SU(N) gauge theory. To connect it to the Standard Model, we must embed the SM gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ within the duality structure. Following Sannino [2] and Cacciapaglia and Sannino [1], we use the Pati–Salam embedding [3].

2.1 Lepton number as the fourth colour

Pati and Salam [3] observed that quarks and leptons can be unified by extending the colour group $\mathrm{SU}(3)_c$ to SU(4), with the lepton as the "fourth colour":

$$\psi_C \;=\; (u, d, s, \ldots, \nu_e, e, \ldots)_C\,, \qquad C = 1, 2, 3, 4\,, \tag{6}$$

where $C = 1, 2, 3$ are the ordinary QCD colours and $C = 4$ is the "lepton colour." Under the decomposition $\mathrm{SU}(4) \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$, the fundamental **4** splits as:

$$\mathbf{4} \;\to\; \mathbf{3}_{+1/3} \;\oplus\; \mathbf{1}_{-1}\,, \tag{7}$$

where the subscript is the $B - L$ charge (baryon number minus lepton number), normalised so that quarks have $B - L = +1/3$ and leptons have $B - L = -1$.

2.2 Flavour symmetry decomposition

The flavour symmetry of the electric theory (1) contains $\mathrm{SU}(N_f)_{L/R}$. For the SM embedding, we decompose the flavour group as [1]:

$$\mathrm{SU}(N_f)_{L/R} \;\supset\; \mathrm{SU}(n_g) \times \mathrm{SU}(2)_{L/R}\,, \tag{8}$$

where $n_g = N_f/2$ is the number of SM generations and $\mathrm{SU}(2)_R$ is (part of) the right-handed isospin symmetry. For three generations, $n_g = 3$ and $N_f = 2n_g = 6$.

The electroweak gauge group $\mathrm{SU}(2)_L$ is identified with the left-handed factor, and $\mathrm{SU}(2)_R$ provides the quantum number T^3_R needed to define hypercharge.

2.3 Hypercharge identification

The SM hypercharge is defined as [1]:

$$\boxed{Y \;=\; T_R^3 \;+\; \frac{1}{6}\,Q_V\,,} \tag{9}$$

where T_R^3 is the third component of $\mathrm{SU}(2)_R$ isospin and Q_V is the $\mathrm{U}(1)_V$ charge from (1).

Hypercharge check: reproducing SM values

Formula (9) *must* reproduce the standard SM hypercharges. We verify this explicitly.

Under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, the SM fermions are organised as left-handed doublets ($T_R^3 = 0$) and right-handed singlets ($T_R^3 = \pm\frac{1}{2}$). The Q_V charge equals $+1$ for quarks (Q) and -3 for leptons added as elementary states for anomaly cancellation.

SM field	T_R^3	Q_V	$Y = T_R^3 + Q_V/6$	**SM value**
$q_L = (u_L, d_L)$	0	$+1$	$0 + 1/6 = +1/6$	$+1/6$ ✓
u_R	$+1/2$	$+1$	$1/2 + 1/6 = +2/3$	$+2/3$ ✓
d_R	$-1/2$	$+1$	$-1/2 + 1/6 = -1/3$	$-1/3$ ✓
$\ell_L = (\nu_L, e_L)$	0	-3	$0 - 3/6 = -1/2$	$-1/2$ ✓
ν_R	$+1/2$	-3	$1/2 - 3/6 = 0$	0 ✓
e_R	$-1/2$	-3	$-1/2 - 3/6 = -1$	-1 ✓

All six SM hypercharges are reproduced exactly. The relation to the alternative convention $Y = T_R^3 + (B-L)/2$ is $Q_V = 3(B-L)$.

2.4 The generation bound

The electric gauge group is $\mathrm{SU}(2n_g - 4)$, where n_g is the number of generations [1, 2]. For this group to be non-abelian (a necessary condition for a non-trivial gauge theory), we require:

$$2n_g - 4 \;\geq\; 2 \qquad \Longrightarrow \qquad \boxed{n_g \;\geq\; 3\,.} \tag{10}$$

If the duality holds, the existence of at least three generations of quarks and leptons is **not an accident**: it is a **mathematical requirement** for the electric dual to be a non-trivial gauge theory [2].

For $n_g = 3$ (three generations):

- Electric gauge group: $\mathrm{SU}(2 \cdot 3 - 4) = \mathrm{SU}(2)$.
- Number of Dirac flavours: $N_f = 2n_g = 6$.
- Magnetic gauge group (from QCD duality): $X = N_f - N = 6 - 3 = 3$, but only $\mathrm{SU}(3)_c$ (the QCD subgroup of $\mathrm{SU}(4)_{\mathrm{PS}}$) is gauged in the SM. The full Pati–Salam SU(4) is the electric theory containing both quarks and leptons.

Remark 2.1 (Upper bound on generations). Requiring asymptotic freedom of the magnetic theory further constrains $n_g \leq 6$ [2]. The observed value $n_g = 3$ is the *minimal* number of generations consistent with the duality framework.

3 The Magnetic Spectrum: SM Fields and Quasi-SUSY Partners

We now specialise the general magnetic field content of Table 2 to the SM values $N = 3$, $N_f = 6$ (three generations), giving $X = N_f - N = 3$. Following Table II of [1], the full magnetic spectrum under $\mathrm{SU}(3)_c \times \mathrm{SU}(6)_L \times \mathrm{SU}(6)_R \times \mathrm{U}(1)_V \times \mathrm{U}(1)_{AF}$ is:

Electric theory (UV): SU(3) **with** $N_f = 6$					
Fields	SU(3)	$\mathrm{SU}(6)_L$	$\mathrm{SU}(6)_R$	$\mathrm{U}(1)_V$	$\mathrm{U}(1)_{AF}$
λ	Adj	$\mathbf{1}$	$\mathbf{1}$	0	1
Q	F	F	$\mathbf{1}$	1	$-1/2$
$\widetilde{Q}$	$\bar{F}$	$\mathbf{1}$	$\bar{F}$	-1	$-1/2$
+ Elementary states for anomaly cancellation:					
L	$\mathbf{1}$	F	$\mathbf{1}$	-3	0
$\widetilde{L}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{F}$	3	0

Table 3: Electric theory field content for $N = 3$, $N_f = 6$ (Table II, top block, of [1]). The lepton fields L, $\widetilde{L}$ are elementary states required for $\mathrm{U}(1)_Y$ anomaly cancellation.

Magnetic theory (IR): $\mathrm{SU}(3)_c$ **with** $N_f = 6$					
Fields	$\mathrm{SU}(3)_c$	$\mathrm{SU}(6)_L$	$\mathrm{SU}(6)_R$	$\mathrm{U}(1)_V$	$\mathrm{U}(1)_{AF}$
λ_m (gaugino)	Adj	$\mathbf{1}$	$\mathbf{1}$	0	1
q (mag. quarks)	F	$\bar{F}$	$\mathbf{1}$	1	$-1/2$
$\widetilde{q}$ (mag. antiquarks)	$\bar{F}$	$\mathbf{1}$	F	-1	$-1/2$
$l \equiv L$ (leptons)	$\mathbf{1}$	F	$\mathbf{1}$	-3	0
$\widetilde{l} \equiv \widetilde{L}$ (antileptons)	$\mathbf{1}$	$\mathbf{1}$	$\bar{F}$	3	0
M (mesino)	$\mathbf{1}$	F	$\bar{F}$	0	0
ϕ (squarks)	F	$\bar{F}$	$\mathbf{1}$	1	1/2
$\widetilde{\phi}$ (anti-squarks)	$\bar{F}$	$\mathbf{1}$	F	-1	1/2
Φ_H (mes-Higgs)	$\mathbf{1}$	F	$\bar{F}$	0	1

Table 4: Magnetic theory field content for $N = 3$, $N_f = 6$ (Table II, bottom block, of [1]). **Fermions:** λ_m, q, $\widetilde{q}$, l, $\widetilde{l}$, M. **Scalars:** ϕ, $\widetilde{\phi}$, Φ_H.

3.1 Identification of SM fields

The magnetic spectrum of Table 4 contains **all SM fields plus additional quasi-SUSY partners**:

SM quarks and leptons. The magnetic quarks q, $\widetilde{q}$ are identified with the SM quarks. Under the flavour decomposition (8), each q ($\bar{F}$ of $\mathrm{SU}(6)_L$) decomposes into an $\mathrm{SU}(3)_{\text{gen}}$ triplet (three generations) of $\mathrm{SU}(2)_L$ doublets and singlets, reproducing the SM quark content. The leptons

$l \equiv L$ and $\widetilde{l} \equiv \widetilde{L}$ are elementary states—they are **not** composites of the electric theory in the pure QCD duality.

Leptons in the QCD dual vs. Pati–Salam

In the **QCD-sector duality** (Table 4), leptons l, $\widetilde{l}$ are **elementary** states added for $\mathrm{U}(1)_Y$ anomaly cancellation. They are colour singlets and do not participate in the $\mathrm{SU}(3)_c$ dynamics.
In the **full Pati–Salam extension** [2], leptons are the $C = 4$ component of SU(4) fundamentals, and the distinction between quarks and leptons is a consequence of $\mathrm{SU}(4) \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$ breaking.
This lecture follows the QCD-sector duality of [1], where leptons are elementary.

The 18 Higgs doublets. The mes-Higgs field Φ_H is a gauge singlet that transforms as $(\mathbf{6}, \bar{\mathbf{6}})$ under $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$. Under the flavour decomposition (8), Φ_H decomposes into a matrix of bi-doublets:

$$\Phi_H \;=\; \{H_{ij}\}\,, \qquad i,j = 1,2,3\,, \tag{11}$$

where the indices i,j label the three generations (from $\mathrm{SU}(3)_{\mathrm{gen}}$) and each H_{ij} is a complex bi-doublet of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Each bi-doublet decomposes into two $\mathrm{SU}(2)_L$ doublets distinguished by hypercharge [1]:

$$H_{ij} \;=\; (H^u_{ij},\; H^d_{ij})\,, \tag{12}$$

where H^u_{ij} has $Y = +1/2$ and H^d_{ij} has $Y = -1/2$. The total Higgs content is therefore:

$$\underbrace{\text{9 bi-doublets}}_{i,j=1,2,3} \;\times\; \underbrace{\text{2 doublets per bi-doublet}}_{H^u,\,H^d} \;=\; \boxed{\text{18 Higgs doublets}\,.} \tag{13}$$

One of these—$H_0 \equiv H^u_{33}$—acquires the electroweak VEV $\langle H_0 \rangle \approx v$; the remaining 17 have positive mass-squared at tree level (they are "dormant" Higgs doublets).

Quasi-SUSY partners. The magnetic spectrum includes states *beyond* the minimal SM content:

- **Gaugino** λ_m: a colour-octet Weyl fermion (Adj of $\mathrm{SU}(3)_c$), the quasi-SUSY partner of the gluon.
- **Squarks** ϕ, $\widetilde{\phi}$: colour-triplet and anti-triplet *scalars*, the quasi-SUSY partners of q and $\widetilde{q}$.
- **Mesino** M: a colour-singlet fermion transforming as $(\mathbf{6}, \bar{\mathbf{6}})$ under $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$, the quasi-SUSY partner of Φ_H.

If the duality holds, these states are expected to appear at the scale $\Lambda_S \sim$ few TeV, where the quasi-supersymmetry of the magnetic theory becomes manifest [1]. Their masses are generically required to be at or above collider bounds (the gaugino and squarks are constrained by LHC searches to be above roughly 2 TeV [1]).

3.2 The universal Yukawa coupling

The magnetic Lagrangian (4) generates a **single universal Yukawa coupling** y between the magnetic quarks and the mes-Higgs field. In terms of the individual Higgs doublets [1]:

$$y\, q\, \widetilde{q}\, \Phi_H \;=\; y \sum_{i,j} \Big(q^i_L\, u^j_R\, H^u_{ij} \;+\; q^i_L\, d^j_R\, H^d_{ij} \Big)\,, \tag{14}$$

where $i, j = 1, 2, 3$ are generation indices. The effective Yukawa matrices are [1]:

$$Y^u_{ij} \;=\; y\,\frac{\langle H^u_{ij}\rangle}{v}\,, \qquad Y^d_{ij} \;=\; y\,\frac{\langle H^d_{ij}\rangle}{v}\,, \tag{15}$$

with the VEV sum rule:

$$v^2 \;=\; \sum_{i,j}\Big(\langle H^u_{ij}\rangle^2 + \langle H^d_{ij}\rangle^2\Big) \;=\; \frac{1}{2}\,(246\text{ GeV})^2\,. \tag{16}$$

Key insight: The fermion mass hierarchy arises *not* from a hierarchy in the Yukawa couplings (they are all equal to y) but from a hierarchy in the Higgs VEVs: $\langle H^u_{33}\rangle \gg \langle H^{u,d}_{ij}\rangle$ for $(i, j) \neq (3, 3)$. This is the "scalar democracy" mechanism [4], which will be developed further in Lecture 8.

4 The Electric Theory

Having identified the magnetic spectrum, we now present the electric theory that, *if the duality holds*, provides the UV-complete description of the SM at energies above the duality scale Λ_D.

4.1 Electric field content for $n_g = 3$

For three generations, the electric gauge group is $\mathrm{SU}(2n_g - 4) = \mathrm{SU}(2)$ (from §2.4). The electric theory contains [2]:

Electric theory: SU(2) **with** $N_f = 6$					
Fields	$\mathrm{SU}(2)_{\mathrm{el}}$	$\mathrm{SU}(6)_L$	$\mathrm{SU}(6)_R$	$\mathrm{U}(1)_V$	$\mathrm{U}(1)_{AF}$
λ	Adj ($= \mathbf{3}$)	$\mathbf{1}$	$\mathbf{1}$	0	1
p	F ($= \mathbf{2}$)	F	$\mathbf{1}$	1	$-1/2$
$\widetilde{p}$	$\bar{F}$ ($= \mathbf{2}$)	$\mathbf{1}$	$\bar{F}$	-1	$-1/2$
L	$\mathbf{1}$	F	$\mathbf{1}$	-3	0
$\widetilde{L}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{F}$	3	0

Table 5: Electric theory for $n_g = 3$: $\mathrm{SU}(2)_{\mathrm{el}}$ with 6 fundamental Weyl fermions p, 6 antifundamental $\widetilde{p}$, and one adjoint λ. For SU(2), the fundamental and antifundamental are equivalent ($\bar{F} \cong F$).

Remark 4.1 (No elementary scalars in the electric theory)*.* The electric theory of Table 5 contains **only fermions and gauge bosons**—no elementary scalars. *If the duality holds*, this means that **the hierarchy problem does not exist** in the fundamental description: there are no scalar masses to protect, because there are no fundamental scalars. The Higgs doublets of the SM are composites $\Phi_H = \langle \widetilde{Q}\lambda\lambda Q\rangle$ (Eq. (5)), and their masses are set by the strong dynamics of the electric theory at the duality scale Λ_D.

4.2 Self-duality of SU(2)

Note that for $n_g = 3$, the electric gauge group is SU(2) with $N_f = 6$ Dirac flavours, and the magnetic gauge group is $\mathrm{SU}(X) = \mathrm{SU}(N_f - N) = \mathrm{SU}(6 - 2) = \mathrm{SU}(4)$. The magnetic SU(4) is the *Pati–Salam* gauge group containing QCD $\mathrm{SU}(3)_c$ as a subgroup. The duality connects:

$$\underbrace{\mathrm{SU}(2)_{\mathrm{el}}}_{\text{electric}} \xleftrightarrow{\text{duality}} \underbrace{\mathrm{SU}(4)_{\mathrm{PS}} \supset \mathrm{SU}(3)_c}_{\text{magnetic (contains SM)}} .$$

Note also that SU(2) is *self-dual* under Seiberg duality with $N_f = 4$ flavours (recalled from Lecture 3). Here we have $N_f = 6$, so the electric and magnetic theories have different gauge groups—the duality is a *non-trivial* map, not a self-duality.

4.3 Energy hierarchy

If the duality holds, the physics is characterised by a hierarchy of energy scales [1]:

1. $E < \Lambda_S \sim$ few TeV: only SM degrees of freedom are active.
2. $\Lambda_S < E < \Lambda_D$: the full magnetic spectrum appears (gaugino, squarks, 17 heavy Higgs doublets, mesino).
3. $E > \Lambda_D \sim 10^{11}$ GeV: the electric theory $\mathrm{SU}(2)_{\mathrm{el}}$ becomes weakly coupled; this is the "electric SM."
4. Optionally, $E > \Lambda_{\mathrm{eGUT}}$: if the three gauge couplings of the electric theory unify, a grand unified theory emerges above Λ_{eGUT}.

At the duality scale Λ_D, the electric and magnetic gauge couplings are related by [1]:

$$\frac{1}{\alpha_e} \sim \alpha_m = \frac{g_m^2}{4\pi} . \tag{17}$$

Strong coupling in one description corresponds to weak coupling in the other—the defining feature of duality (recall Lecture 3, §1).

5 Anomaly Matching for $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

The central non-trivial consistency check of the dualised SM construction is 't Hooft anomaly matching (Lecture 1) between the electric and magnetic descriptions. We now compute all independent anomaly conditions for the gauged subgroup $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ of the global symmetry.

Remark 5.1 (Anomaly matching as consistency check)*.* Recall from Lecture 1: 't Hooft anomaly matching constrains the massless fermion spectrum by requiring that all anomaly coefficients for global symmetries match between UV and IR descriptions. Here, we treat $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as the gauged symmetry whose anomalies must *cancel* (gauge anomaly cancellation), and we verify that both the electric and magnetic theories satisfy the same anomaly conditions.

5.1 Independent anomaly conditions

For a gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, the independent anomaly conditions for gauge anomaly cancellation are [6]:

(i) $\mathrm{SU}(3)^3$: the cubic $\mathrm{SU}(3)_c$ anomaly.

(ii) $\mathrm{SU}(2)^3$: the cubic $\mathrm{SU}(2)_L$ anomaly.

(iii) $\mathrm{SU}(3)^2\,\mathrm{U}(1)_Y$: the mixed colour–hypercharge anomaly.

(iv) $\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y$: the mixed weak–hypercharge anomaly.

(v) $\mathrm{U}(1)_Y^3$: the cubic hypercharge anomaly.

(vi) $\mathrm{U}(1)_Y$–gravitational: the mixed hypercharge–gravitational anomaly.

5.2 Condition (ii): $\mathrm{SU}(2)^3$ vanishes trivially

The cubic Casimir d_{abc} vanishes for SU(2):

$$A_{\mathrm{SU}(2)^3} \;=\; \sum_{\mathrm{Weyl}} \mathrm{Tr}\big[T^a_{\mathrm{SU}(2)}\{T^b_{\mathrm{SU}(2)}, T^c_{\mathrm{SU}(2)}\}\big] \;=\; 0 \qquad \text{(all representations)}\,. \tag{18}$$

This is a group-theoretic identity for SU(2) and provides no constraint on the spectrum.

5.3 Computing the anomaly coefficients

For the remaining five non-trivial conditions, we need the SM quantum numbers of all magnetic fermions. Using the flavour decomposition (8) with $n_g = 3$ and the hypercharge formula (9), the magnetic **fermion** content (only fermions contribute to anomalies) decomposes as follows.

Magnetic fermions and their SM quantum numbers:

Field	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	Y	**Weyl count**	**Type**
			SM quarks (from q, $\widetilde{q}$):		
q^i_L	$\mathbf{3}$	$\mathbf{2}$	$+1/6$	$3\times 2 = 6$	Weyl
u^i_R	$\mathbf{3}$	$\mathbf{1}$	$+2/3$	3	Weyl
d^i_R	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	3	Weyl
$\widetilde{q}^i_L$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-1/6$	6	Weyl
$\widetilde{u}^i_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	3	Weyl
$\widetilde{d}^i_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$+1/3$	3	Weyl
			SM leptons (from l, $\widetilde{l}$):		
ℓ^i_L	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	6	Weyl
ν^i_R	$\mathbf{1}$	$\mathbf{1}$	0	3	Weyl
e^i_R	$\mathbf{1}$	$\mathbf{1}$	-1	3	Weyl
$\widetilde{\ell}^i_L$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$	6	Weyl
$\widetilde{\nu}^i_R$	$\mathbf{1}$	$\mathbf{1}$	0	3	Weyl
$\widetilde{e}^i_R$	$\mathbf{1}$	$\mathbf{1}$	$+1$	3	Weyl
			Gaugino (from λ_m):		
λ_m	$\mathbf{8}$	$\mathbf{1}$	0	8	Weyl
			Mesino (from M):		
M_{ij}	$\mathbf{1}$	various	various	36	Weyl

Table 6: SM quantum numbers of all magnetic Weyl fermions. Scalars (ϕ, $\widetilde{\phi}$, Φ_H) do *not* contribute to anomalies.

A crucial observation is that the magnetic fermion content can be decomposed into two sectors:

1. **Vector-like pairs:** The SM quarks and leptons come in left-right pairs (q, $\widetilde{q}$) and (l, $\widetilde{l}$) with opposite $\mathrm{U}(1)_V$ charges and conjugate gauge representations. Each left-right pair contributes zero net anomaly to any purely gauge anomaly condition.
2. **Extra states:** The gaugino λ_m and mesino M have specific $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$ quantum numbers.

5.4 Condition (i): $\mathrm{SU}(3)^3$

The cubic $\mathrm{SU}(3)_c$ anomaly coefficient is:

$$A_{\mathrm{SU}(3)^3} = \sum_{\text{Weyl}} A(R_{\text{colour}})\,, \tag{19}$$

where $A(\mathbf{3}) = 1$, $A(\bar{\mathbf{3}}) = -1$, $A(\mathbf{8}) = 0$ (adjoint is real), and $A(\mathbf{1}) = 0$.

SM quarks: Per generation, the SM quark contribution is $A(q_L)+A(u_R)+A(d_R) = 2\cdot 1+1+1 = 4$ from left-handed quarks (two $\mathrm{SU}(2)_L$ doublet components) and $A(\widetilde{q}_L) + A(\widetilde{u}_R) + A(\widetilde{d}_R) = 2\cdot(-1)+(-1)+(-1) = -4$ from the conjugate quarks. Total per generation: $4 + (-4) = 0$.

Extra states: $A(\lambda_m) = A(\mathbf{8}) = 0$ (adjoint is real). The mesino M is a colour singlet: $A(\mathbf{1}) = 0$.

$$\boxed{A_{\mathrm{SU}(3)^3} = 0\,.} \qquad \text{(Vector-like pairs cancel generation by generation.)} \tag{20}$$

5.5 Condition (iii): $\mathrm{SU}(3)^2\,\mathrm{U}(1)_Y$

This anomaly coefficient sums $Y \cdot T(R_{\text{colour}})$ over all colour-charged Weyl fermions:

$$A_{\mathrm{SU}(3)^2\,\mathrm{U}(1)_Y} = \sum_{\text{colour-charged Weyl}} Y \cdot T(R_{\text{colour}})\,. \tag{21}$$

With $T(\mathbf{3}) = T(\bar{\mathbf{3}}) = 1/2$ and $T(\mathbf{8}) = 3$:

SM quarks (per generation):

$$\begin{aligned} &q_L:\ 2\times\tfrac{1}{6}\times\tfrac{1}{2} = \tfrac{1}{6}\,, \quad u_R:\ \tfrac{2}{3}\times\tfrac{1}{2} = \tfrac{1}{3}\,, \quad d_R:\ (-\tfrac{1}{3})\times\tfrac{1}{2} = -\tfrac{1}{6}\,,\\ &\text{Sum} = \tfrac{1}{6}+\tfrac{1}{3}-\tfrac{1}{6} = \tfrac{1}{3}\,. \end{aligned} \tag{22}$$

Conjugate quarks (per generation):

$$\begin{aligned} &\widetilde{q}_L:\ 2\times(-\tfrac{1}{6})\times\tfrac{1}{2} = -\tfrac{1}{6}\,, \quad \widetilde{u}_R:\ (-\tfrac{2}{3})\times\tfrac{1}{2} = -\tfrac{1}{3}\,, \quad \widetilde{d}_R:\ \tfrac{1}{3}\times\tfrac{1}{2} = \tfrac{1}{6}\,,\\ &\text{Sum} = -\tfrac{1}{6}-\tfrac{1}{3}+\tfrac{1}{6} = -\tfrac{1}{3}\,. \end{aligned} \tag{23}$$

Quarks total (3 generations): $3\times(\tfrac{1}{3}-\tfrac{1}{3}) = 0$.

Gaugino: λ_m has $Y = 0$ (colour octet, $\mathrm{SU}(2)_L$ singlet, $Q_V = 0$), so $Y \cdot T(\mathbf{8}) = 0\times 3 = 0$.

$$\boxed{A_{\mathrm{SU}(3)^2\,\mathrm{U}(1)_Y} = 0\,.} \qquad \text{(Vector-like quark pairs cancel; gaugino is neutral.)} \tag{24}$$

5.6 Condition (iv): $\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y$

This anomaly coefficient sums $Y \cdot T(R_{\text{weak}})$ over all $\mathrm{SU}(2)_L$ doublet Weyl fermions:

$$A_{\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y} \;=\; \sum_{\mathrm{SU}(2)_L\text{-doublet Weyl}} Y \cdot T(\mathbf{2})\,. \tag{25}$$

With $T(\mathbf{2}) = 1/2$ for the $\mathrm{SU}(2)_L$ fundamental:

SM quark doublets (per generation): $q_L :\; Y = +1/6,\ T = 1/2$, colour multiplicity 3: contribution $= 3 \times \frac{1}{6} \times \frac{1}{2} = \frac{1}{4}$.

Conjugate quark doublets (per generation): $\widetilde{q}_L :\; Y = -1/6,\ T = 1/2$, colour multiplicity 3: contribution $= 3 \times (-\frac{1}{6}) \times \frac{1}{2} = -\frac{1}{4}$.

Lepton doublets (per generation): $\ell_L :\; Y = -1/2,\ T = 1/2$, colour multiplicity 1: contribution $= 1 \times (-\frac{1}{2}) \times \frac{1}{2} = -\frac{1}{4}$.

Conjugate lepton doublets (per generation): $\widetilde{\ell}_L :\; Y = +1/2,\ T = 1/2$, colour multiplicity 1: contribution $= 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Total per generation: $\frac{1}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = 0$.

Gaugino: $\mathrm{SU}(2)_L$ singlet—does not contribute.

Mesino: The mesino M transforms under $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$. Under the flavour decomposition (8), M contains components that are $\mathrm{SU}(2)_L$ doublets with various hypercharges. However, M transforms as $(F, \bar{F})$ under $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$ with $Q_V = 0$. Its $\mathrm{SU}(2)_L$ doublet components come in conjugate pairs (from the $\mathrm{SU}(2)_L \subset \mathrm{SU}(6)_L$ and $\mathrm{SU}(2)_R \subset \mathrm{SU}(6)_R$ decomposition), which contribute with opposite signs to the anomaly. The mesino's net contribution to $\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y$ vanishes.

$$\boxed{A_{\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y} \;=\; 0\,.} \qquad \text{(All contributions cancel pairwise.)} \tag{26}$$

5.7 Condition (v): $\mathrm{U}(1)_Y^3$

The cubic hypercharge anomaly sums Y^3 over all Weyl fermions:

$$A_{\mathrm{U}(1)_Y^3} \;=\; \sum_{\text{all Weyl}} \dim(R_{\text{colour}}) \cdot \dim(R_{\text{weak}}) \cdot Y^3\,. \tag{27}$$

SM quarks (per generation):

$$\begin{aligned}
&q_L :\; 3 \times 2 \times (1/6)^3 = 6/216 = 1/36\,,\\
&u_R :\; 3 \times 1 \times (2/3)^3 = 3 \times 8/27 = 24/27 = 8/9\,,\\
&d_R :\; 3 \times 1 \times (-1/3)^3 = 3 \times (-1/27) = -3/27 = -1/9\,.
\end{aligned} \tag{28}$$

Conjugate quarks (per generation):

$$\begin{aligned}
&\widetilde{q}_L :\; 3 \times 2 \times (-1/6)^3 = -1/36\,,\\
&\widetilde{u}_R :\; 3 \times 1 \times (-2/3)^3 = -8/9\,,\\
&\widetilde{d}_R :\; 3 \times 1 \times (1/3)^3 = 1/9\,.
\end{aligned} \tag{29}$$

Quark + conjugate quark total per generation: 0 (each term cancels its partner).

SM leptons (per generation):

$$\begin{aligned}
&\ell_L: \ 1 \times 2 \times (-1/2)^3 = -1/4\,, \\
&\nu_R: \ 1 \times 1 \times 0^3 = 0\,, \\
&e_R: \ 1 \times 1 \times (-1)^3 = -1\,.
\end{aligned} \tag{30}$$

Conjugate leptons (per generation):

$$\begin{aligned}
&\widetilde{\ell}_L: \ 1 \times 2 \times (1/2)^3 = 1/4\,, \\
&\widetilde{\nu}_R: \ 1 \times 1 \times 0^3 = 0\,, \\
&\widetilde{e}_R: \ 1 \times 1 \times (1)^3 = 1\,.
\end{aligned} \tag{31}$$

Lepton + conjugate lepton total per generation: 0.

Gaugino: $Y = 0$, so $Y^3 = 0$.

Mesino: M has $Q_V = 0$, and its $\mathrm{SU}(2)_R$ components come in pairs with $T^3_R = \pm 1/2$ and $T^3_R = 0$. Since $Q_V = 0$, $Y = T^3_R$ for the mesino. The $T^3_R = +1/2$ and $T^3_R = -1/2$ components are related by the $\mathrm{SU}(2)_R$ structure and have equal multiplicities, so their Y^3 contributions cancel: $(+1/2)^3 + (-1/2)^3 = 0$. The $T^3_R = 0$ components contribute $0^3 = 0$.

$$\boxed{A_{\mathrm{U}(1)^3_Y} \ = \ 0\,.} \qquad \text{(Vector-like pairs and SU(2)}_R\text{ pairing enforce cancellation.)} \tag{32}$$

5.8 Condition (vi): $\mathrm{U}(1)_Y$–gravitational

The gravitational anomaly sums Y over all Weyl fermions:

$$A_{\mathrm{U}(1)_Y\text{-grav}} \ = \ \sum_{\text{all Weyl}} \dim(R_{\text{colour}}) \cdot \dim(R_{\text{weak}}) \cdot Y\,. \tag{33}$$

SM quarks (per generation):

$$\begin{aligned}
&q_L: \ 3 \times 2 \times 1/6 = 1\,, \\
&u_R: \ 3 \times 1 \times 2/3 = 2\,, \\
&d_R: \ 3 \times 1 \times (-1/3) = -1\,.
\end{aligned}$$

Conjugate quarks (per generation):

$$\begin{aligned}
&\widetilde{q}_L: \ 3 \times 2 \times (-1/6) = -1\,, \\
&\widetilde{u}_R: \ 3 \times 1 \times (-2/3) = -2\,, \\
&\widetilde{d}_R: \ 3 \times 1 \times (1/3) = 1\,.
\end{aligned}$$

Quark + conjugate total per generation: $(1 + 2 - 1) + (-1 - 2 + 1) = 0$.

SM leptons (per generation): $\ell_L : 2 \times (-1/2) = -1$; $\nu_R : 0$; $e_R : -1$. **Conjugate:** $\widetilde{\ell}_L : 2 \times (+1/2) = +1$; $\widetilde{\nu}_R : 0$; $\widetilde{e}_R : +1$. Total per generation: $(-1 + 0 - 1) + (1 + 0 + 1) = 0$.

Gaugino: $Y = 0$, so contribution $= 8 \times 1 \times 0 = 0$.

Mesino: By the same $\mathrm{SU}(2)_R$ pairing argument as in Condition (v), the mesino contributions cancel: $(+1/2) + (-1/2) = 0$ for each doublet pair, and 0 for singlets.

$$\boxed{A_{\mathrm{U}(1)_Y\text{-grav}} \ = \ 0\,.} \qquad \text{(Vector-like structure ensures vanishing.)} \tag{34}$$

5.9 Summary of anomaly matching

Anomaly condition	**Magnetic (SM + partners)**	**Electric**	**Status**
$\mathrm{SU}(3)^3$	0	0	✓
$\mathrm{SU}(2)^3$	0 (trivial: $d_{abc} = 0$)	0	✓
$\mathrm{SU}(3)^2\,\mathrm{U}(1)_Y$	0	0	✓
$\mathrm{SU}(2)^2\,\mathrm{U}(1)_Y$	0	0	✓
$\mathrm{U}(1)_Y^3$	0	0	✓
$\mathrm{U}(1)_Y$-grav	0	0	✓

Table 7: Summary of anomaly matching for $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. All six conditions are satisfied.

The key mechanism is the **vector-like structure** of the magnetic theory: quarks and conjugate quarks $(q, \widetilde{q})$ carry opposite quantum numbers, as do leptons and conjugate leptons $(l, \widetilde{l})$. The extra states (gaugino, mesino) either are neutral under $\mathrm{U}(1)_Y$ (gaugino) or come in $\mathrm{SU}(2)_R$ pairs that cancel (mesino).

5.10 Electric side: why the anomalies also vanish

In the electric theory, $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is a *subgroup of the global symmetry* (it is gauged only at lower energies, in the magnetic description). The 't Hooft anomaly coefficients of this subgroup must match between the electric and magnetic descriptions.

The electric fermions (Table 5) decompose under $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ using the same embedding (8) and hypercharge (9):

- Q (SU(3) triplet, $\mathrm{SU}(6)_L$ fundamental, $Q_V = +1$): decomposes into 3 generations of $\mathrm{SU}(2)_L$ doublets with $Y = +1/6$, each carrying an $\mathrm{SU}(2)_{\mathrm{el}}$ fundamental index (multiplicity 2).
- $\widetilde{Q}$ ($\bar{\mathbf{3}}$, $\mathrm{SU}(6)_R$ antifundamental, $Q_V = -1$): decomposes into 3 generations of $\mathrm{SU}(2)_R$ doublets. The $T_R^3 = +1/2$ component has $Y = 1/2 - 1/6 = 1/3$ and the $T_R^3 = -1/2$ component has $Y = -1/2 - 1/6 = -2/3$. Each carries $\mathrm{SU}(2)_{\mathrm{el}}$ antifundamental (multiplicity 2).
- λ (SU(3) octet, flavour singlet, $Q_V = 0$, $Y = 0$): carries $\mathrm{SU}(2)_{\mathrm{el}}$ adjoint (multiplicity 3).
- L (SU(3) singlet, $\mathrm{SU}(6)_L$ fundamental, $Q_V = -3$): decomposes into 3 generations of $\mathrm{SU}(2)_L$ doublets with $Y = -1/2$. These are $\mathrm{SU}(2)_{\mathrm{el}}$ singlets.
- $\widetilde{L}$ (SU(3) singlet, $\mathrm{SU}(6)_R$ antifundamental, $Q_V = +3$): $\mathrm{SU}(2)_R$ doublets with $Y = +1$ and $Y = 0$. $\mathrm{SU}(2)_{\mathrm{el}}$ singlets.

Crucially, the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers of the electric fermions are **identical** to those of the corresponding magnetic fermions (compare with Table 6)—the only difference is the $\mathrm{SU}(2)_{\mathrm{el}}$ multiplicity factor. Since every anomaly coefficient of a product group factorises as $A = \dim(R_{\mathrm{el}}) \times A_{\mathrm{SM}}$, and $A_{\mathrm{SM}} = 0$ for each condition (as computed above), the electric anomalies also vanish:

$$A_{\mathrm{electric}} \;=\; \dim(R_{\mathrm{SU}(2)_{\mathrm{el}}}) \times \underbrace{A_{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y}}_{=0} \;=\; 0\,. \tag{35}$$

This confirms the anomaly matching between electric and magnetic descriptions: $A_{\text{electric}} = A_{\text{magnetic}} = 0$ for all six conditions.

Anomaly matching is necessary but not sufficient

The anomaly matching demonstrated in Table 7 is a **necessary condition** for the conjectured duality between the electric SU(2) theory and the magnetic SM, not a **sufficient** one. It constrains the field content of the magnetic theory but does not prove the dynamical equivalence of the two descriptions.

If the anomaly matching had failed—if any coefficient were non-zero on one side and zero on the other—the duality as stated would be **inconsistent** and the field content would need modification. The fact that all conditions are satisfied is **evidence** for the construction's consistency, but the construction remains **conjectural** (recall the central vulnerability of Lecture 6, §**??**).

5.11 Recovery of SM anomaly cancellation

As a cross-check, we verify that when the extra magnetic states are "removed" (their contributions set to zero), the standard SM anomaly cancellation is recovered.

The extra magnetic fermions are:

- Gaugino λ_m: $Y = 0$ for all anomaly conditions. Removing it changes nothing.
- Mesino M: contributes zero to all anomaly conditions (by the $\mathrm{SU}(2)_R$ pairing argument). Removing it changes nothing.
- Conjugate quarks $\widetilde{q}$ and conjugate leptons $\widetilde{l}$: these form exact vector-like pairs with q and l. Removing both $\widetilde{q}$ and $\widetilde{l}$ leaves exactly the SM quark and lepton content.

After removing all extra states, the remaining fields are exactly three generations of SM quarks and leptons: $\{q_L, u_R, d_R, \ell_L, \nu_R, e_R\} \times 3$, and the standard SM anomaly cancellation from Lecture 1 is recovered. This consistency check confirms that the anomaly matching of the full magnetic theory is a *refinement* of the standard SM result, not a replacement.

6 Summary

Lecture 7: Summary

1. **General duality framework:** An SU(N) gauge theory with N_f Dirac fundamentals and one adjoint Weyl fermion has, *if the conjectured non-SUSY duality holds*, a magnetic dual SU(X) with $X = N_f - N$ and a quasi-SUSY spectrum: gaugino, magnetic quarks, squarks, mesino, and mes-Higgs (§1).

2. **Pati–Salam embedding:** Embedding quarks and leptons in SU(4) (lepton = fourth colour, Pati–Salam [3]), the flavour symmetry decomposes as $\mathrm{SU}(N_f)_{L/R} \supset \mathrm{SU}(n_g) \times \mathrm{SU}(2)_{L/R}$, and hypercharge is identified as $Y = T_R^3 + Q_V/6$ (§2). All six SM hypercharges are reproduced exactly.

3. **Generation bound:** $2n_g - 4 \geq 2$ gives $n_g \geq 3$. *If the duality holds*, three or more generations are a mathematical necessity (§2.4).

4. **Magnetic spectrum:** For $n_g = 3$, the magnetic theory contains all SM fields plus quasi-SUSY partners (gaugino, squarks, mesino) and 18 Higgs doublets (9 bi-doublets $\times$ 2), of which one acquires the electroweak VEV (§3).

5. **Electric theory:** The UV description is SU(2) with 6 Dirac flavours and an adjoint—*no elementary scalars.* All scalars in the magnetic theory are composites (§4).

6. **Anomaly matching:** All six independent anomaly conditions for $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ are satisfied. The vector-like structure of the magnetic theory ensures cancellation. This is necessary but not sufficient for the duality (§5).

Preview: Lecture 8

In Lecture 8, we will develop the quantitative consequences of the dualised SM:

- The scalar democracy mechanism: universal Yukawa coupling y combined with hierarchical Higgs VEVs produces the observed fermion mass hierarchy.
- Scale matching: the duality scale $\Lambda_D \sim 10^{11}$ GeV, the Fermi scale $v \approx 246$ GeV, and $\Lambda_{\mathrm{QCD}} \approx 200$ MeV from the dual description.
- Collider signatures: the quasi-SUSY spectrum at the TeV scale.

Every prediction derived in Lecture 8 is conditional on the conjectured non-SUSY duality being correct.

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