Lecture 10: Physical Mesons as Dual Mesons

The Dualised Standard Model — Part III Lecture Notes

Lent Term 2027

Contents

Conventions for Lecture 10

All conventions from Lectures 1–9 remain in force. This lecture introduces conventions for quark flavour labelling, meson naming, and the chiral perturbation theory (ChPT) field parametrisation.

Quantity	**Convention**	**Comment**
Units	$\hbar = c = 1$ (natural)	
Metric	$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$	mostly minus
Dynkin index	$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\,\delta^{ab}$	fundamental of $\mathrm{SU}(N_c)$
Hypercharge	$Y = T^3_R + \frac{1}{6}\,Q_V$	Lecture 7
Universal Yukawa	y (single coupling)	Lecture 7
Epistemic	"if the duality holds"	all non-SUSY claims (P1)
New in this lecture:		
Flavour ordering	$(u, d, s, c, b, t) = (1, 2, 3, 4, 5, 6)$	PDG standard
Meson naming	PDG 2024 conventions	[3]
ChPT field	$U = \exp(2i\,\pi^a T^a/f_\pi)$	Gasser–Leutwyler [5]
f_π	92.1 MeV (physical convention)	$f_\pi = F_\pi\sqrt{2}$ if $F_\pi \approx 65$ MeV

Epistemic convention. Every statement asserting or assuming non-supersymmetric duality carries an explicit conjectural qualifier. The meson identification tables are standard QCD flavour physics; the *duality interpretation*—that these mesons *are* the composites of the magnetic theory—is conjectural.

1 The Meson Matrix in QCD and SQCD

In Lecture 3, we studied Seiberg duality for $\mathrm{SU}(N_c)$ SQCD with N_f flavours. The magnetic theory contains an elementary gauge-singlet field [1]

$$M^i{}_j \;=\; Q^i\,\widetilde{Q}_j\,, \qquad i, j = 1, \ldots, N_f\,, \tag{1}$$

which, in the electric theory, corresponds to the composite quark bilinear $Q^i\,\widetilde{Q}_j/\Lambda$. The meson M appears in the magnetic superpotential:

$$W_{\mathrm{mag}} \;=\; \frac{1}{\mu}\,M^i{}_j\,q_i\,\widetilde{q}^j\,, \tag{2}$$

where q_i, $\widetilde{q}^j$ are the magnetic quarks and μ is a mass scale related to the dynamical scale Λ.

The meson M is a chiral superfield

In the SUSY theory, $M^i{}_j$ is a **chiral superfield**. Its lowest component is a **complex scalar** with spin-parity $J^P = 0^+$. However, the best-known physical mesons—the pions, kaons, and η—are **pseudoscalars** with $J^P = 0^-$. Resolving this mismatch is the central conceptual challenge of this lecture.

In QCD (without supersymmetry), the analogous object is the quark bilinear

$$\mathcal{M}_{ij} \;\sim\; \bar{q}_{R\,j}\,q_{L\,i}\,, \tag{3}$$

which transforms as $(N_f, \bar{N}_f)$ under the chiral symmetry group $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$. For $N_f = 6$ quark flavours (u, d, s, c, b, t), this is a 6×6 matrix with 36 complex entries.

The structure of this matrix is the same whether we view it from the electric theory (where it is a composite operator) or the magnetic theory (where, *if the duality holds*, it is an elementary field). The quantum numbers of each entry $\mathcal{M}_{ij}$ are determined purely by the quark content $q_i\,\bar{q}_j$ and are independent of whether the meson is composite or elementary.

2 The 6×6 Meson Matrix

We now construct the full 6×6 flavour meson matrix $\mathcal{M}_{ij}$ explicitly. The row index i labels the quark flavour; the column index j labels the antiquark flavour. Each entry $\mathcal{M}_{ij}$ corresponds to a meson with quark content $q_i\,\bar{q}_j$.

2.1 Quark quantum numbers

The six quark flavours carry the following additive quantum numbers (all values for quarks; antiquarks carry opposite signs):

	u	d	s	c	b	t
Q_{em}	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
S	0	0	-1	0	0	0
C	0	0	0	$+1$	0	0
B_q	0	0	0	0	-1	0
T	0	0	0	0	0	$+1$

(4)

Here S is strangeness, C is charm, B_q is bottomness (beauty), and T is topness. The electric charge satisfies the generalised Gell-Mann–Nishijima formula:

$$Q_{\mathrm{em}} \;=\; I_3 + \frac{1}{2}(B + S + C + B_q + T)\,, \tag{5}$$

where $B = 1/3$ for quarks. For a meson $q_i\,\bar{q}_j$ with $B = 0$, the charge is simply:

$$Q_{\mathrm{em}}(\mathcal{M}_{ij}) \;=\; Q_{\mathrm{em}}(q_i) - Q_{\mathrm{em}}(q_j)\,. \tag{6}$$

2.2 The light-quark sector: the 3×3 nonet

The upper-left 3×3 block of $\mathcal{M}_{ij}$ involves the light quarks (u, d, s) and corresponds to the well-known pseudoscalar and scalar meson nonets. In the pseudoscalar sector, the physical mesons and their PDG masses [3] are:

$q_i \backslash \bar{q}_j$	$\bar{u}$	$\bar{d}$	$\bar{s}$
u	$\pi^0/\eta/\eta'$ (mix)	π^+ (139.6)	K^+ (493.7)
d	π^- (139.6)	$\pi^0/\eta/\eta'$ (mix)	K^0 (497.6)
s	K^- (493.7)	$\bar{K}^0$ (497.6)	η_s (mix with η, η')

(7)

All masses are in MeV. The diagonal entries $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ do not correspond to individual physical mesons; instead, they mix to produce π^0, η, and η'. The η–η' system also involves the $\mathrm{U}(1)_A$ anomaly (the resolution of the "U(1) problem" [12]).

In the **scalar** sector ($J^P = 0^+$), the corresponding nonet is [3]:

$$\begin{array}{c|ccc} q_i\backslash\bar{q}_j & \bar{u} & \bar{d} & \bar{s} \\ \hline u & a_0(980)/f_0\ (\text{mix}) & a_0^+\ (980) & K_0^{*+}\ (700) \\ d & a_0^-\ (980) & a_0(980)/f_0\ (\text{mix}) & K_0^{*0}\ (700) \\ s & K_0^{*-}\ (700) & \bar{K}_0^{*0}\ (700) & f_0(980)\ (\text{mix}) \end{array} \tag{8}$$

The light scalar mesons (sometimes denoted σ, κ, a_0, f_0) have been controversial for decades; the interpretation of the $f_0(500)$ (formerly σ) as a $q\bar{q}$ state versus a tetraquark or molecular state remains debated [3].

2.3 The full 6×6 matrix

The complete meson matrix includes heavy quarks. We display the off-diagonal entries with their standard PDG names and masses [3]:

Table 1: Physical meson identification for the 6×6 flavour matrix $\mathcal{M}_{ij} = q_i\,\bar{q}_j$. Masses in MeV. Diagonal entries (shaded) involve flavour mixing. The 6th row and column (top quark) are formal only—see Section 2.4. Only the lightest pseudoscalar or ground-state meson is shown for each entry.

$q_i\backslash\bar{q}_j$	$\bar{u}$	$\bar{d}$	$\bar{s}$	$\bar{c}$	$\bar{b}$	$\bar{t}$
u	π^0/η	π^+ 139.6	K^+ 493.7	$\bar{D}^0$ 1864.8	B^+ 5279.3	—
d	π^- 139.6	π^0/η	K^0 497.6	D^- 1869.7	B^0 5279.7	—
s	K^- 493.7	$\bar{K}^0$ 497.6	η_s/ϕ	D_s^- 1968.3	B_s^0 5366.9	—
c	D^0 1864.8	D^+ 1869.7	D_s^+ 1968.3	$\eta_c/J/\psi$	B_c^+ 6274.5	—
b	B^- 5279.3	$\bar{B}^0$ 5279.7	$\bar{B}_s^0$ 5366.9	B_c^- 6274.5	η_b/Υ	—
t	—	—	—	—	—	—

Remark 2.1 (Quark content conventions). The PDG convention for charmed mesons is: $D^0 = c\bar{u}$ (row c, column $\bar{u}$; entry $\mathcal{M}_{41}$), $D^+ = c\bar{d}$ ($\mathcal{M}_{42}$), $D_s^+ = c\bar{s}$ ($\mathcal{M}_{43}$). For bottom mesons: $B^+ = u\bar{b}$ ($\mathcal{M}_{15}$), $B^0 = d\bar{b}$ ($\mathcal{M}_{25}$), $B_s^0 = s\bar{b}$ ($\mathcal{M}_{35}$), $B_c^+ = c\bar{b}$ ($\mathcal{M}_{45}$). Note that $\mathcal{M}_{ij}$ and $\mathcal{M}_{ji}$ correspond to charge-conjugate mesons: $\mathcal{M}_{15} = B^+$ while $\mathcal{M}_{51} = B^-$.

2.4 The top-quark caveat

Top-quark mesons are formal entries only

The top quark decays via $t \to bW^+$ with a lifetime

$$\tau_t \approx \frac{1}{\Gamma_t} \approx \frac{1}{1.4\ \text{GeV}} \approx 5 \times 10^{-25}\ \text{s}, \tag{9}$$

which is **much shorter** than the hadronisation timescale

$$\tau_{\text{had}} \sim \frac{1}{\Lambda_{\text{QCD}}} \sim \frac{1}{0.3\ \text{GeV}} \sim 3 \times 10^{-24}\ \text{s}. \tag{10}$$

Since $\Gamma_t \gg \Lambda_{\text{QCD}}$, the top quark decays **before** it can form a bound state. Consequently, the entries in the 6th row and 6th column of the meson matrix—$\mathcal{M}_{6j}$ and $\mathcal{M}_{i6}$—do **not** correspond to physical hadrons. They are formal entries, included for algebraic completeness of the 6×6 flavour matrix. No "top meson" (sometimes called Θ in the literature) has been or will be observed.

3 Quantum Number Verification

As a consistency check, we verify the electric charge of each meson identified in Table 1 using the quark charges from Eq. (4) and the charge formula (6).

Table 2: Electric charge verification for representative mesons. All charges match the known values.

Meson	Entry	Quark content	$Q(q_i)$	$Q(\bar{q}_j)$	Q_{meson}
π^+	$\mathcal{M}_{12}$	$u\bar{d}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓
K^+	$\mathcal{M}_{13}$	$u\bar{s}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓
D^0	$\mathcal{M}_{41}$	$c\bar{u}$	$+\frac{2}{3}$	$-\frac{2}{3}$	0 ✓
D^+	$\mathcal{M}_{42}$	$c\bar{d}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓
B^+	$\mathcal{M}_{15}$	$u\bar{b}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓
B_s^0	$\mathcal{M}_{35}$	$s\bar{b}$	$-\frac{1}{3}$	$+\frac{1}{3}$	0 ✓
B_c^+	$\mathcal{M}_{45}$	$c\bar{b}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓
D_s^+	$\mathcal{M}_{43}$	$c\bar{s}$	$+\frac{2}{3}$	$+\frac{1}{3}$	$+1$ ✓

All eight representative mesons have the correct electric charge. The reader may verify the remaining entries as Exercise 8.1.

Remark 3.1 (Flavour quantum number conservation)*.* For each off-diagonal entry $\mathcal{M}_{ij}$, the net flavour quantum numbers are $(S, C, B_q, T) = (S_i - S_j,\ C_i - C_j,\ B_{q,i} - B_{q,j},\ T_i - T_j)$, where the subscripts refer to the quark (i) and antiquark (j). For example, $D_s^+ = c\bar{s}$ has $C = +1$, $S = +1$ (since $\bar{s}$ carries $S = +1$), and all other flavour numbers zero. This is consistent with the PDG assignment [3].

4 The Scalar–Pseudoscalar Question

This section addresses the central conceptual challenge of this lecture: the lowest component of the SUSY chiral superfield $M^i{}_j$ is a *scalar* ($J^P = 0^+$), but the most prominent physical mesons (π, K, η) are *pseudoscalars* ($J^P = 0^-$). How can the dual meson be identified with the observed spectrum?

4.1 The problem stated precisely

In the magnetic theory of Lecture 3, the meson superfield $M^i{}_j$ has the component expansion (schematically):

$$M^i{}_j \;=\; \phi^i{}_j \;+\; \sqrt{2}\,\theta\,\psi^i{}_j \;+\; \theta^2\,F^i{}_j\,, \tag{11}$$

where $\phi^i{}_j$ is a **complex scalar** ($J^P = 0^+$), $\psi^i{}_j$ is a Weyl fermion (the mesino), and $F^i{}_j$ is the auxiliary field. The physical meson spectrum, however, includes:

- **Pseudoscalar mesons** ($J^P = 0^-$): $\pi^\pm$, π^0, $K^\pm$, K^0, $\bar{K}^0$, η, η', etc. These are the pseudo-Goldstone bosons of chiral symmetry breaking.
- **Scalar mesons** ($J^P = 0^+$): $f_0(500)$ (formerly σ), $a_0(980)$, $K_0^*(700)$ (formerly κ), $f_0(980)$, etc.

The pseudoscalars *cannot* be identified with the lowest component $\phi^i{}_j$ of the chiral superfield, because ϕ transforms as a scalar, not a pseudoscalar, under parity.

4.2 The resolution: chiral symmetry breaking and the linear sigma model

The resolution comes from the pattern of chiral symmetry breaking. In QCD, the chiral condensate

$$\langle \bar{q}_{R\,j}\, q_{L\,i} \rangle \;=\; -v_0\,\delta_{ij}\,, \qquad v_0 \sim (270\ \text{MeV})^3/f_\pi^2\,, \tag{12}$$

breaks $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$. In the SUSY theory, the corresponding statement is that the meson field acquires a vacuum expectation value:

$$\langle M^i{}_j \rangle \;\neq\; 0\,. \tag{13}$$

We now decompose the meson field into fluctuations around the condensate. Writing the lowest component of the meson superfield as a complex matrix field:

$$\boxed{\phi_{ij} \;=\; \frac{1}{\sqrt{2}}\big(v_{ij} + \sigma_{ij} + i\,\pi_{ij}\big)\,,} \tag{14}$$

where:

- v_{ij} is the vacuum expectation value (the chiral condensate);
- σ_{ij} is the **radial** (amplitude) fluctuation: a **scalar** meson with $J^P = 0^+$;
- π_{ij} is the **angular** (phase) fluctuation: a **pseudoscalar** meson with $J^P = 0^-$.

This is precisely the linear sigma model decomposition [7], now applied to the SUSY meson field.

The pion is NOT the lowest component of M

The pseudoscalar mesons π_{ij} arise as **phase fluctuations** of the condensate $\langle\phi_{ij}\rangle$. They are the (pseudo-)Goldstone bosons of the spontaneously broken chiral symmetry. The **lowest component** ϕ_{ij} of the chiral superfield M_{ij} is a complex scalar field that *contains* both the scalar mode σ_{ij} and the pseudoscalar mode π_{ij}. One must **not** write "$\pi^+ = M_{12}$" without this qualification. The correct statement is:

The pseudoscalar meson π^+ corresponds to the phase fluctuation of the condensate $\langle\phi_{12}\rangle$ in the chiral symmetry breaking pattern $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$.

4.3 Physical identification of the two sectors

After the decomposition (14), the physical mesons are identified as follows.

Scalar sector (σ_{ij}, $J^P = 0^+$). The radial modes σ_{ij} correspond to the scalar mesons. These are the fields whose quantum numbers match the lowest component of the SUSY chiral superfield directly:

Entry	Scalar meson	Mass (MeV)
$\sigma_{11} - \sigma_{22}$ (isovector)	$a_0(980)^0$	~ 980
σ_{12}	$a_0(980)^+$	~ 980
σ_{13}	$K_0^*(700)^+$	~ 700
$(\sigma_{11} + \sigma_{22})/\sqrt{2}$	$f_0(500)$ (σ)	~ 400–550
σ_{33}	$f_0(980)$	~ 990

The light scalar meson spectrum is experimentally challenging and theoretically controversial: the $f_0(500)$ is a broad resonance, and there is ongoing debate about whether the light scalars are $q\bar{q}$ states, tetraquarks, or meson-meson molecules [3]. We note this caveat but adopt the $q\bar{q}$ interpretation for consistency with the duality picture.

Pseudoscalar sector (π_{ij}, $J^P = 0^-$). The angular modes π_{ij} correspond to the pseudo-Goldstone bosons of chiral symmetry breaking. These are the familiar light mesons of the pseudoscalar nonet:

Entry	Pseudoscalar meson	Mass (MeV)
π_{12}	π^+	139.6
π_{21}	π^-	139.6
$\pi_{11} - \pi_{22}$	π^0	135.0
π_{13}	K^+	493.7
π_{23}	K^0	497.6
π_{33}	η_s (mixes into η, η')	—

The pseudoscalar mesons are *light* (compared to the chiral symmetry breaking scale $\Lambda_\chi \sim 1$ GeV) precisely because they are pseudo-Goldstone bosons: they would be exactly massless if the quark masses were zero. Their non-zero masses arise from the *explicit* breaking of chiral symmetry by the quark mass terms $m_q\,\bar{q}\,q$.

4.4 The Sannino–Schechter mechanism

How does the SUSY meson field M_{ij} give rise to the physical QCD mesons when supersymmetry is broken? Sannino and Schechter [8] showed that the answer lies in the **auxiliary fields** of the SUSY effective Lagrangian.

In the SUSY effective theory, the auxiliary F-term of the meson superfield is:

$$F^{i}_{Mj} \sim \frac{\partial W}{\partial M^{j}{}_{i}} . \tag{15}$$

Sannino and Schechter observed that the F-term components contain the information about the QCD meson fields. When SUSY is broken (as it is in the real world), the F-term components mix with the scalar component ϕ_{ij}, and the physical spectrum reorganises into the scalar and pseudoscalar sectors described above.

Remark 4.1 (Limitations of the mechanism). Sannino and Schechter themselves described their approach [8] as a "toy model" for the transition from the SUSY effective theory to QCD. The mechanism demonstrates *how* the QCD mesons can emerge from the SUSY meson field, but it does not provide quantitative control over the hard SUSY-breaking regime relevant to real QCD. The detailed dynamics of the SUSY-to-QCD transition remain an open problem.

4.5 Komargodski's bridge: hidden local symmetry

A complementary perspective was provided by Komargodski [9], who showed that the **hidden local symmetry** (HLS) framework—the standard effective theory for vector meson dominance in QCD [10]—is *rigorously realised* within SQCD.

The key insight is:

> *The magnetic gauge symmetry of Seiberg's dual theory is identified with the hidden local symmetry of the meson effective action.*

In this picture:

- The magnetic gauge bosons of $\mathrm{SU}(N_c - N_f)$ map to the vector mesons (ρ, K^*, ϕ, etc.) of QCD;
- The scalar mesons arise from the amplitude fluctuations of the meson condensate;
- The pseudoscalar mesons arise from the phase fluctuations (Goldstone bosons).

This provides a conceptual bridge between the SUSY duality and hadronic physics. However, it is important to note that Komargodski's result is exact only in the SUSY limit; the extension to the non-supersymmetric regime of QCD remains conjectural.

What the duality adds

The standard QCD description of mesons as $q\bar{q}$ bound states and Goldstone bosons does not require Seiberg duality. *If the duality holds*, the dual perspective provides a structural explanation: the meson matrix is an elementary field in the magnetic theory, and the hidden local symmetry that governs vector meson interactions is the magnetic gauge symmetry. This is a *conceptual* insight, not a new quantitative prediction for meson masses.

5 Connection to Chiral Perturbation Theory

5.1 The ChPT Lagrangian

The low-energy dynamics of the pseudoscalar mesons is described by chiral perturbation theory (ChPT) [4,5]. The leading-order Lagrangian is:

$$\mathcal{L}_2 \;=\; \frac{f_\pi^2}{4}\,\mathrm{Tr}\big(\partial_\mu U^\dagger\,\partial^\mu U\big) \;+\; \frac{f_\pi^2}{4}\,\mathrm{Tr}\big(\chi^\dagger U + U^\dagger \chi\big)\,, \tag{16}$$

where the chiral field U parametrises the Goldstone bosons of the spontaneous chiral symmetry breaking $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$:

$$\boxed{U \;=\; \exp\!\Big(\frac{2i\,\pi^a T^a}{f_\pi}\Big)}\,, \tag{17}$$

with T^a the generators of $\mathrm{SU}(N_f)$ and $f_\pi \approx 92.1$ MeV the pion decay constant [3]. The mass term is

$$\chi \;=\; 2B_0\,\mathcal{M}_q\,, \tag{18}$$

where $\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s, \ldots)$ is the quark mass matrix and B_0 is related to the chiral condensate by $\langle \bar{q}\,q\rangle = -f_\pi^2 B_0$.

5.2 The bridge to the dual meson matrix

The connection to the meson matrix of Section 1 is now clear. The pseudoscalar fields π^a appearing in the ChPT field U are **exactly** the angular (phase) modes π_{ij} of the meson condensate from the decomposition (14):

$$\pi^a T^a \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \tag{19}$$

for $N_f = 3$. The chiral symmetry breaking in QCD gives the meson condensate a VEV, and the phase fluctuations of this VEV are precisely the pion, kaon, and eta fields of ChPT.

In the dual picture, *if the duality holds*, the meson condensate $\langle M^i{}_j\rangle$ is the elementary meson field acquiring a VEV, and the ChPT pion fields are the Goldstone bosons of the same symmetry breaking pattern. The identification is:

$$\text{Phase of } \langle\phi_{ij}\rangle \;\longleftrightarrow\; \text{ChPT field } U = e^{2i\pi^a T^a/f_\pi}\,. \tag{20}$$

5.3 Goldstone boson counting

For $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$, the number of broken generators is $N_f^2 - 1$ (each factor $\mathrm{SU}(N_f)$ has $N_f^2 - 1$ generators, the diagonal subgroup has $N_f^2 - 1$, so the number of broken generators is $2(N_f^2-1) - (N_f^2-1) = N_f^2 - 1$).

N_f	Goldstone bosons	Physical identification
2	3	π^+, π^-, π^0
3	8	π^+, π^-, π^0, K^+, K^-, K^0, $\bar{K}^0$, η
6	35	Formal only (see below)

For $N_f = 6$, the 35 formal Goldstone bosons would include pseudoscalars with heavy quark content (D, B, B_c, and top mesons). However, the ChPT expansion in powers of p^2/Λ_χ^2 and m_q/Λ_χ is valid only for the light quarks (u, d, s), where $m_q \ll \Lambda_\chi \sim 1$ GeV. The charm and bottom quarks are too heavy for ChPT to apply: their masses $m_c \approx 1.27$ GeV and $m_b \approx 4.18$ GeV are comparable to or larger than the chiral symmetry breaking scale.

5.4 The GMOR relation

The Gell-Mann–Oakes–Renner (GMOR) relation [6] connects the pion mass to the quark masses and the chiral condensate:

$$\boxed{m_\pi^2\, f_\pi^2 \;=\; -(m_u + m_d)\,\langle \bar{q}\, q\rangle\,.} \tag{21}$$

Using the standard inputs [3, 11]:

$$f_\pi \;=\; 92.1 \text{ MeV}\,, \tag{22}$$

$$\langle \bar{q}\, q\rangle \;\approx\; -(270 \text{ MeV})^3\,, \tag{23}$$

$$m_u + m_d \;\approx\; 8.5 \text{ MeV} \quad (\overline{\text{MS}},\ \mu = 2 \text{ GeV})\,, \tag{24}$$

we compute:

$$\begin{aligned} m_\pi^2 \;&=\; \frac{(m_u + m_d)\,|\langle \bar{q}\, q\rangle|}{f_\pi^2} \;=\; \frac{8.5 \times (270)^3}{(92.1)^2}\ \text{MeV}^2 \\ &=\; \frac{8.5 \times 1.97 \times 10^7}{8482}\ \text{MeV}^2 \;=\; \frac{1.67 \times 10^8}{8482}\ \text{MeV}^2 \;\approx\; 19{,}700 \text{ MeV}^2\,, \end{aligned} \tag{25}$$

giving:

$$m_\pi \;\approx\; \sqrt{19{,}700}\ \text{MeV} \;\approx\; 140 \text{ MeV}\,. \tag{26}$$

This matches the observed pion mass $m_{\pi^\pm} = 139.6$ MeV to within the uncertainties of the input parameters ($m_u + m_d$ and $\langle \bar{q}\, q\rangle$ each carry $\sim 10\%$ uncertainties [11]).

Remark 5.1 (What the GMOR relation confirms). The GMOR relation is a statement about standard QCD and chiral symmetry breaking. It does *not* depend on the duality. Its role here is to verify that the identification π_{ij} = pseudo-Goldstone boson of $\langle \phi_{ij}\rangle$ is consistent with the observed pion mass, using the standard QCD parameters. The duality provides the *structural* interpretation (the condensate is the VEV of an elementary magnetic field); the *numerical* value of m_π comes from QCD alone.

5.5 Heavy mesons and the limits of ChPT

The ChPT description is limited to the light quark sector. For mesons containing one heavy quark (c or b) and one light quark, the appropriate effective theory is heavy quark effective theory (HQET) [13], which expands in $1/m_Q$ where $m_Q \gg \Lambda_{\text{QCD}}$.

Meson type	**Examples**	**Effective theory**	**Expansion parameter**
Light–light	π, K, η	ChPT	m_q/Λ_χ, p^2/Λ_χ^2
Heavy–light	D, B, B_s	HQET	Λ_{QCD}/m_Q
Heavy–heavy	J/ψ, Υ, B_c	NRQCD / pNRQCD	$v/c \sim \alpha_s$
Top–anything	$t\bar{q}$	Not applicable	$\Gamma_t/\Lambda_{\text{QCD}} \gg 1$

In the dual picture, the entries $\mathcal{M}_{ij}$ with i or $j \in \{4, 5\}$ (charm or bottom) still correspond to physical mesons, but their dynamics cannot be captured by ChPT. The duality, *if it holds*, provides the flavour matrix structure but not the dynamical details that determine individual masses—those are governed by QCD at the appropriate energy scale.

6 Connection to the Dualised Standard Model

6.1 The Higgs matrix of Lecture 7

In Lecture 7, following Cacciapaglia and Sannino [2], we constructed the mes-Higgs field Φ_H of the dualised Standard Model. This field decomposes into 18 Higgs doublets organised by generation (Lecture 7, Eq. (7)):

$$\Phi_H \;=\; \{H_{ij}\}\,, \qquad i,j = 1,2,3\,, \tag{27}$$

where each $H_{ij} = (H^u_{ij},\, H^d_{ij})$ is a pair of $\mathrm{SU}(2)_L$ doublets distinguished by hypercharge ($Y = +1/2$ for H^u, $Y = -1/2$ for H^d).

6.2 From generation indices to flavour indices

The generation indices $i, j = 1, 2, 3$ in H_{ij} correspond to the three generations of quarks. In the Pati–Salam decomposition used in Lecture 7 [18], each generation contains one up-type and one down-type quark:

$$\text{Gen. 1: } (u,d)\,, \qquad \text{Gen. 2: } (c,s)\,, \qquad \text{Gen. 3: } (t,b)\,. \tag{28}$$

When $\mathrm{SU}(2)_L$ acts on the left-handed doublets and $\mathrm{SU}(2)_R$ acts on the right-handed fields, the H^u and H^d components of H_{ij} couple to the up-type and down-type sectors respectively (Lecture 7, Eq. (12)):

$$y \sum_{i,j} \big(q^i_L\, u^j_R\, H^u_{ij} + q^i_L\, d^j_R\, H^d_{ij}\big)\,. \tag{29}$$

The 3×3 generation-indexed Higgs matrix H_{ij} is therefore related to the 6×6 flavour-indexed meson matrix $\mathcal{M}_{ij}$ as follows:

- The H^u_{ij} components ($i, j = 1, 2, 3$) correspond to the up-type flavour block: combinations involving $(u, c, t) \times (\bar{u}, \bar{c}, \bar{t})$.
- The H^d_{ij} components correspond to the down-type flavour block: combinations involving $(d, s, b) \times (\bar{d}, \bar{s}, \bar{b})$.
- The off-diagonal mixing between up-type and down-type quarks (the charged-current mesons) arises from the $\mathrm{SU}(2)_L$ doublet structure of the left-handed quarks.

Remark 6.1 (The Higgs-meson correspondence). The 3×3 Higgs matrix $\Phi_H = \{H_{ij}\}$ from Lecture 7 and the 6×6 meson matrix $\mathcal{M}_{ij}$ from this lecture are **different presentations of the same underlying object**: the meson composite of the electric theory, which becomes an elementary field in the magnetic theory. The Higgs matrix uses generation indices and separates the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ structure explicitly; the meson matrix uses flavour indices and makes the quark content of each entry manifest.

6.3 The universal Yukawa and the meson–quark coupling

From Eq. (29), the Yukawa coupling between quarks and Higgs fields is **universal**: the same coupling y appears for every generation pair. In the meson language, this means that the coupling of a quark to "its" composite meson is generation-independent.

The effective Yukawa matrices (Lecture 7, Eq. (14)):

$$Y^u_{ij} = y\,\frac{\langle H^u_{ij}\rangle}{v}\,, \qquad Y^d_{ij} = y\,\frac{\langle H^d_{ij}\rangle}{v}\,, \tag{30}$$

show that the mass hierarchy arises entirely from the VEV hierarchy:

$$\langle H^u_{33} \rangle \gg \langle H^u_{22} \rangle \gg \langle H^u_{11} \rangle \,, \tag{31}$$

which, as shown in Lecture 8, is generated by Planck-scale operators [2].

6.4 What the dualised SM adds beyond standard QCD

It is essential to state clearly what the dualised SM framework contributes to our understanding of the meson spectrum, and what it does *not*.

What the dualised SM provides:

(a) **Structural origin of the meson matrix.** In standard QCD, the meson flavour matrix $q_i \bar{q}_j$ is a phenomenological classification scheme. In the dualised SM, *if the duality holds*, this matrix is inherited from the UV dual theory: the meson field is *elementary* in the magnetic description, and its $N_f \times N_f$ structure follows from the gauge and flavour symmetries of the electric theory.

(b) **Universal Yukawa coupling.** The single coupling y for all generation pairs explains why there is a universal meson–quark interaction at the level of the magnetic Lagrangian. The mass hierarchy comes from the VEV hierarchy, not from different couplings.

(c) **Connection to the Higgs sector.** The 18 Higgs doublets $H^{u,d}_{ij}$ of Lecture 7 *are* the meson composites after Pati–Salam decomposition. This unifies the electroweak symmetry breaking sector (Higgs) with the hadronic spectrum (mesons) within a single framework—the mes-Higgs field Φ_H.

(d) **Scalar democracy.** The VEV hierarchy $\langle H^u_{33} \rangle \gg \langle H^u_{22} \rangle \gg \langle H^u_{11} \rangle$ from Planck-scale operators (Lecture 8) provides a framework for the fermion mass hierarchy that goes beyond the Standard Model's arbitrary Yukawa couplings.

What the dualised SM does not provide:

(a) **New quantitative predictions for meson masses.** The masses of π, K, D, B, etc. are determined by QCD dynamics (quark masses plus confinement). The dualised SM does not provide an independent calculation of these quantities.

(b) **A derivation of f_π or $\langle \bar{q}\, q \rangle$ from the dual theory.** These are QCD quantities determined by the strong dynamics at Λ_{QCD}.

(c) **A proof that the SUSY-to-QCD transition preserves the meson identification.** This is the central unresolved question. The Sannino–Schechter mechanism (Section 4.4) and Komargodski's bridge (Section 4.5) provide qualitative arguments, but a rigorous proof is lacking.

Honest framing

The dualised Standard Model provides a **dual perspective** on the meson spectrum that is **consistent with**, but does not improve upon, standard QCD. The added value is **conceptual**—why the meson matrix has the structure it does—and **structural**—the connection between the Higgs sector and the hadronic spectrum via scalar democracy. It is **not** a new prediction for the meson spectrum.

7 Summary and Outlook

7.1 Summary of the 10-lecture course

This lecture completes a 10-lecture course on non-perturbative gauge dynamics and the dualised Standard Model. The arc of the course is:

Lectures	Topic
1–2	Foundations: anomaly matching, SUSY introduction
3–4	Seiberg duality: SU(N_c) SQCD, $\mathcal{N} = 2$ origin
5–6	Non-perturbative dynamics: tumbling, complementarity, non-SUSY extensions
7–8	The dualised Standard Model: construction, spectrum, scales
9–10	Extensions: the duality landscape, physical meson identification

The key conceptual arc runs from abstract duality (Lecture 3) through its conjectured non-SUSY extension (Lectures 5–6) to the physical particle spectrum (Lectures 7–10). Along the way, we have:

- Verified anomaly matching as the consistency condition for dualities (Lectures 1–3);
- Constructed the Seiberg duality map and its evidence from holomorphic dynamics and the conformal window (Lectures 3–4);
- Explored the landscape of SUSY dualities for SU, SO, and Sp gauge groups (Lecture 9);
- Applied the non-SUSY duality conjecture to construct the dualised Standard Model (Lectures 7–8);
- Connected the abstract meson matrix of the dual theory to the observed physical meson spectrum (this lecture).

7.2 Open questions

Several important questions remain open:

1. **Does the dualised SM predict anything new about collider physics?** The quasi-SUSY partners (gaugino, squarks, mesino) predicted in Lecture 7 should appear at the scale $\Lambda_S \sim$ few TeV, *if the duality holds*. Their discovery—or non-discovery—at the LHC or future colliders would provide decisive evidence for or against the framework.

2. **Can the SUSY-breaking transition be made quantitative?** The Sannino–Schechter toy model (Section 4.4) and Komargodski's bridge (Section 4.5) provide qualitative arguments, but a quantitative treatment of the hard SUSY-breaking regime ($m_{\text{soft}} \sim \Lambda_{\text{QCD}}$) remains an open challenge.

3. **What is the role of the duality cascade (Lecture 9) in the UV completion?** The Klebanov–Strassler cascade provides a natural mechanism for reducing the rank of the gauge group through successive duality steps. Whether this mechanism can be applied to the dualised SM's UV completion is an open question.

4. **Can lattice simulations constrain the $\mathcal{O}(1)$ coefficients?** The $\mathcal{O}(1)$ coefficients c^{abcd} in the Planck-scale operators (Lecture 8) are currently free parameters. Lattice simulations of the electric theory could, in principle, constrain these coefficients by computing the meson spectrum directly.

7.3 Suggestions for further reading

- **Seiberg duality:** Intriligator and Seiberg [14] (comprehensive review); Peskin [15] (TASI lectures, pedagogical); Terning [16] (textbook treatment).
- **Dualised Standard Model:** Cacciapaglia and Sannino [2] (foundational paper).
- **Chiral perturbation theory:** Gasser and Leutwyler [4,5] (original papers); Scherer [17] (review).
- **Hidden local symmetry and vector mesons:** Bando, Kugo, and Yamawaki [10] (original framework); Komargodski [9] (SQCD connection).

Exercises

Exercise 7.1 (Charge verification)**.** Verify the electric charge of every off-diagonal entry in the 6×6 meson matrix (Table 1) using $Q_{\text{em}} = Q(q_i) - Q(q_j)$. Confirm that each matches the known meson charge from the PDG [3].

Exercise 7.2 (Goldstone boson counting)**.** For $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \to \mathrm{SU}(N_f)_V$:

(a) Show that the number of Goldstone bosons is $N_f^2 - 1$.

(b) For $N_f = 2$, list the three Goldstone bosons and their quantum numbers.

(c) For $N_f = 3$, list the eight Goldstone bosons (the pseudoscalar octet).

(d) For $N_f = 6$, how many formal Goldstone bosons would there be? Why is this count physical only for the light sector?

Exercise 7.3 (The GMOR relation for kaons)**.** Using the GMOR relation for the kaon:

$$m_K^2\, f_\pi^2 \;=\; -(m_u + m_s)\,\langle \bar{q}\, q\rangle\,,$$

and the inputs $m_u + m_s \approx 103$ MeV (at $\mu = 2$ GeV), $f_\pi = 92.1$ MeV, $\langle \bar{q}\, q\rangle \approx -(270\text{ MeV})^3$:

(a) Compute m_K and compare with the PDG value $m_{K^\pm} = 493.7$ MeV.

(b) Comment on the accuracy of the GMOR relation for strange quarks. Why might you expect larger corrections than for pions?

Exercise 7.4 (Linear sigma model decomposition)**.** Starting from the complex field $\phi = (v + \sigma + i\pi)/\sqrt{2}$ with the potential $V(\phi) = \lambda(|\phi|^2 - v^2/2)^2$:

(a) Show that σ acquires a mass $m_\sigma^2 = 2\lambda v^2$ while π is massless (the Goldstone boson).

(b) Add an explicit symmetry-breaking term $\epsilon\,\mathrm{Re}(\phi)$ and show that π acquires a mass $m_\pi^2 = \epsilon/v$.

(c) Identify ϵ with the quark mass term and recover the GMOR relation.

Exercise 7.5 (The Higgs–meson correspondence)**.** Using the generation-to-flavour map of Eq. (28):

(a) Write out the 3×3 matrix H^u_{ij} explicitly, identifying each entry with a specific up-type meson combination.

(b) Do the same for H^d_{ij}.

(c) Show that the total number of independent meson entries in the 6×6 matrix exceeds 18 (the number of Higgs doublets), and explain what happens to the additional entries (charged-current mixing between up-type and down-type quarks).

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