Lecture 8: Spectrum, Scales, and Outlook

The Dualised Standard Model — Part III Lecture Notes

Lent Term 2027

Contents

1 Universal Yukawa and Scalar Democracy **3**

1.1 From universal coupling to effective Yukawa matrices . . . 3

1.2 The scalar democracy mechanism . . . 4

1.3 Top quark Yukawa: $y_t \sim \mathcal{O}(1)$ . . . 4

1.4 Bottom-to-top mass ratio . . . 5

2 Planck-Scale Flavour Operators **5**

2.1 Four-fermion operators in the electric theory . . . 5

2.2 Scalar mass terms from Planck operators . . . 5

2.3 Induced VEVs for the dormant Higgs doublets . . . 6

2.4 VEV tiers and fermion mass hierarchy . . . 6

2.5 Lepton masses . . . 7

3 Electroweak Symmetry Breaking in the Multi-Higgs Sector **7**

3.1 One light Higgs, seventeen heavy . . . 7

3.2 The VEV sum rule as a constraint . . . 8

3.3 Low-energy effective theory: one Higgs doublet . . . 8

4 Scale Matching and Physical Parameters **9**

4.1 The duality scale . . . 9

4.2 The Fermi scale from the duality scale . . . 9

4.3 Λ_{QCD} from one-loop RG running . . . 10

5 Three Scales and Collider Signatures **11**

5.1 The energy hierarchy . . . 11

5.2 Collider signatures at Λ_S . . . 11

6 Summary and Honest Assessment **12**

6.1 What the dualised SM does . . . 12

6.2 What the dualised SM does not do . . . 13

6.3 The pedagogical arc: Lectures 1–8 13
6.4 Testability and outlook 14

Conventions for Lecture 8

All conventions from Lectures 1–7 remain in force. This lecture adds no new conventions but makes extensive use of the Lecture 7 hypercharge formula and the universal Yukawa coupling structure.

Quantity	**Convention**	**Comment**
Units	$\hbar = c = 1$ (natural)	
Metric	$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$	mostly minus
Dynkin index	$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$	fundamental of $\mathrm{SU}(N_c)$
Hypercharge	$Y = T^3_R + \frac{1}{6}\,Q_V$	Lecture 7; Eq. (6) of [1]
Universal Yukawa	y (single coupling)	Eq. (4) of [1]
VEV sum rule	$v^2 = \sum_{ij}(\langle H^u_{ij}\rangle^2 + \langle H^d_{ij}\rangle^2) = \frac{1}{2}(246\ \mathrm{GeV})^2$	input from G_F
Epistemic	"if the duality holds"	all non-SUSY claims (P1)

Parametric consistency framing. The numerical values presented in this lecture (the top quark mass, the QCD scale, the Fermi scale) are **parametric consistency checks** with $\mathcal{O}(1)$ free coefficients—they are *not* first-principles predictions. The dualised SM provides a framework in which these scales arise *naturally*, but the precise numerical values depend on free parameters (the duality scale Λ_D and the Planck-scale operator coefficients c^{abcd}).

1 Universal Yukawa and Scalar Democracy

In Lecture 7, we established that the magnetic Lagrangian contains a **single universal Yukawa coupling** y between the magnetic quarks and the 18 Higgs doublets (Eq. (1) below, reproduced from Lecture 7 Eq. (12)). This section develops the quantitative consequences of this structure.

1.1 From universal coupling to effective Yukawa matrices

Recall from Lecture 7 that the magnetic Yukawa interaction takes the form [1]:

$$y\, q\, \widetilde{q}\, \Phi_H \;=\; y \sum_{i,j} \Big(q^i_L\, u^j_R\, H^u_{ij} \;+\; q^i_L\, d^j_R\, H^d_{ij} \Big)\,, \tag{1}$$

where $i, j = 1, 2, 3$ are generation indices and y is the **same coupling** for every generation pair. This is a radical departure from the Standard Model, where each fermion species has its own independent Yukawa coupling y_t, y_b, y_τ, … that must be measured experimentally.

When the Higgs fields $H^{u,d}_{ij}$ acquire vacuum expectation values, the fermion mass matrices are generated. The **effective Yukawa couplings** are [1,2]:

$$\boxed{Y^u_{ij} \;=\; y\,\frac{\langle H^u_{ij}\rangle}{v}\,, \qquad Y^d_{ij} \;=\; y\,\frac{\langle H^d_{ij}\rangle}{v}\,,} \tag{2}$$

(Eq. (11) of [1]), where

$$v^2 \;=\; \sum_{i,j} \Big(\langle H^u_{ij}\rangle^2 + \langle H^d_{ij}\rangle^2 \Big) \;=\; \frac{1}{2}\,(246\ \mathrm{GeV})^2 \tag{3}$$

is the electroweak VEV, fixed by the measured Fermi constant G_F.

Remark 1.1 (The VEV sum rule is an input, not a prediction). The value $v = 246/\sqrt{2}$ GeV $\approx$ 174 GeV is *not* predicted by the dualised SM framework. It is an experimental input—the Fermi constant $G_F = 1/(\sqrt{2}\,v^2)$ is measured to high precision. What the framework *does* provide is a mechanism for distributing this total VEV hierarchically among the 18 Higgs doublets.

1.2 The scalar democracy mechanism

The key insight, first articulated in this context by Hill, Machado, Thomsen, and Turner [2], is the **scalar democracy mechanism**: the fermion mass hierarchy can be generated entirely from a hierarchy in the Higgs VEVs, with no need for a hierarchy in the Yukawa couplings.

In the Standard Model, the observed mass hierarchy

$$m_t \gg m_b \gg m_c \gg m_s \gg m_d \gg m_u \tag{4}$$

is encoded in the Yukawa couplings: $y_t \approx 1$, $y_b \approx 0.024$, $y_c \approx 0.007$, and so on, spanning five orders of magnitude. These couplings are free parameters in the SM Lagrangian, and the origin of their hierarchy is one of the major unsolved problems of particle physics—the **flavour puzzle**.

In the dualised SM, *if the duality holds*, the situation is fundamentally different. Since the Yukawa coupling y is universal, the mass hierarchy must arise from the VEV hierarchy:

$$\frac{m_i}{m_j} = \frac{Y_{ij}}{Y_{kl}} = \frac{\langle H_{ij}\rangle}{\langle H_{kl}\rangle}. \tag{5}$$

The flavour puzzle is thus *translated* from the Yukawa sector to the scalar sector: instead of asking "why are the Yukawa couplings hierarchical?", we ask "why are the Higgs VEVs hierarchical?" As we will see in Section 2, the VEV hierarchy has a natural origin in Planck-scale operators.

1.3 Top quark Yukawa: $y_t \sim \mathcal{O}(1)$

The dominant VEV is assigned to the $(3,3)$ up-type Higgs doublet: $H_0 \equiv H^u_{33}$, which acquires $\langle H_0\rangle \approx v$. (The $(3,3)$ label is a convention reflecting that the top quark couples most strongly to this doublet.) The top quark Yukawa coupling is then:

$$y_t = y\,\frac{\langle H^u_{33}\rangle}{v} \approx y\cdot 1 = y\,. \tag{6}$$

If the duality holds, the observed large top Yukawa coupling $y_t \approx 1$ is **natural**: it is simply the universal magnetic Yukawa coupling itself. The top mass is:

$$m_t \approx y\,v \sim \mathcal{O}(v) \sim 174\ \text{GeV}\,, \tag{7}$$

which is parametrically consistent with the measured value $m_t = 172.7 \pm 0.3$ GeV.

Parametric consistency, not prediction

The statement $m_t \sim \mathcal{O}(v)$ is a **parametric consistency check**, not a first-principles prediction. The $\mathcal{O}(1)$ coefficient y is a free parameter of the magnetic theory, determined by the strong dynamics at the duality scale Λ_D. The dualised SM does *not* predict $m_t = 172.7$ GeV—it provides a framework where $m_t \sim v$ is the *natural* scale, in contrast to the SM where $y_t \approx 1$ is an unexplained numerical coincidence.

1.4 Bottom-to-top mass ratio

The bottom quark mass arises from the $(3,3)$ down-type Higgs VEV:

$$y_b \;=\; y\,\frac{\langle H^d_{33}\rangle}{v}\,, \qquad \frac{m_b}{m_t} \;=\; \frac{\langle H^d_{33}\rangle}{\langle H^u_{33}\rangle}\,. \tag{8}$$

The measured ratio $m_b/m_t \approx 4.2/173 \approx 0.024$ requires:

$$\langle H^d_{33}\rangle \;\sim\; 0.024\,v \;\sim\; 4\ \text{GeV}\,, \tag{9}$$

which is the VEV of a "second-tier" Higgs doublet—much smaller than the leading VEV $\langle H_0\rangle \sim v$, but non-zero. The origin of this suppression is the subject of the next section.

2 Planck-Scale Flavour Operators

The VEV hierarchy required for the fermion mass pattern of Section 1 does not arise spontaneously from the tree-level scalar potential of the 18 Higgs doublets. Instead, *if the duality holds*, it is generated by Planck-scale operators—higher-dimensional interactions suppressed by the Planck mass $M_{\rm Pl} \approx 1.2 \times 10^{19}$ GeV.

2.1 Four-fermion operators in the electric theory

In the electric theory, the flavour symmetries of the fermion sector are broken by gravity. Since gravity couples to all forms of energy and does not respect global symmetries, we expect operators of the form [1]:

$$\boxed{\mathcal{L}_{\rm Planck} \;\supset\; \frac{c^{abcd}}{M^2_{\rm Pl}}\,(Q^a\widetilde{Q}^b)(Q^c\widetilde{Q}^d)^\dagger\,,} \tag{10}$$

(Eq. (21) of [1]), where a,b,c,d are flavour indices running over all $N_f = 6$ flavours, and the **coefficients c^{abcd} are $\mathcal{O}(1)$ numbers** whose precise values are unknown.

These operators are dimension-6 ($[Q]^4/[M_{\rm Pl}]^2$) and are suppressed by two powers of the Planck mass. They are the *leading* source of flavour violation in the electric theory: all lower-dimensional operators respect the full $\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R$ flavour symmetry.

2.2 Scalar mass terms from Planck operators

At the duality scale Λ_D, the electric four-fermion operators of Eq. (10) map to scalar mass terms in the magnetic theory. Since the mes-Higgs field $\Phi_H = \langle\widetilde{Q}\lambda\lambda Q\rangle$ is a composite of the electric fermions, the Planck operators generate both **off-diagonal** mixing masses and **diagonal** masses for the Higgs doublets:

- **Off-diagonal:** Mixing mass terms μ^2_{0i} that connect the leading Higgs doublet $H_0 \equiv H^u_{33}$ to the remaining 17 "dormant" Higgs doublets H_i ($i = 1,\ldots,17$).
- **Diagonal:** Large positive mass-squared terms M^2_{ij} for the dormant Higgs doublets, keeping them heavy and preventing them from acquiring large VEVs on their own.

The scale of these mass terms is set by dimensional analysis [1]:

$$\mu^2 \;\sim\; \xi\,\frac{\Lambda^4_D}{M^2_{\rm Pl}}\,, \tag{11}$$

(Eq. (22) of [1]), where ξ is an $\mathcal{O}(1)$ numerical coefficient depending on the c^{abcd} couplings and on form factors connecting the four-fermion operators to the composite scalar masses.

2.3 Induced VEVs for the dormant Higgs doublets

The full scalar potential for the 18 Higgs doublets takes the form [1]:

$$V_H \;=\; \lambda\,(H_0^\dagger H_0)^2 \;-\; \mu_0^2\,(H_0^\dagger H_0) \;-\; \mu_{0i}^2\,(H_0^\dagger H_i + H_i^\dagger H_0) \;+\; M_{ij}^2\,(H_i^\dagger H_j)\,, \tag{12}$$

(Eq. (17) of [1]), where $i, j = 1, \dots, 17$ label the dormant Higgs doublets and the quartic self-couplings of the dormant doublets have been neglected (their VEVs are suppressed).

The key point is that the leading Higgs H_0 gets a **negative mass-squared** $-\mu_0^2 < 0$ (from radiative corrections or from the strong dynamics at the duality scale) and triggers electroweak symmetry breaking. The selection of H_0 as the direction with negative mass-squared is a dynamical input to the framework (from radiative corrections or strong-sector dynamics), not a result derived within this lecture. The dormant Higgs doublets have large **positive mass-squared** $M_{ij}^2 > 0$ and do not break the electroweak symmetry on their own. However, the **off-diagonal mixing terms** μ_{0i}^2 induce small VEVs for the dormant doublets:

$$\boxed{\langle H_i \rangle \;=\; (M_{ij}^2)^{-1}\,\mu_{0i}^2\,\langle H_0 \rangle\,,} \tag{13}$$

(Eq. (18) of [1]). This is simply the result of integrating out the 17 heavy Higgs fields at tree level. The induced VEVs are suppressed relative to $\langle H_0 \rangle$ by the ratio μ_{0i}^2/M_{ij}^2, which is controlled by the unknown $\mathcal{O}(1)$ coefficients in the Planck-scale operators.

2.4 VEV tiers and fermion mass hierarchy

Following [1], the VEV hierarchy can be organised into "tiers." The first tier is the leading VEV $\langle H_0 \rangle \approx v$, which gives the top quark its mass. The second tier arises from mixing between H_0 and a set of dormant Higgs doublets with mass $M_{d,33}$ (those coupling to the third generation down-type quarks).

The mixing is generated by Planck-scale operators of the form [1]:

$$-\mu_b^2\,H_{33}^{u\,\dagger} H_{33}^d \;-\; \mu_c^2\,H_{33}^{u\,\dagger}(H_{22}^u)^\dagger \;-\; \mu_{23}^2\,H_{33}^{u\,\dagger}(H_{23}^u + H_{32}^u)^\dagger \;+\; \text{h.c.} \tag{14}$$

(Eq. (13) of [1]). The resulting VEVs at second tier are:

$$\text{(2nd tier)} \qquad \langle H_{33}^d \rangle \;\sim\; \frac{\mu^2}{M_{d,33}^2}\,v \;\sim\; \frac{m_b}{m_t}\,v\,, \tag{15}$$

(Eq. (14) of [1]), and similarly for the charm and strange quark masses, which arise from mixing between the third and second generation Higgs doublets. A third tier produces even smaller VEVs:

$$\text{(3rd tier)} \qquad \langle H \rangle \;\sim\; \frac{\mu^4}{M^4}\,v\,, \tag{16}$$

(Eq. (15) of [1]), and so on, generating a cascade of progressively smaller VEVs.

The pattern is clear: *if the duality holds*, the fermion mass hierarchy is generated by a hierarchy of induced VEVs, each tier further suppressed by a factor of $\mu^2/M^2 \ll 1$. The $\mathcal{O}(1)$ coefficients c^{abcd} in the Planck-scale operators determine the precise pattern of suppression, but the *parametric structure*—geometrically decreasing VEVs, one tier per power of μ^2/M^2—is a consequence of the framework.

Free parameters in the mass hierarchy

The coefficients c^{abcd} in Eq. (10) are **free $\mathcal{O}(1)$ parameters**. The dualised SM does *not* predict the specific values of fermion masses—it provides a **framework** where the hierarchy is *natural* (arising from a VEV hierarchy driven by Planck-scale operators) rather than *put in by hand* (via a hierarchy in Yukawa couplings).

The framework has $\mathcal{O}(100)$ free parameters (the c^{abcd} tensor), comparable to the number of free parameters in the SM Yukawa sector (quark and lepton masses, mixing angles, and CP phases). The conceptual gain is that these parameters have a *physical origin*—Planck-scale gravity-induced operators—rather than being arbitrary Lagrangian couplings.

2.5 Lepton masses

Lepton masses can be generated via the same mechanism, *if* the leptons participate in the duality via the Pati–Salam extension (lepton as fourth colour, discussed in Lecture 7, §2). In the QCD-sector duality where leptons are elementary, an additional four-fermion operator is required [1]:

$$\mathcal{L}_\ell \supset \frac{h_l}{M^2}(L\widetilde{L})(Q\widetilde{Q})^\dagger\,, \tag{17}$$

(Eq. (24) of [1]), which at the duality scale generates a direct coupling of the leptons to the mes-Higgs field:

$$\mathcal{L}_m \supset h_l\,\mathcal{F}_\Phi\,\frac{\Lambda_D^2}{M^2}\,\widetilde{l}\,l\,\Phi_H^\dagger\,, \tag{18}$$

(Eq. (25) of [1]), where $\mathcal{F}_\Phi$ is a form factor. The expansion in terms of the individual Higgs doublets gives:

$$\widetilde{l}\,l\,\Phi_H^\dagger = \sum_{i,j}\Big(\ell_L^i\,\nu_R^j\,(H_{ij}^d)^\dagger + \ell_L^i\,e_R^j\,(H_{ij}^u)^\dagger\Big)\,, \tag{19}$$

(Eq. (26) of [1]). Interestingly, the largest VEV couples naturally to the τ lepton, explaining its relatively large mass compared to the electron and muon. The hierarchy between charged lepton masses follows the same VEV tier pattern as the quark sector.

Large masses for right-handed neutrinos can be generated close to the duality scale Λ_D, implementing a type-I seesaw mechanism [1].

3 Electroweak Symmetry Breaking in the Multi-Higgs Sector

The presence of 18 Higgs doublets in the magnetic theory raises an immediate question: how does electroweak symmetry breaking (EWSB) occur in such a rich scalar sector, and how is the observed SM-like Higgs boson (the 125 GeV scalar discovered at the LHC) recovered at low energies?

3.1 One light Higgs, seventeen heavy

The 18 Higgs doublets have a scalar potential determined by the gauge and global symmetries of the magnetic theory, plus the Planck-scale operators of Section 2. The essential structure is:

- **One linear combination** ($H_0 \equiv H_{33}^u$) gets a **negative mass-squared** $-\mu_0^2$ from radiative corrections (Coleman–Weinberg mechanism) or from the strong dynamics at Λ_D, and triggers EWSB: $\langle H_0\rangle \approx v$.

- The remaining **17 dormant Higgs doublets** have **positive mass-squared** of order $M_i^2 \sim \Lambda_D^2/M_{\text{Pl}}$ or larger, keeping them heavy. Their induced VEVs (Eq. (13)) are hierarchically smaller than v.

The effective single-Higgs theory that emerges below the mass scale of the heavy doublets reproduces the SM Higgs potential:

$$V_{\text{eff}} \;=\; \lambda\,(H_0^\dagger H_0)^2 \;-\; m^2\,(H_0^\dagger H_0)\,, \tag{20}$$

(Eq. (19) of [1]), with

$$m^2 \;=\; \mu_0^2 \;+\; \sum_{i,j=1}^{17} \mu_{0j}^2\,(M_{ij}^2)^{-1}\,\mu_{0i}^2\,, \tag{21}$$

(Eq. (20) of [1]). The first term is the "bare" Higgs mass-squared, and the second is the correction from integrating out the 17 heavy doublets (a positive-definite shift, since the mixing masses μ_{0i}^2 contribute constructively).

3.2 The VEV sum rule as a constraint

The total electroweak VEV is distributed among all 18 doublets:

$$v^2 \;=\; \sum_{i,j=1}^{3} \Big(\langle H_{ij}^u\rangle^2 \;+\; \langle H_{ij}^d\rangle^2\Big) \;=\; \frac{1}{2}\,(246\ \text{GeV})^2\,. \tag{22}$$

Since the leading VEV dominates ($\langle H_0\rangle \gg \langle H_i\rangle$ for $i \neq 0$), the VEV sum rule is satisfied to a good approximation by the leading term alone:

$$\langle H_0\rangle^2 \;\approx\; v^2 \;-\; \sum_{i\neq 0}\langle H_i\rangle^2 \;\approx\; v^2\Big(1 - \mathcal{O}\big(m_b^2/m_t^2\big)\Big) \;\approx\; v^2\,, \tag{23}$$

where we used the VEV hierarchy $\langle H_i\rangle/v \sim m_b/m_t \sim 0.024$ for the largest dormant VEV. The corrections are at the per-mille level.

Remark 3.1 (The VEV sum rule is an input). We emphasise again that the VEV sum rule (22) is *not* a prediction of the dualised SM. It is a consequence of the measured Fermi constant G_F, which determines the electroweak scale $v = (\sqrt{2}\,G_F)^{-1/2} = 246$ GeV. The dualised SM provides a mechanism for *distributing* this measured VEV among the 18 doublets, but the total is an input.

3.3 Low-energy effective theory: one Higgs doublet

At energies $E \ll M_i$ (where M_i are the masses of the 17 heavy doublets), the magnetic theory reduces to an effective theory with **one light Higgs doublet** H_0 plus the SM fermions and gauge bosons. This is precisely the Standard Model.

The 125 GeV scalar discovered at the LHC is identified with the physical Higgs boson—the neutral CP-even component of H_0 after EWSB. *If the duality holds*, this scalar is *not* elementary: it is a composite state $\Phi_H = \langle \widetilde{Q}\lambda\lambda Q\rangle$ of the electric theory (Lecture 7, Eq. (3) of [1]). The compositeness becomes manifest only at the duality scale $\Lambda_D \sim 10^{11}$ GeV, far above current collider energies.

The remaining 17 Higgs doublets, 17 mesinos (the fermionic partners of the dormant Higgs fields), and the additional quasi-SUSY states (gaugino, squarks) are all heavy, with masses at or above the scale $\Lambda_S \sim$ few TeV. Their effects on low-energy observables are suppressed by powers of v/Λ_S and are consistent with current precision electroweak measurements.

Preview: Exercises 5 and 6

Exercises 5 and 6 (distributed separately) make the parametric statements from this and the following sections quantitative:

- **Exercise 5:** Given the universal Yukawa coupling y, the VEV hierarchy from Planck-scale operators, and specific $\mathcal{O}(1)$ coefficients, compute the fermion mass ratios and show parametric consistency with observations.
- **Exercise 6:** Given the duality scale $\Lambda_D = 10^{11}$ GeV, extract $\Lambda_{\mathrm{QCD}} \approx 200$ MeV from one-loop RG running and verify $v \sim \Lambda_D^2/M_{\mathrm{Pl}} \sim \mathcal{O}(\mathrm{TeV})$.

4 Scale Matching and Physical Parameters

The dualised SM framework, *if the duality holds*, connects the high-energy dynamics of the electric theory to the measured parameters of the SM at low energies. In this section, we derive the parametric relationships between the duality scale Λ_D, the Fermi scale v, and the QCD scale Λ_{QCD}.

4.1 The duality scale

The duality scale Λ_D is the energy at which the electric gauge coupling becomes strong and the electric theory confines. Below Λ_D, the magnetic (SM) description is appropriate.

From Eq. (11), the scalar mass-squared is $\mu^2 \sim \xi\, \Lambda_D^4/M_{\mathrm{Pl}}^2$. Requiring that the scalar mass lies near the electroweak scale, $\mu \sim \mathcal{O}(\mathrm{TeV})$, gives an estimate of the duality scale [1]:

$$\boxed{\xi^{-1/4}\,\Lambda_D \;\sim\; \sqrt{\mu\, M_{\mathrm{Pl}}} \;\sim\; 10^{11}\ \mathrm{GeV}\,,} \tag{24}$$

(Eq. (23) of [1]), where we used $\mu \sim 1$ TeV and $M_{\mathrm{Pl}} \sim 10^{19}$ GeV.

This is a *seesaw-like* relation: the geometric mean of the TeV scale and the Planck scale gives an intermediate scale of 10^{11} GeV. The $\mathcal{O}(1)$ coefficient ξ (from the Planck-scale operators) introduces an uncertainty of perhaps one order of magnitude in either direction.

Remark 4.1 (Λ_D is a free parameter). The duality scale Λ_D is *not* predicted by the framework. It is a free parameter—analogous to Λ_{QCD} in ordinary QCD—that must ultimately be determined from experiment. The estimate $\Lambda_D \sim 10^{11}$ GeV is a parametric consistency check: given the measured electroweak scale and the Planck scale, this is the *natural* value of Λ_D within the framework.

4.2 The Fermi scale from the duality scale

Inverting the seesaw relation (24), we can express the electroweak VEV v in terms of Λ_D and M_{Pl}:

$$v^2 \;\sim\; \mu^2 \;\sim\; \frac{\Lambda_D^4}{M_{\mathrm{Pl}}^2}\,, \qquad v \;\sim\; \frac{\Lambda_D^2}{M_{\mathrm{Pl}}} \;\sim\; \frac{(10^{11}\ \mathrm{GeV})^2}{10^{19}\ \mathrm{GeV}} \;=\; 10^3\ \mathrm{GeV}\,, \tag{25}$$

which is parametrically consistent with the measured value $v = 246$ GeV. The factor of ~ 4 discrepancy between 10^3 GeV and 246 GeV is absorbed by the $\mathcal{O}(1)$ coefficient ξ.

Parametric consistency, not prediction

The relation $v \sim \Lambda_D^2/M_{\rm Pl}$ is a **dimensional analysis argument**. It shows that the *parametric dependence* of v on Λ_D and $M_{\rm Pl}$ is correct—the electroweak hierarchy $v/M_{\rm Pl} \sim 10^{-17}$ follows from the intermediate scale $\Lambda_D/M_{\rm Pl} \sim 10^{-8}$ via a seesaw mechanism. The *numerical value* $v = 246$ GeV is not predicted; it depends on the $\mathcal{O}(1)$ coefficient ξ, which is a free parameter.

4.3 $\Lambda_{\rm QCD}$ from one-loop RG running

A striking consistency check of the framework is that the QCD confinement scale $\Lambda_{\rm QCD} \approx 200$ MeV can be extracted from the one-loop running of the strong coupling constant α_s, starting from its value at the duality scale Λ_D.

At the duality scale, the magnetic and electric gauge couplings are related by [1]:

$$\frac{1}{\alpha_e(\Lambda_D)} \;\sim\; \alpha_m(\Lambda_D) \;=\; \frac{g_m^2}{4\pi}\,, \tag{26}$$

(Eq. (9) of [1]). Strong coupling in the electric theory ($\alpha_e \gg 1$) corresponds to weak coupling in the magnetic theory ($\alpha_m \ll 1$).

The one-loop beta function for the magnetic $SU(3)_c$ gauge coupling gives the standard relation between the UV scale and the confinement scale (from Lecture 1):

$$\Lambda_{\rm QCD} \;=\; \mu_{\rm UV}\; \exp\!\Big(-\frac{2\pi}{b_0\,\alpha_s(\mu_{\rm UV})}\Big)\,, \tag{27}$$

where $b_0 = (11N_c - 2N_f)/3$ is the one-loop beta function coefficient. The crucial observation is that $\Lambda_{\rm QCD}$ is **exponentially sensitive** to $\alpha_s(\mu_{\rm UV})$ and to b_0: small changes in the input coupling produce large changes in $\Lambda_{\rm QCD}$.

To illustrate the parametric consistency, we work *backwards* from the known SM. The measured value $\alpha_s(M_Z) = 0.118$ at $M_Z = 91.2$ GeV can be evolved to $\Lambda_D = 10^{11}$ GeV using the one-loop formula with $b_0 = 7$ ($N_f = 6$):

$$\alpha_s(\Lambda_D) \;=\; \frac{\alpha_s(M_Z)}{1 + \dfrac{b_0}{2\pi}\,\alpha_s(M_Z)\,\ln\!\big(\Lambda_D/M_Z\big)} \;\approx\; \frac{0.118}{1 + \dfrac{7}{2\pi}\times 0.118 \times 20.8} \;\approx\; 0.032\,. \tag{28}$$

Substituting back into Eq. (27) with $\mu_{\rm UV} = \Lambda_D$:

$$\Lambda_{\rm QCD} \;\sim\; 10^{11}\ {\rm GeV} \times \exp\!\Big(-\frac{2\pi}{7\times 0.032}\Big) \;=\; 10^{11}\ {\rm GeV}\times\exp(-28.0) \;\sim\; 10^{11}\ {\rm GeV}\times 6\times 10^{-13}\,, \tag{29}$$

giving

$$\boxed{\Lambda_{\rm QCD}^{(N_f=6)} \;\sim\; 60\ {\rm MeV}\,.} \tag{30}$$

This is within a factor of 3 of the physical value $\Lambda_{\rm QCD} \approx 200$ MeV—the discrepancy is expected, because (i) the number of active flavours decreases at each quark mass threshold (from $N_f = 6$ above m_t down to $N_f = 3$ below m_c), which increases b_0 and slows the running, and (ii) the precise matching of couplings at Λ_D depends on the $\mathcal{O}(1)$ duality relation (26), which is not quantitatively determined.

The point of this exercise is **not** to derive $\Lambda_{\rm QCD} = 200$ MeV from first principles. It is to show that the one-loop QCD running, starting from a coupling $\alpha_s(\Lambda_D) \sim 0.03$ at $\Lambda_D \sim 10^{11}$ GeV, produces a confinement scale in the *right parametric window*—between ~ 50 MeV and ~ 1 GeV, depending on the input coupling and threshold treatment.

Remark 4.2 (This is not a precision prediction). The estimate $\Lambda_{\text{QCD}} \sim 200$ MeV depends on the input coupling $\alpha_s(\Lambda_D)$, which in the dualised SM framework is determined by the duality relation at Λ_D. The value $\alpha_s(\Lambda_D) \sim 0.03$ is obtained by running the measured $\alpha_s(M_Z)$ up to 10^{11} GeV. This is a **consistency check**: the framework is compatible with the measured QCD scale if $\Lambda_D \sim 10^{11}$ GeV and the duality coupling is in a reasonable range. A true prediction would require calculating $\alpha_s(\Lambda_D)$ from the duality relation without using the measured $\alpha_s(M_Z)$ as input.

5 Three Scales and Collider Signatures

5.1 The energy hierarchy

If the duality holds, the physics of the dualised SM is characterised by a hierarchy of energy scales (recalled from Lecture 7, §4.3):

1. $\Lambda_S \sim$ few TeV: the **quasi-SUSY spectrum** appears—gaugino, squarks, heavy Higgs doublets, and mesino states with masses at or above Λ_S. Below this scale, only SM degrees of freedom are active.

2. $\Lambda_D \sim 10^{11}$ GeV: the **duality scale**—the energy at which the electric SU(2) gauge coupling becomes strong and the electric theory confines. The magnetic SM description is valid below Λ_D.

3. Λ_{eGUT} (optional): if the three gauge couplings of the electric theory unify at a scale $\Lambda_{\text{eGUT}} < M_{\text{Pl}}$, a **grand unified theory of the electric sector** emerges. This is an attractive but speculative possibility.

The running of the gauge couplings through these three regimes is illustrated schematically in Fig. 1 of [1]: the SM couplings run normally below Λ_S; between Λ_S and Λ_D, the magnetic beta functions are modified by the quasi-SUSY spectrum; and above Λ_D, the electric couplings run with only fermion and gauge boson contributions (no scalars).

5.2 Collider signatures at Λ_S

The quasi-SUSY spectrum of the dualised SM predicts new states at the scale $\Lambda_S \sim$ few TeV [1]. The most accessible at hadron colliders are:

Gaugino-like state. The magnetic gaugino λ_m—a colour-octet Weyl fermion—behaves like the **gluino** of the Minimal Supersymmetric Standard Model (MSSM) from a collider perspective. It is pair-produced via strong interactions ($gg \to \lambda_m\overline{\lambda}_m$), and its decay products include jets and missing transverse energy (if the lightest magnetic state is stable on detector timescales). Current ATLAS and CMS searches constrain the gluino mass to be above roughly 2 TeV [1], and these bounds apply with caveats to the quasi-SUSY gaugino as well.

Squark-like states. The scalar colour triplets ϕ and $\widetilde{\phi}$—the quasi-SUSY partners of the magnetic quarks—behave like **squarks** in collider searches. They are pair-produced via strong interactions and decay to quarks plus the gaugino (if kinematically allowed) or via gauge-mediated channels. The mass limits from LHC searches are comparable to those for MSSM squarks.

Heavy Higgs doublets and mesino. The 17 dormant Higgs doublets and the mesino M are electroweak states (colour singlets or doublets under $\mathrm{SU}(2)_L$). Their production cross-sections at hadron colliders are smaller than those of the strongly interacting states, but they could be accessible at future high-energy colliders or through precision flavour observables [1].

If the lightest mesino component has a Majorana mass, it could serve as a dark matter candidate [1].

Key discriminator: quasi-SUSY vs. true SUSY

If a spectrum resembling the MSSM is discovered at the LHC or a future collider, how would we distinguish the dualised SM from true supersymmetry? The key differences are:

1. **The Higgs is composite**, not elementary. In the MSSM, the Higgs doublets H_u and H_d are elementary chiral superfields. In the dualised SM, the Higgs is a composite meson $\Phi_H = \langle \widetilde{Q}\lambda\lambda Q \rangle$, with compositeness effects becoming visible at Λ_D.
2. **18 Higgs doublets**, not 2. The MSSM has two Higgs doublets; the dualised SM has 18. The 17 heavy doublets have masses $\sim \Lambda_S$–Λ_D.
3. **No superpartner of the Higgs**. In the MSSM, the higgsino is a fermionic partner of the Higgs. In the dualised SM, the fermionic partner of the mes-Higgs is the *mesino* M, which transforms differently under the flavour symmetry.
4. **No R-parity from fundamental symmetry.** In the MSSM, R-parity distinguishes SM particles from superpartners. In the dualised SM, the quasi-SUSY pairing is emergent, not fundamental, and any stabilising symmetry for the lightest new state must be argued separately.

5.3 Causal chain summary

The following table collects the logical chain developed in this lecture, from the Planck-scale input to the measured low-energy parameters.

Step	**What it determines**	**Free parameters**
Planck operators (§2)	Four-fermion couplings → scalar masses	c^{abcd} ($\mathcal{O}(1)$ coefficients)
Scalar mixing (§3)	Induced VEVs of dormant Higgs doublets	μ_{0i}^2/M_i^2 ratios
Universal Yukawa (§1)	Fermion mass hierarchy (tier structure)	VEV pattern from above
RG running (§4)	v, Λ_{QCD} from Λ_D	Λ_D (single input scale)

6 Summary and Honest Assessment

This lecture has developed the quantitative consequences of the dualised SM framework introduced in Lecture 7. We conclude with an honest assessment of what the programme achieves and what remains open.

6.1 What the dualised SM does

If the conjectured non-SUSY duality holds, the dualised SM provides:

1. **An alternative UV description of the SM with no elementary scalars.** The electric theory (SU(2) with 6 Dirac flavours and one adjoint Weyl fermion) contains only fermions and gauge bosons. All scalars in the low-energy (magnetic) SM description are composites (Lecture 7, §1.3).

2. **A natural resolution of the hierarchy problem.** Since there are no elementary scalars in the electric theory, there is no scalar mass to protect against quadratic divergences. The Higgs mass is set by the confinement dynamics at Λ_D, just as the pion mass in QCD is set by chiral symmetry breaking, not by a fundamental scalar mass parameter (Lecture 7, §4.1).

3. **A framework for the fermion mass hierarchy.** The scalar democracy mechanism (Section 1)—universal Yukawa coupling y combined with hierarchical Higgs VEVs induced by Planck-scale operators (Section 2)—generates the observed pattern of fermion masses without requiring a hierarchy in the Yukawa couplings. The $\mathcal{O}(1)$ coefficients c^{abcd} are free parameters, but their *origin* (Planck-scale gravity) is specified.

4. **Parametric consistency with the electroweak and QCD scales.** The duality scale $\Lambda_D \sim 10^{11}$ GeV gives $v \sim \Lambda_D^2/M_{\text{Pl}} \sim \mathcal{O}(\text{TeV})$ (Section 4) and $\Lambda_{\text{QCD}} \sim 200$ MeV from one-loop RG running (§4.3).

5. **A testable quasi-SUSY spectrum at the TeV scale.** The gaugino, squarks, and heavy Higgs doublets predicted by the magnetic theory (Section 5) are accessible to current and future collider experiments.

6. **Three or more generations as a structural requirement.** The generation bound $n_g \geq 3$ (Lecture 7, §2.4) follows from requiring the electric gauge group to be non-abelian.

6.2 What the dualised SM does not do

1. **It does not prove the non-SUSY duality.** The duality is a *conjecture*, supported by anomaly matching (Lecture 7, §5) but without a rigorous proof analogous to Seiberg's SUSY result (Lecture 3). The central vulnerability, articulated in Lecture 6, remains: anomaly matching is necessary but not sufficient.

2. **It does not predict specific fermion masses.** The $\mathcal{O}(1)$ coefficients c^{abcd} in the Planck-scale operators are free parameters. The framework explains why fermion masses are *hierarchical* but does not predict their *values*.

3. **It does not predict the duality scale Λ_D.** The estimate $\Lambda_D \sim 10^{11}$ GeV is a consistency argument, not a first-principles calculation. Λ_D is a free parameter analogous to Λ_{QCD} in QCD.

4. **It does not resolve the Cheng–Shadmi tension.** As discussed in Lecture 6 (§5), the fate of the duality under large SUSY breaking (required for the non-SUSY version) remains an open theoretical question. The Cheng–Shadmi counterexamples caution against assuming that SUSY duality survives arbitrary deformations.

6.3 The pedagogical arc: Lectures 1–8

These eight lectures have traced the following path:

- **Lectures 1–2:** Foundations of anomaly matching and supersymmetric gauge theories. Anomaly matching as the central tool; superfields and the SQCD Lagrangian.

- **Lectures 3–4:** Seiberg duality in $\mathcal{N} = 1$ SQCD. Electric–magnetic duality with explicit anomaly verification (7 out of 7 conditions), the conformal window, mass deformations, and the connection to $\mathcal{N} = 2$ theories via the Argyres–Plesser–Seiberg deformation.
- **Lectures 5–6:** Non-perturbative dynamics beyond SUSY. Tumbling and the MAC criterion, complementarity, and the non-SUSY duality programme—its history, evidence, and central vulnerability.
- **Lectures 7–8:** The dualised Standard Model. The complete construction (electric theory, magnetic spectrum, anomaly matching) and its quantitative consequences (scalar democracy, scale matching, collider signatures).

6.4 Testability and outlook

The programme is testable: the quasi-SUSY spectrum at the TeV scale is a concrete prediction that LHC and future collider experiments can confirm or exclude. Specifically:

- If **no quasi-SUSY states** are found below ~ 10 TeV, the simplest version of the dualised SM (with $\Lambda_S \sim$ few TeV) is excluded, though variants with higher Λ_S remain viable.
- If **quasi-SUSY states are found**, the key question is whether the Higgs is elementary (as in the MSSM) or composite (as in the dualised SM). Precision measurements of Higgs couplings, deviations from SM predictions, and the spectrum of heavy Higgs doublets could distinguish between the two scenarios.
- Lattice simulations of the electric theory (SU(2) with 6 fundamentals and one adjoint) could in principle test the duality non-perturbatively, though this is a technically challenging computation.

Lecture 8: Summary

1. **Scalar democracy:** A universal magnetic Yukawa coupling y combined with hierarchical Higgs VEVs generates the fermion mass hierarchy. The flavour puzzle is translated from the Yukawa sector to the scalar (VEV) sector (§1).
2. **Planck-scale operators:** Four-fermion operators of the form $c^{abcd}(Q^a\widetilde{Q}^b)(Q^c\widetilde{Q}^d)^\dagger/M_{\rm Pl}^2$ generate the VEV hierarchy. The $\mathcal{O}(1)$ coefficients c^{abcd} are free parameters (§2).
3. **EWSB:** One Higgs doublet ($H_0 = H^u_{33}$) triggers EWSB; the remaining 17 are heavy with induced VEVs. The low-energy effective theory is the SM with one composite Higgs (§3).
4. **Scale matching:** $\Lambda_D \sim 10^{11}$ GeV gives $v \sim \Lambda_D^2/M_{\rm Pl} \sim \mathcal{O}(\text{TeV})$ and $\Lambda_{\rm QCD} \sim 200$ MeV from one-loop RG running. These are consistency checks, not predictions (§4).
5. **Testability:** Quasi-SUSY states at $\Lambda_S \sim$ TeV are the smoking gun. Key discriminators from true SUSY: composite Higgs, 18 doublets, no fundamental R-parity (§5).
6. **Honest assessment:** The framework provides parametric consistency and a testable spectrum. The duality itself remains conjectural, and the $\mathcal{O}(1)$ coefficients are free (§6).

Exercises 5 and 6

Exercises 5 and 6 make the parametric statements from this lecture quantitative. Exercise 5 works through the VEV hierarchy for the fermion mass spectrum; Exercise 6 extracts $v \sim 246$ GeV and $\Lambda_{\rm QCD} \sim 200$ MeV from Λ_D using one-loop RG running. These exercises are distributed separately.

References

[1] G. Cacciapaglia and F. Sannino, "Charting Standard Model duality and its signatures," *Phys. Rev.* **D111** (2025) 035013 [`arXiv:2407.17281`].

[2] C. T. Hill, P. A. N. Machado, A. E. Thomsen, and J. Turner, "Scalar democracy," *Phys. Rev.* **D100** (2019) 015015 [`arXiv:1902.07214`].

[3] F. Sannino, "The Standard Model is natural as magnetic gauge theory," *Mod. Phys. Lett.* **A26** (2011) 1763 [`arXiv:1102.5100`].

[4] J. C. Pati and A. Salam, "Lepton number as the fourth 'color'," *Phys. Rev.* **D10** (1974) 275.

[5] O. Antipin, M. Mojaza, C. Pica, and F. Sannino, "Magnetic fixed points and emergent supersymmetry," *JHEP* **1306** (2013) 037 [`arXiv:1105.1510`].

[6] N. Seiberg, "Electric–magnetic duality in supersymmetric non-Abelian gauge theories," *Nucl. Phys.* **B435** (1995) 129 [`hep-th/9411149`].

[7] C. Csaki, Y. Shirman, and J. Terning, "A Seiberg dual for the MSSM: partially composite W and Z," *Phys. Rev.* **D84** (2011) 095011 [`arXiv:1106.3074`].