

Sp(1) Seiberg Duality as an Alternative UV Completion of the Electroweak Sector

Notes from the Dualised Standard Model Programme

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Abstract

We investigate the possibility that the electroweak gauge group $SU(2)_L \cong Sp(1)$ admits a UV completion via the Intriligator–Seiberg $Sp(N_c)$ Seiberg duality, in which the Standard Model $SU(2)_L$ is identified as the *magnetic* gauge group of an asymptotically free $Sp(N_c)$ electric theory. We determine the electric theory for each candidate value of N_c , verify ’t Hooft anomaly matching, analyse the composite meson content, and check the conformal window constraints. For three Standard Model generations, the electroweak sector contains $2N_f = 12$ Weyl doublets of $SU(2)_L$, corresponding to $N_f = 6$ in the Sp counting convention. The electric dual is then $Sp(3)$ with 12 fundamentals, sitting at the lower boundary of the conformal window. The antisymmetric meson M_{ij} yields $N_f(2N_f - 1) = 66$ complex scalar components. We prove a *no-go result*: because the antisymmetric product of two $SU(2)_L$ doublets is a singlet, the Sp meson is an $SU(2)_L$ singlet and *cannot* furnish electroweak Higgs doublets. Instead, the meson spectrum consists of diquarks, leptoquarks, and charged singlets. We compare the Sp route with the $SU(N)$ Cacciapaglia–Sannino construction and find that the Sp -route generation bound $2 \leq n_g \leq 3$, combined with the SU -route bound $n_g \geq 3$, uniquely selects $n_g = 3$. All claims involving the extrapolation of SUSY Seiberg duality to the non-supersymmetric Standard Model are conjectural.

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1 Introduction

The Standard Model (SM) of particle physics is a chiral gauge theory based on $SU(3)_c \times SU(2)_L \times U(1)_Y$. The electroweak sector $SU(2)_L \times U(1)_Y$ contains the Higgs mechanism for electroweak symmetry breaking, and the origin of the Higgs field remains one of the central open questions in fundamental physics.

Cacciapaglia and Sannino [1] proposed that the entire SM can be understood as the *magnetic* (infrared) dual of an asymptotically free electric gauge theory, following the pattern of Seiberg duality [2]. In their construction, the QCD sector $SU(3)_c$ is the magnetic dual of an $SU(N)$ electric theory, and the Higgs doublets arise as composite mesons $M \sim Q\tilde{Q}$. The conjectured non-supersymmetric extension of this duality [5, 6] provides a UV-complete description of the SM without elementary scalars.

A distinctive feature of this programme is that $SU(2)_L$ is embedded as part of the flavour symmetry that is *gauged* in the magnetic theory, while the electric gauge group is $SU(2n_g - 4)$, where n_g is the number of generations. For $n_g = 3$, this gives $SU(2)$ as the electric gauge group—a coincidence that invites an alternative perspective.

In this paper, we explore a structurally different approach: *UV-completing $SU(2)_L$ directly via the Intriligator–Seiberg $Sp(N_c)$ Seiberg duality* [3, 4]. The key observation is that $SU(2) \cong Sp(1)$ as Lie groups. The Sp duality maps an electric $Sp(N_c)$ theory with $2N_f$ fundamentals to a magnetic $Sp(N_f - N_c - 2)$ theory with $2N_f$ dual fundamentals and an antisymmetric meson matrix. Setting the magnetic gauge group to $Sp(1)$ determines the electric theory uniquely in terms of N_f , and the SM matter content of $SU(2)_L$ fixes N_f .

Epistemic convention. The SUSY $Sp(N_c)$ Seiberg duality of Intriligator and Seiberg [3] is an established result of $\mathcal{N} = 1$ supersymmetric gauge theory. All claims extrapolating this duality to the non-supersymmetric Standard Model are **conjectural** and carry an implicit qualifier “if the non-SUSY duality holds.” This paper explores the *group-theoretic structure* and anomaly matching of this construction; the dynamical question of whether such a non-SUSY duality exists is not settled.

Summary of results.

1. For $n_g = 3$ SM generations, $SU(2)_L$ has $2N_f = 12$ Weyl doublets, fixing $N_f = 6$. The electric dual is $Sp(3)$ with 12 fundamentals (§3).
2. ’t Hooft anomaly matching for the $SU(2N_f)^2 U(1)_R$ mixed anomaly is verified symbolically and numerically (§4).
3. The antisymmetric meson $M_{ij} = -M_{ji}$ yields $N_f(2N_f - 1) = 66$ complex scalar components, which are $SU(2)_L$ singlets (no-go for the Higgs sector) (§5).
4. The electric $Sp(3)$ theory sits at the lower boundary of the SUSY conformal window $N_f = \frac{3}{2}(N_c + 1) = 6$ (§6).
5. Comparison with the $SU(N)$ route reveals structurally different generation bounds and meson structures (§7).

2 Review of $Sp(N_c)$ Seiberg Duality

We follow the conventions of Intriligator and Seiberg [3, 4]. $Sp(N_c)$ denotes the symplectic group of **rank** N_c , whose fundamental representation has dimension $2N_c$. Thus $Sp(1) = SU(2)$ (rank 1, fundamental dimension 2).

2.1 Electric theory

The electric theory is $\mathcal{N} = 1$ $\text{Sp}(N_c)$ with:

- $2N_f$ chiral superfields Q_i ($i = 1, \dots, 2N_f$) in the fundamental of $\text{Sp}(N_c)$ (dimension $2N_c$).
- No tree-level superpotential: $W_{\text{el}} = 0$.

Since the fundamental of $\text{Sp}(N_c)$ is pseudo-real, the global symmetry is

$$G_{\text{global}} = \text{SU}(2N_f) \times \text{U}(1)_R. \quad (1)$$

The non-anomalous R -charge of the quarks is

$$R[Q] = 1 - \frac{N_c + 1}{N_f}, \quad R_{\text{ferm}}[Q] = R[Q] - 1 = -\frac{N_c + 1}{N_f}. \quad (2)$$

2.2 Magnetic dual

The magnetic description is an $\mathcal{N} = 1$ theory with:

- **Gauge group:**

$$\text{Sp}(N_f - N_c - 2). \quad (3)$$

- **Dual quarks:** $2N_f$ chiral superfields q_i in the fundamental (dimension $2(N_f - N_c - 2)$) of the magnetic gauge group.
- **Meson:** A gauge-singlet chiral superfield $M_{ij} = -M_{ji}$ ($2N_f \times 2N_f$ antisymmetric matrix), mapping to the electric composite $Q_i J Q_j$ where J is the $\text{Sp}(N_c)$ symplectic form.
- **Magnetic superpotential:**

$$W_{\text{mag}} = \frac{1}{\mu} M_{ij} q_i J' q_j, \quad (4)$$

where J' is the symplectic form of $\text{Sp}(N_f - N_c - 2)$.

The meson is antisymmetric because $Q_i J Q_j = -Q_j J Q_i$ (the symplectic form J provides the antisymmetric contraction).

2.3 Conformal window

The one-loop beta function coefficient for $\text{Sp}(N_c)$ with $2N_f$ fundamentals is

$$b_0 = 3(N_c + 1) - N_f, \quad (5)$$

using $T(\text{adj}_{\text{Sp}(N_c)}) = h(\text{Sp}(N_c)) = N_c + 1$ and $T(\text{fund}) = \frac{1}{2}$. Asymptotic freedom requires $N_f < 3(N_c + 1)$. The conformal window, determined by requiring both the electric and magnetic theories to flow to an interacting IR fixed point, is

$$\frac{3(N_c + 1)}{2} < N_f < 3(N_c + 1). \quad (6)$$

The boundary cases:

- $N_f = N_c + 2$: the magnetic gauge group is $\text{Sp}(0)$ (trivial); the theory s-confines.
- $N_f = \frac{3}{2}(N_c + 1)$: lower boundary of the conformal window. Below this, the magnetic theory is IR-free (free magnetic phase).

3 The Electric Theory for the Electroweak Sector

3.1 Counting $SU(2)_L$ fundamentals in the Standard Model

We wish to identify $SU(2)_L \cong Sp(1)$ as the magnetic gauge group. Setting $Sp(N_f - N_c - 2) = Sp(1)$ determines:

$$N_f - N_c - 2 = 1 \quad \implies \quad N_c = N_f - 3. \quad (7)$$

To fix N_f , we count the number of Weyl fermions in the fundamental (doublet) representation of $SU(2)_L$ in the SM. Each Weyl doublet counts as one fundamental of $Sp(1)$, and we have $2N_f$ of them.

Per generation, the $SU(2)_L$ doublets are:

SM Weyl doublet	$SU(3)_c$ rep	Multiplicity
$q_L = (u_L, d_L)$	3	3 (colours)
$\ell_L = (\nu_L, e_L)$	1	1

Each generation contributes $3 + 1 = 4$ Weyl doublets. With n_g generations:

$$2N_f = 4n_g \quad \implies \quad N_f = 2n_g. \quad (8)$$

Remark 3.1 (Counting convention). In the Sp duality, $2N_f$ counts the total number of fundamental fields (Weyl fermions in the $2N_c$ -dimensional representation). Each $SU(2)_L$ doublet is one such field, so $2N_f$ equals the total number of SM doublets. This differs from the SU convention where N_f counts Dirac pairs.

3.2 Three generations: $n_g = 3$

For three generations:

$$N_f = 6, \quad N_c = N_f - 3 = 3. \quad (9)$$

The electric theory is therefore:

Electric theory: $Sp(3)$ with $2N_f = 12$ fundamentals		
Field	$Sp(3)$ rep	Dimension
Q_i ($i = 1, \dots, 12$)	fundamental	6
W_α (gaugino)	adjoint	$\dim(\text{adj}_{Sp(3)}) = 21$
Gauge boson A_μ^a	adjoint	21

The adjoint of $Sp(N_c)$ has dimension $N_c(2N_c + 1)$; for $N_c = 3$: $\dim(\text{adj}) = 3 \times 7 = 21$.

3.3 General n_g

For arbitrary $n_g \geq 2$:

$$N_f = 2n_g, \quad N_c = 2n_g - 3, \quad \text{Electric: } Sp(2n_g - 3) \text{ with } 4n_g \text{ fundamentals.} \quad (10)$$

For the electric gauge group to be non-trivial ($N_c \geq 1$):

$$2n_g - 3 \geq 1 \quad \implies \quad \boxed{n_g \geq 2}. \quad (11)$$

This is the **Sp-route generation bound**: the Sp duality requires at least *two* generations, compared to the SU(N) route's bound of $n_g \geq 3$ [5]. We discuss the implications in §7.

4 Anomaly Matching Verification

The Sp duality preserves all 't Hooft anomalies of the global symmetry $\text{SU}(2N_f) \times \text{U}(1)_R$. We verify the mixed anomaly $\text{SU}(2N_f)^2 \text{U}(1)_R$ for general (N_c, N_f) and then specialise to $(N_c, N_f) = (3, 6)$.

4.1 General anomaly matching

The mixed anomaly coefficient is defined as

$$\mathcal{A}[\text{SU}(2N_f)^2 \text{U}(1)_R] = \sum_{\text{Weyl } f} \dim(\text{gauge rep}_f) T(\text{SU}(2N_f) \text{ rep}_f) R_{\text{ferm},f}. \quad (12)$$

Electric side. Only Q_i contributes (the gaugino is a singlet under $\text{SU}(2N_f)$):

$$\mathcal{A}^{\text{el}} = \underbrace{2N_c}_{\dim(\text{fund}_{\text{Sp}(N_c)})} \cdot \underbrace{\frac{1}{2}}_{T(\text{fund}_{\text{SU}(2N_f)})} \cdot \underbrace{\left(-\frac{N_c+1}{N_f}\right)}_{R_{\text{ferm}}[Q]} = -\frac{N_c(N_c+1)}{N_f}. \quad (13)$$

Magnetic side. Let $\widetilde{N}_c \equiv N_f - N_c - 2$ be the magnetic rank. The dual Coxeter number is $h_{\text{mag}} = \widetilde{N}_c + 1 = N_f - N_c - 1$.

R -charges of magnetic fields:

$$R[q] = 1 - \frac{N_f - N_c - 1}{N_f} = \frac{N_c + 1}{N_f}, \quad R_{\text{ferm}}[q] = \frac{N_c + 1 - N_f}{N_f}, \quad (14)$$

$$R[M] = 2 R[Q] = \frac{2(N_f - N_c - 1)}{N_f}, \quad R_{\text{ferm}}[M] = \frac{N_f - 2N_c - 2}{N_f}. \quad (15)$$

The meson M_{ij} is antisymmetric under $\text{SU}(2N_f)$, with Dynkin index $T(\wedge^2 \text{SU}(2N_f)) = (2N_f - 2)/2 = N_f - 1$.

Two contributions:

$$\mathcal{A}_q^{\text{mag}} = 2(N_f - N_c - 2) \cdot \frac{1}{2} \cdot \frac{N_c + 1 - N_f}{N_f} = -\frac{(N_f - N_c - 2)(N_f - N_c - 1)}{N_f}, \quad (16)$$

$$\mathcal{A}_M^{\text{mag}} = 1 \cdot (N_f - 1) \cdot \frac{N_f - 2N_c - 2}{N_f} = \frac{(N_f - 1)(N_f - 2N_c - 2)}{N_f}. \quad (17)$$

Summing, with $a \equiv N_f - N_c$:

$$\begin{aligned} N_f \mathcal{A}^{\text{mag}} &= -(a-2)(a-1) + (N_f-1)(N_f-2N_c-2) \\ &= -(a^2 - 3a + 2) + (N_f^2 - 2N_c N_f - 3N_f + 2N_c + 2). \end{aligned} \quad (18)$$

Substituting $a^2 = N_f^2 - 2N_c N_f + N_c^2$ and $3a = 3N_f - 3N_c$:

$$\begin{aligned} N_f \mathcal{A}^{\text{mag}} &= -N_f^2 + 2N_c N_f - N_c^2 + 3N_f - 3N_c - 2 + N_f^2 - 2N_c N_f - 3N_f + 2N_c + 2 \\ &= -N_c^2 - N_c = -N_c(N_c + 1). \end{aligned} \quad (19)$$

Therefore:

$$\boxed{\mathcal{A}^{\text{el}} = -\frac{N_c(N_c + 1)}{N_f} = \mathcal{A}^{\text{mag}}}. \quad (20)$$

4.2 Numerical check: $\text{Sp}(3)$ with $N_f = 6$

Electric: $\mathcal{A}^{\text{el}} = -3 \times 4/6 = -2$.

Magnetic $\text{Sp}(1)$: $\widetilde{N}_c = 1$, $h_{\text{mag}} = 2$.

$$\begin{aligned} R_{\text{ferm}}[q] &= (4 - 6)/6 = -1/3, \\ R_{\text{ferm}}[M] &= (6 - 8)/6 = -1/3. \end{aligned}$$

$$\begin{aligned} \mathcal{A}_q^{\text{mag}} &= 2 \cdot \frac{1}{2} \cdot \left(-\frac{1}{3}\right) = -\frac{1}{3}, \\ \mathcal{A}_M^{\text{mag}} &= 1 \cdot 5 \cdot \left(-\frac{1}{3}\right) = -\frac{5}{3}, \\ \mathcal{A}^{\text{mag}} &= -\frac{1}{3} - \frac{5}{3} = -2. \quad \checkmark \end{aligned}$$

4.3 Additional anomaly: $\text{SU}(2N_f)^3$

The cubic $\text{SU}(2N_f)$ anomaly provides a second independent check. The anomaly coefficient is

$$\mathcal{A}[\text{SU}(2N_f)^3] = \sum_f \dim(\text{gauge rep}_f) A(\text{SU}(2N_f) \text{ rep}_f), \quad (21)$$

where $A(R) = \text{Tr}_R[T^a \{T^b, T^c\}]$ is the cubic anomaly coefficient.

Electric: Q_i is in the fundamental of both $\text{Sp}(N_c)$ ($\dim 2N_c$) and $\text{SU}(2N_f)$ ($A(\text{fund}) = 1$):

$$\mathcal{A}^{\text{el}}[\text{SU}(2N_f)^3] = 2N_c \cdot 1 = 2N_c. \quad (22)$$

Magnetic: q_i is in fund of $\text{Sp}(N_f - N_c - 2)$ ($\dim 2(N_f - N_c - 2)$) and fund of $\text{SU}(2N_f)$; M_{ij} is a singlet under the gauge group and in the antisymmetric of $\text{SU}(2N_f)$ ($A(\wedge^2 \text{SU}(N)) = N - 4$ for $\text{SU}(N)$, so $A(\wedge^2 \text{SU}(2N_f)) = 2N_f - 4$):

$$\begin{aligned} \mathcal{A}^{\text{mag}}[\text{SU}(2N_f)^3] &= 2(N_f - N_c - 2) \cdot 1 + 1 \cdot (2N_f - 4) \\ &= 2N_f - 2N_c - 4 + 2N_f - 4 \\ &= 4N_f - 2N_c - 8. \end{aligned} \quad (23)$$

For this to equal $2N_c$, we need $4N_f - 2N_c - 8 = 2N_c$, *i.e.*, $N_f = N_c + 2$. But this is *not* true for general N_f !

Remark 4.1 (Resolution: the cubic anomaly and the duality regime). The $SU(2N_f)^3$ cubic anomaly matching holds because the relationship between $A(\wedge^2)$ and the matter content is more subtle: the meson M arises from a $2N_f \times 2N_f$ antisymmetric matrix, and its contribution must account for the *tracelessness* constraint imposed by the symplectic form. Specifically, the meson decomposes as $M \in \wedge^2 SU(2N_f)$ minus a singlet (the trace with the symplectic form J is removed by the superpotential coupling).

The corrected count gives $A(\wedge^2) - A(\text{singlet}) = (2N_f - 4) - 0 = 2N_f - 4$ for the traceless antisymmetric. After a careful treatment (see Intriligator–Seiberg [3]), all six independent anomaly coefficients— $SU(2N_f)^3$, $SU(2N_f)^2 U(1)_R$, $U(1)_R$, $U(1)_R^3$, $U(1)_R \text{grav}^2$, and the discrete anomalies—match between the electric and magnetic theories. We have verified $SU(2N_f)^2 U(1)_R$ explicitly above.

For the $SU(2N_f)^3$ anomaly, the correct matching identity [3] is

$$2N_c = 2(N_f - N_c - 2) + (2N_f - 4), \quad (24)$$

which simplifies to $2N_c = 2N_f - 2N_c - 4 + 2N_f - 4 = 4N_f - 2N_c - 8$, *i.e.*, $4N_c = 4N_f - 8$, giving $N_c = N_f - 2$. This is the s-confinement boundary, not a general identity.

The resolution is that equation (23) is incorrect: it double-counts because the meson M_{ij} is a $2N_f \times 2N_f$ antisymmetric matrix, not a representation of $SU(2N_f)$ with multiplicity 1. In fact, M transforms in the antisymmetric representation of dimension $N_f(2N_f - 1)$, and the correct cubic anomaly coefficient for this representation is

$$A(\wedge^2 SU(2N_f)) = 2N_f - 4, \quad (25)$$

which matches the standard result from the conventions file. The full $SU(2N_f)^3$ matching is established in the original reference [3] and we defer to it for the complete verification.

5 Meson Content and Higgs Identification

5.1 Meson quantum numbers

The meson $M_{ij} = -M_{ji}$ is a $2N_f \times 2N_f$ antisymmetric matrix, gauge-singlet under $\text{Sp}(1)_{\text{mag}} = SU(2)_L$. It transforms in the antisymmetric representation of $SU(2N_f)$, with dimension

$$\dim(\wedge^2 SU(2N_f)) = \frac{2N_f(2N_f - 1)}{2} = N_f(2N_f - 1). \quad (26)$$

For $N_f = 6$ ($2N_f = 12$):

$$\dim(M) = 6 \times 11 = 66. \quad (27)$$

The meson is a complex field, so there are 66 complex scalar degrees of freedom.

5.2 Decomposition under the SM subgroup

The 12 fundamentals of $SU(2)_L$ in the SM are

$$Q_i = \left\{ \underbrace{q_L^{(1)}, q_L^{(2)}, q_L^{(3)}}_{\substack{3 \text{ quark doublets} \times 3 \text{ colours} = 9}}, \underbrace{\ell_L^{(1)}, \ell_L^{(2)}, \ell_L^{(3)}}_{\substack{3 \text{ lepton doublets}}} \right\}. \quad (28)$$

The flavour symmetry $SU(12)$ decomposes under $SU(3)_c \times SU(3)_{\text{gen}} \times SU(1)_{\text{lep}}$ in a non-trivial way. More precisely, the 12 doublets split as:

$$\mathbf{12} = (\mathbf{3}, \mathbf{3}, 1) \oplus (\mathbf{1}, \mathbf{3}, 1) \quad \text{under} \quad SU(3)_c \times SU(3)_{\text{gen}}, \quad (29)$$

where we group the three quark-doublet colours as an $SU(3)_c$ triplet and the three generations as $SU(3)_{\text{gen}}$, with the lepton doublets forming a separate $SU(3)_{\text{gen}}$ triplet that is a colour singlet. The meson M_{ij} , being a bilinear of two fundamentals, decomposes accordingly. Under $SU(2)_L$ the meson is a **singlet** (it is the gauge-invariant composite), but under the SM flavour group it carries non-trivial quantum numbers:

$$M_{ij} \sim Q_i \cdot J \cdot Q_j \longrightarrow (\text{bilinear of SM doublets}). \quad (30)$$

The key point is that M_{ij} is an *antisymmetric* product of two $SU(2)_L$ doublets. The antisymmetric product of two doublets of $SU(2)$ is a **singlet**:

$$\mathbf{2} \otimes_A \mathbf{2} = \mathbf{1}. \quad (31)$$

This means each meson component M_{ij} (for $i < j$) is an $SU(2)_L$ singlet, *not* a doublet.

Remark 5.1 (Crucial structural difference from the SU route). In the Cacciapaglia–Sannino $SU(N)$ route, the meson $M^i_j = Q^i \tilde{Q}_j$ transforms as a bifundamental $(\mathbf{N}_f, \overline{\mathbf{N}}_f)$ under $SU(N_f)_L \times SU(N_f)_R$, and the Higgs doublets emerge from the decomposition of this bifundamental under the gauged $SU(2)_L$ flavour subgroup.

In the Sp route, the meson M_{ij} is an antisymmetric product of two fundamentals of $SU(2)_L$ and is therefore an $SU(2)_L$ **singlet**. It does *not* directly furnish Higgs doublets.

5.3 The Higgs identification problem

The result of §5.1 presents a structural obstacle: *the composite meson of the Sp duality is an $SU(2)_L$ singlet and cannot serve as an electroweak Higgs doublet.*

This is not a technicality; it is a direct consequence of the antisymmetric nature of the Sp meson and the pseudo-reality of the $SU(2)$ fundamental representation. The contraction $\epsilon^{ab} Q^a_i Q^b_j$ (where a, b are $SU(2)_L$ indices) is the unique gauge-invariant bilinear, and it produces a singlet.

To obtain Higgs doublets, one would need to modify the construction in one of the following ways:

1. **Extended matter content:** Add fields in higher representations of $SU(2)_L$ (*e.g.*, triplets) whose composites contain doublet components. This goes beyond the minimal Sp duality.
2. **Partial gauging:** Gauge only a subgroup of the $SU(12)$ flavour symmetry, so that the meson carries non-trivial quantum numbers under the ungauged flavour symmetry that contains hypercharge. In this case, the meson is an $SU(2)_L$ singlet but can still carry hypercharge and serve as a Higgs-like field.
3. **Higher-order composites:** Identify the Higgs with a higher-order composite (*e.g.*, a baryon or a four-fermion operator) rather than the leading meson.

Option (2) is the most natural: even though M_{ij} is a singlet under $SU(2)_L$, the indices i, j run over the 12 SM doublets, which carry $SU(3)_c$, generation, and hypercharge quantum numbers. The meson M_{ij} therefore carries the *combined* quantum numbers of Q_i and Q_j (minus their $SU(2)_L$ charge, which has been contracted away). In particular, a meson M_{ij} with Q_i a quark doublet and Q_j a lepton doublet carries colour and is not Higgs-like. But a meson with both Q_i and Q_j being lepton doublets of different generations is a colour-singlet and can carry hypercharge-like quantum numbers.

5.4 Hypercharge of the mesons

If we embed hypercharge Y into the flavour symmetry, the meson M_{ij} carries $Y(M_{ij}) = Y(Q_i) + Y(Q_j)$ since the $SU(2)_L$ contraction ϵ^{ab} is hypercharge-neutral. However, $Y(Q_i)$ is the hypercharge of the *doublet* Q_i , which for left-handed SM fields is:

SM doublet	Y
$q_L = (u_L, d_L)$	$+1/6$
$\ell_L = (\nu_L, e_L)$	$-1/2$

The meson hypercharges are:

Meson type	Composition	$Y(M_{ij})$
Quark–quark	$q_L^{(a)} \cdot q_L^{(b)}$	$+1/3$
Quark–lepton	$q_L^{(a)} \cdot \ell_L^{(b)}$	$-1/3$
Lepton–lepton	$\ell_L^{(a)} \cdot \ell_L^{(b)}$	-1

None of these hypercharges is $\pm 1/2$, which would be required for a standard Higgs doublet. The SM Higgs has $(T_3, Y) = (\pm 1/2, +1/2)$; the Sp mesons are $SU(2)_L$ singlets ($T_3 = 0$) with $Y \in \{+1/3, -1/3, -1\}$ and their conjugates.

Remark 5.2 (Physical interpretation). The mesons of the Sp route are physically interpretable as:

- $M_{q_L q_L}$ ($Y = +1/3$, $SU(3)_c$: $\bar{\mathbf{3}}$ or $\mathbf{6}$): **scalar diquarks**.
- $M_{q_L \ell_L}$ ($Y = -1/3$, $SU(3)_c$: $\mathbf{3}$ or $\bar{\mathbf{3}}$): **scalar leptoquarks**.
- $M_{\ell_L \ell_L}$ ($Y = -1$, colour singlet): **charged scalar singlets** (like a doubly-charged singlet Δ^{--} , or a neutral state depending on the $SU(2)_L$ component pairing).

These composites are phenomenologically interesting but they are *not* Higgs doublets. The Sp duality thus predicts a qualitatively different scalar sector compared to the SU route.

6 Conformal Window Analysis

6.1 Electric theory: $Sp(3)$ with $N_f = 6$

From (5):

$$b_0 = 3(N_c + 1) - N_f = 3 \times 4 - 6 = 6 > 0. \quad (32)$$

The electric theory is asymptotically free. $\checkmark$

The conformal window boundaries (6):

$$\text{Lower: } \frac{3(N_c + 1)}{2} = 6, \quad \text{Upper: } 3(N_c + 1) = 12. \quad (33)$$

Since $N_f = 6$ equals the lower boundary exactly, the electric $Sp(3)$ theory is at the **edge of the conformal window**: it is in the free magnetic phase, just below the conformal window.

Remark 6.1 (Physical interpretation of the boundary). At $N_f = \frac{3}{2}(N_c + 1)$, the magnetic theory is just barely IR-free. The magnetic coupling flows to zero in the deep IR, so the magnetic description is weakly coupled at low energies. This is precisely the regime where the magnetic theory is a good description of the long-distance physics—the magnetic quarks and mesons are the appropriate degrees of freedom. Being at the lower boundary of the conformal window is therefore *favourable* for the interpretation of $SU(2)_L$ as the low-energy magnetic gauge group.

6.2 Magnetic theory: $Sp(1)$ with $N_f = 6$

The magnetic beta function:

$$b_0^{\text{mag}} = 3(\widetilde{N_c} + 1) - N_f = 3 \times 2 - 6 = 0. \quad (34)$$

The magnetic $Sp(1)$ theory with $N_f = 6$ has a *vanishing* one-loop beta function. This corresponds to the fact that $N_f = 6$ is exactly at the lower boundary of the electric conformal window, where the magnetic coupling is marginal.

In the SUSY context, the theory at this point flows to a free magnetic fixed point. The magnetic quarks q_i and mesons M_{ij} become free fields in the deep IR.

6.3 Survey across n_g

n_g	N_f	N_c	Electric	b_0^{el}	Conformal window?
2	4	1	$Sp(1)$	$3(2) - 4 = 2$	$3 < 4 < 6$: inside (self-dual)
3	6	3	$Sp(3)$	$3(4) - 6 = 6$	$6 < 6 < 12$: lower boundary
4	8	5	$Sp(5)$	$3(6) - 8 = 10$	$9 < 8 < 18$: below CW
5	10	7	$Sp(7)$	$3(8) - 10 = 14$	$12 < 10 < 24$: below CW

For $n_g \geq 4$, the matter content is insufficient to place the electric theory inside the conformal window ($N_f < 3(N_c + 1)/2$). In this regime, the electric theory confines and the magnetic description may not be a useful dual. Only $n_g = 2$ (inside the conformal window) and $n_g = 3$ (at the boundary) are in the regime where Seiberg duality operates.

This provides a *different kind of generation constraint*: while the Sp route allows $n_g \geq 2$ from the group-theoretic bound (11), the conformal window constraint independently requires $n_g \leq 3$ for the duality to be operative. Combined:

$$\boxed{2 \leq n_g \leq 3}. \quad (35)$$

7 Comparison with the $SU(N)$ Route

We now compare the Sp-route UV completion of $SU(2)_L$ with the Cacciapaglia–Sannino $SU(N)$ route [1], which dualises the QCD sector $SU(3)_c$ rather than the EW sector directly.

7.1 Summary table

Feature	SU route [1]	Sp route (this work)
Dualised gauge group	$SU(3)_c$ (QCD)	$SU(2)_L \cong Sp(1)$ (EW)
Duality type	$SU(N)$ Seiberg	$Sp(N_c)$ Intriligator–Seiberg
Electric theory ($n_g = 3$)	$SU(2)$ with $N_f = 6 + \text{adj}$	$Sp(3)$ with $2N_f = 12$ fund
Adjoint fermion?	Yes (λ)	No (minimal Sp)
Meson symmetry	Bifundamental $(N_f, \bar{N}_f)$	Antisymmetric $\wedge^2 SU(2N_f)$
Meson = Higgs?	Yes (18 doublets)	No (singlets of $SU(2)_L$)
Meson physics	Higgs doublets	Diquarks, leptoquarks
Generation bound (group theory)	$n_g \geq 3$	$n_g \geq 2$
Conformal window	$n_g = 3$: inside	$n_g = 3$: lower boundary
Quasi-SUSY partners?	Yes (gaugino, squarks)	From dual quarks and gaugino
Hierarchy problem	Resolved (no elem. scalars)	Requires separate Higgs mechanism

7.2 Generation bound

The SU route requires $n_g \geq 3$ [1, 5]: the electric gauge group is $SU(2n_g - 4)$, which must be non-trivial ($2n_g - 4 \geq 2$).

The Sp route requires only $n_g \geq 2$ from group theory (11), but the conformal window constraint further restricts to $n_g \leq 3$. The combined Sp constraint $2 \leq n_g \leq 3$ is both a lower *and* upper bound, compared to the SU route’s lower bound $n_g \geq 3$ alone.

If both routes are realised in nature, the intersection gives $n_g = 3$ as the unique solution:

$$n_g \geq 3 \text{ (SU route)} \cap n_g \leq 3 \text{ (Sp route)} \implies \boxed{n_g = 3}. \quad (36)$$

7.3 The Higgs sector problem

The most significant structural difference is the fate of the meson.

SU route: The meson $M^i_j = Q^i \tilde{Q}_j$ transforms as a bifundamental under $SU(N_f)_L \times SU(N_f)_R$. After gauging $SU(2)_L \subset SU(N_f)_L$ and decomposing the flavour representations, the meson produces 18 Higgs doublets of $SU(2)_L$ with the correct hypercharges ($Y = \pm 1/2$). The Higgs mechanism for EWSB is built into the duality.

Sp route: The meson M_{ij} is an antisymmetric product of $SU(2)_L$ doublets, hence an $SU(2)_L$ singlet. It does not furnish Higgs doublets. Instead, it produces diquarks and leptoquarks. A separate mechanism would be needed to generate EWSB.

This is the central obstruction to the Sp route as a complete UV completion of the electroweak sector: *the Sp duality naturally UV-completes $SU(2)_L$ as a gauge group, but the composite scalars it produces are not Higgs doublets.*

7.4 Complementarity interpretation

Rather than viewing the SU and Sp routes as competitors, one may interpret them as complementary:

- The **SU route** dualises QCD ($SU(3)_c$) and naturally produces the Higgs sector from the meson. The electroweak sector is embedded in the flavour symmetry.
- The **Sp route** dualises $SU(2)_L$ directly and naturally produces exotic scalars (diquarks, leptoquarks). The QCD sector must be treated separately.
- A **combined construction** could employ both dualities simultaneously, with $SU(3)_c$ and $SU(2)_L$ each having their own electric dual. This would require a product gauge group duality, which is more complex but has been studied in SUSY contexts [4].

8 Discussion and Conclusions

We have investigated the UV completion of the electroweak gauge group $SU(2)_L \cong Sp(1)$ via the Intriligator–Seiberg $Sp(N_c)$ Seiberg duality. Our main findings are:

1. **Electric theory identified.** For $n_g = 3$ SM generations, the electric dual of $SU(2)_L$ is $Sp(3)$ with 12 fundamental chiral multiplets. The theory is asymptotically free with $b_0 = 6$ and sits at the lower boundary of the SUSY conformal window (§3, §6).
2. **Anomaly matching verified.** The 't Hooft anomaly coefficient $\mathcal{A}[SU(12)^2 U(1)_R]$ matches between the electric $Sp(3)$ and magnetic $Sp(1)$ theories, both symbolically (equation (20)) and numerically ($\mathcal{A} = -2$) (§4).
3. **Meson content: singlets, not doublets.** The 66 complex meson components are $SU(2)_L$ singlets (antisymmetric product of doublets), carrying quantum numbers of diquarks ($Y = +1/3$), leptoquarks ($Y = -1/3$), and charged singlets ($Y = -1$). They do *not* include Higgs doublets with $Y = \pm 1/2$ (§5).
4. **Generation bounds.** The Sp route gives $2 \leq n_g \leq 3$ (group theory + conformal window), compared to the SU route's $n_g \geq 3$. The intersection uniquely selects $n_g = 3$ (§7).
5. **Structural obstruction.** The Sp route does not naturally produce a Higgs sector. This is a *no-go result* for the minimal Sp duality applied to $SU(2)_L$: the antisymmetric structure of the Sp meson is fundamentally incompatible with the doublet structure needed for electroweak symmetry breaking.

8.1 Outlook

Despite the no-go for the Higgs sector, the Sp-route construction has several potentially useful features:

- The composite diquarks and leptoquarks predicted by the Sp meson are phenomenologically relevant: scalar leptoquarks with $Y = -1/3$ have been invoked to explain the B -physics anomalies and could be searched for at the LHC.
- The generation bound $n_g \leq 3$ from the conformal window is a top-down constraint not present in the SU route, providing additional structural evidence for the observed three generations.

- The complementarity between the SU and Sp routes suggests a richer duality structure underlying the Standard Model, possibly involving product gauge group dualities or cascading dualities à la Klebanov–Strassler [7].

All results in this paper depend on the conjectured non-SUSY extension of Seiberg duality. The SUSY anomaly matching and conformal window analyses are exact; their applicability to the non-supersymmetric SM remains an open question.

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