Lattice Signatures of Emergent Supersymmetry at the Conformal Window Boundary

Notes from the Dualised Standard Model Programme

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Abstract

Antipin, Mojaza, Pica, and Sannino showed that supersymmetric fixed points can emerge from non-supersymmetric RG flow in the magnetic dual of QCD-like theories, at least in the Veneziano limit. If this emergent-SUSY phenomenon persists at finite N_c, it makes specific predictions for the spectrum and operator dimensions of SU(3) gauge theories near the lower boundary N_f^* of the conformal window. We identify five lattice-measurable observables that discriminate between emergent SUSY and ordinary confining or conformal behaviour: (i) the scalar-to-pseudoscalar mass ratio m_σ/m_π, (ii) the glueball–fermion-bilinear mass relation, (iii) the mass anomalous dimension γ_m at the fixed point, (iv) a lattice SUSY Ward identity violation parameter, and (v) the effective potential for the chiral condensate. For each observable, we give the predicted value in the emergent-SUSY scenario versus the ordinary (non-SUSY) scenario for SU(3) with $N_f = 8$, 10, and 12 fundamental Dirac flavours. Existing lattice data from the LatKMI and LSD collaborations are compared with these predictions where available. We propose a concrete measurement programme to test the emergent-SUSY hypothesis. All predictions from the emergent-SUSY framework are conjectural.

Contents

1 Introduction

The conformal window of a non-abelian gauge theory is the range of matter content for which the theory possesses an infrared-stable fixed point: the theory is asymptotically free in the UV but flows to a conformal field theory in the IR. For SU(N_c) with N_f Dirac fundamental flavours, asymptotic freedom is lost at

$$N_f^{\mathrm{AF}} = \frac{11N_c}{2}\,, \tag{1}$$

giving $N_f^{\mathrm{AF}} = 16.5$ for SU(3). Below N_f^{AF}, asymptotic freedom holds. As N_f is decreased, the theory undergoes a transition at some critical value N_f^*: for $N_f < N_f^*$ the theory confines and breaks chiral symmetry spontaneously, while for $N_f^* < N_f < N_f^{\mathrm{AF}}$ the theory flows to a conformal fixed point (the Banks–Zaks fixed point [1, 2]). The value of N_f^* for SU(3) is not known analytically and is the subject of active lattice investigation, with current estimates placing it in the range $8 \lesssim N_f^* \lesssim 12$ [3–6].

In a remarkable paper, Antipin, Mojaza, Pica, and Sannino [7] demonstrated that the magnetic dual of a QCD-like theory, when augmented by Yukawa and quartic couplings, possesses perturbative fixed points where all couplings simultaneously take the values predicted by $\mathcal{N} = 1$ supersymmetry. These SUSY fixed points are reached from generic non-supersymmetric initial conditions—no fine-tuning is required—and have finite basins of attraction. The result was established in the Veneziano limit ($N_c, N_f \to \infty$ with N_f/N_c fixed), where the fixed-point couplings are perturbatively small.

If emergent SUSY persists at finite N_c and governs the physics near N_f^*, then the IR dynamics of SU(3) theories at the conformal window boundary inherits SUSY predictions. These predictions are qualitatively different from those of ordinary confining or conformal behaviour. The purpose of this paper is to identify lattice observables that sharply discriminate between the two scenarios, derive the specific SUSY predictions for SU(3) with $N_f = 8$, 10, and 12, and compare with existing lattice data.

Epistemic status. The emergent-SUSY hypothesis is a conjecture supported by perturbative evidence in the Veneziano limit. Whether it survives at $N_c = 3$ is an open question. All predictions labelled "emergent SUSY" in this paper are conditional on the conjecture being correct. The purpose of our proposed measurements is precisely to *test* this conjecture, not to assume it.

2 The non-SUSY conformal window and lattice status

2.1 Perturbative constraints on the conformal window

The one-loop coefficient of the beta function for SU(N_c) with N_f Dirac fundamental flavours is

$$b_0 = \frac{11N_c - 2N_f}{3}\,. \tag{2}$$

Asymptotic freedom ($b_0 > 0$) requires $N_f < 11N_c/2$. The two-loop coefficient is

$$b_1 = \frac{1}{3}\left(34N_c^2 - N_f\left(10N_c + \frac{3(N_c^2-1)}{N_c}\right)\right). \tag{3}$$

A perturbative (Banks–Zaks) fixed point exists when $b_0 > 0$ and $b_1 < 0$, which for SU(3) gives $N_f > 8.05$ [1]. However, the two-loop estimate for the lower boundary is unreliable for the actual non-perturbative transition; it merely provides a rough guide.

2.2 Non-perturbative estimates

Several approaches give estimates for N_f^*:

(i) **Schwinger–Dyson (ladder approximation):** Chiral symmetry breaking occurs when the mass anomalous dimension γ_m exceeds unity. Combined with the two-loop running, this gives $N_f^* \approx 4N_c = 12$ for SU(3) [12, 13].

(ii) **All-orders conjectured beta function (Ryttov–Sannino):** Using a conjectured non-perturbative beta function inspired by the NSVZ form, Ryttov and Sannino [11] estimated $N_f^* \approx (11/4)N_c = 8.25$ for SU(3), with the lower boundary corresponding to $\gamma_m = 2$.

(iii) **Functional RG and other methods:** Various functional RG and truncated Schwinger–Dyson studies give $N_f^* \in [8, 12]$ [14, 15].

The spread of estimates underscores the need for non-perturbative lattice calculations.

2.3 Lattice status for SU(3)

Several collaborations have performed large-scale lattice simulations of SU(3) with varying numbers of fundamental flavours:

$\boldsymbol{N_f = 8}$**:** The LatKMI collaboration [16] and the LSD collaboration [17] have studied this theory extensively. The spectrum shows a light scalar (σ) state with mass comparable to the pseudoscalar (π), reminiscent of a dilaton or pseudo-Nambu–Goldstone boson of approximate scale invariance. The mass anomalous dimension is measured to be $\gamma_m \approx 0.4$–1.0, depending on the analysis method. The consensus is that $N_f = 8$ is *likely below but near* the conformal window boundary, with slow ("walking") running of the coupling.

$\boldsymbol{N_f = 10}$**:** Fewer lattice studies exist at $N_f = 10$. The available data are consistent with either a conformal fixed point or very slow walking [19, 20]. This is the critical region where N_f^* is expected to lie.

$\boldsymbol{N_f = 12}$**:** This case has generated significant debate. The LatKMI collaboration [21] found evidence for a conformal fixed point with $\gamma_m \approx 0.4$–0.5, while Fodor *et al.* [6] argued for chiral symmetry breaking. The current consensus leans toward conformality, with the theory being inside or at the edge of the conformal window.

Table 1: Current lattice status for SU(3) with fundamental Dirac flavours. The mass anomalous dimension γ_m is defined via $\dim(\bar{\psi}\psi) = 3 - \gamma_m$.

N_f	Status	γ_m (lattice)	Key references
8	Confining/walking	0.4–1.0	LatKMI [16], LSD [17]
10	Uncertain	—	[19, 20]
12	Likely conformal	0.4–0.5	LatKMI [21], [6]

3 SUSY predictions for the conformal window boundary

In this section we derive the predictions that follow from the emergent-SUSY hypothesis. **All results in this section are conditional on the conjecture that SUSY governs the IR fixed point near N_f^*.**

3.1 The NSVZ beta function and the SUSY conformal window

In $\mathcal{N}=1$ SQCD with gauge group $\mathrm{SU}(N_c)$ and N_f fundamental chiral flavour pairs $(Q_i, \widetilde{Q}_i)$, the exact NSVZ beta function [8] is

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\,\frac{3N_c - N_f(1-\gamma_Q(\alpha))}{1 - N_c\,\alpha/(2\pi)}\,, \tag{4}$$

where γ_Q is the anomalous dimension of the fundamental chiral superfield Q. The denominator is non-singular in the conformal window. The fixed point $\beta(\alpha_*)=0$ requires

$$\gamma_Q^* = 1 - \frac{3N_c}{N_f}\,. \tag{5}$$

The SUSY conformal window is the range where the fixed point exists and respects the unitarity bound $\dim(Q) \geq 1$, *i.e.* $\gamma_Q^* \geq -1/2$ (using $\dim(Q) = 1 + \gamma_Q/2$ in the convention where the free-field dimension is 1). The resulting window is

$$\frac{3}{2}N_c < N_f < 3N_c\,. \tag{6}$$

For SU(3): $4.5 < N_f < 9$.

3.2 The mass anomalous dimension

The key quantity for lattice comparison is the mass anomalous dimension γ_m, defined by the scaling dimension of the fermion bilinear:

$$\dim(\bar\psi\psi) = 3 - \gamma_m\,. \tag{7}$$

In the SUSY theory, the fermion bilinear is the lowest component of the meson superfield $M^i{}_j = Q^i\widetilde{Q}_j/\Lambda$. At a superconformal fixed point, the dimension of a chiral primary operator $\mathcal{O}$ with R-charge $R(\mathcal{O})$ is fixed by the superconformal algebra [9, 10]:

$$\dim(\mathcal{O}) = \frac{3}{2}\,R(\mathcal{O})\,. \tag{8}$$

The meson has R-charge

$$R(M) = 2\,R(Q) = \frac{2(N_f - N_c)}{N_f}\,, \tag{9}$$

so its scaling dimension at the fixed point is

$$\dim(M) = \frac{3(N_f - N_c)}{N_f}\,. \tag{10}$$

Identifying $\dim(M) = \dim(\bar\psi\psi) = 3 - \gamma_m$, the SUSY prediction for the mass anomalous dimension at the fixed point is

$$\boxed{\gamma_m^{\mathrm{SUSY}} = 3 - \frac{3(N_f - N_c)}{N_f} = \frac{3N_c}{N_f}\,.} \tag{11}$$

Remark 3.1 (Consistency check: conformal window boundaries)*.* At the upper boundary $N_f = 3N_c$: $\gamma_m^{\mathrm{SUSY}} = 1$. The fixed point merges with free-field behaviour. At the lower boundary $N_f = \frac{3}{2}N_c$: $\gamma_m^{\mathrm{SUSY}} = 2$, which saturates the unitarity bound $\dim(\bar\psi\psi) \geq 1$, *i.e.* $\gamma_m \leq 2$. Below this point the theory confines. Both limiting behaviours are correct.

3.3 Numerical predictions for SU(3)

The emergent-SUSY hypothesis asserts that, near N_f^* in the non-SUSY theory, the IR fixed point acquires SUSY-like properties and the mass anomalous dimension approaches the SUSY value (11). For SU(3):

$$\gamma_m^{\rm SUSY}(N_f) = \frac{9}{N_f} \qquad \Longrightarrow \qquad \gamma_m^{\rm SUSY} = \begin{cases} 9/8 = 1.125 & (N_f = 8) \\ 9/10 = 0.900 & (N_f = 10) \\ 9/12 = 0.750 & (N_f = 12) \end{cases} . \tag{12}$$

These should be compared with the non-SUSY expectation. In the ladder Schwinger–Dyson approximation, the lower boundary corresponds to $\gamma_m = 1$; in the Ryttov–Sannino all-orders beta function, it corresponds to $\gamma_m = 2$ [11]. The SUSY values are distinct from both. Specifically:

Table 2: Predicted mass anomalous dimension γ_m at the fixed point for SU(3), under different hypotheses for the conformal window boundary. "BZ" denotes the perturbative Banks–Zaks prediction using the two-loop beta function. The SUSY values use (11); they apply only *if* emergent SUSY governs the fixed point. The Schwinger–Dyson (SD) value marks the onset of chiral symmetry breaking.

N_f	$\gamma_m^{\rm SUSY}$	$\gamma_m^{\rm BZ}$ (2-loop)	$\gamma_m^{\rm SD}$	$\gamma_m^{\rm lattice}$
8	1.125	0.46	~ 1	0.4–1.0
10	0.900	0.26	~ 1	—
12	0.750	0.13	< 1 (conformal)	0.4–0.5

The two-loop Banks–Zaks values are much smaller than the SUSY predictions for all three cases. The Schwinger–Dyson prediction of $\gamma_m \to 1$ at N_f^* differs from the SUSY value of $9/N_f^*$. For $N_f^* = 10$, the distinction is $\gamma_m = 1$ (SD) versus $\gamma_m = 0.9$ (SUSY)—a 10% difference that would require high-precision lattice measurements. For $N_f^* = 8$, the SUSY value $\gamma_m = 1.125$ exceeds the SD value of 1, and the unitarity bound $\gamma_m \leq 2$ is far from saturated.

3.4 The SUSY spectrum: boson–fermion degeneracies

At a superconformal fixed point, operators organise into superconformal multiplets. The scalar meson σ (the real part of the chiral condensate) and the pseudoscalar meson π (the imaginary part) are related by a superconformal transformation to the fermionic component of the same superfield. The key spectral predictions are:

(i) **Scalar–pseudoscalar degeneracy:** If the σ and π mesons belong to the same superconformal multiplet, they must have the same scaling dimension. In a confining theory deformed slightly away from the fixed point (to give the states a mass), this implies [24]

$$\frac{m_\sigma}{m_\pi} \xrightarrow{\text{emergent SUSY}} 1 \,. \tag{13}$$

In contrast, in ordinary QCD-like confining theories, $m_\sigma/m_\pi \gg 1$. In real QCD ($N_c = 3$, $N_f = 2$), $m_{f_0(500)}/m_\pi \approx 3.6$.

(ii) **Glueball–gluinoball degeneracy:** In $\mathcal{N} = 1$ super Yang–Mills theory, the glueball (a 0^{++} state built from $F_{\mu\nu}F^{\mu\nu}$) and the gluinoball (a fermionic state built from $\lambda\lambda$, where λ is the gluino) belong to the same supermultiplet. If emergent SUSY holds in a theory with an adjoint Weyl fermion (as in Sannino's magnetic dual), the glueball and the adjoint-fermion bilinear state should become degenerate:

$$\frac{m_{0^{++}}}{m_{\lambda\lambda}} \xrightarrow{\text{emergent SUSY}} 1\,. \tag{14}$$

4 Proposed lattice observables

We now identify five lattice-measurable quantities that discriminate between emergent SUSY, ordinary confinement, and conformal behaviour without emergent SUSY. For each observable, we give the prediction in each scenario.

4.1 Observable 1: scalar-to-pseudoscalar mass ratio m_σ/m_π

The lightest scalar (σ, quantum numbers 0^{++}) and pseudoscalar (π, quantum numbers 0^{-+}) mesons are the most directly accessible hadronic states in lattice simulations.

Confining (below window): Chiral symmetry is spontaneously broken. The pseudoscalar is a (pseudo-)Nambu–Goldstone boson with $m_\pi \ll m_\sigma$. In the chiral limit:

$$\frac{m_\sigma}{m_\pi} \gg 1 \qquad \text{(confining)}\,. \tag{15}$$

In QCD ($N_f = 2$): $m_\sigma/m_\pi \approx 3.6$. For walking theories ($N_f$ just below N_f^*), the ratio is expected to decrease but remain significantly above unity.

Conformal without emergent SUSY: At a non-SUSY conformal fixed point, the σ and π operators have scaling dimensions determined by the non-SUSY fixed-point dynamics. In general, $\dim(\sigma) \neq \dim(\pi)$ because there is no symmetry relating scalar and pseudoscalar bilinears at a non-SUSY fixed point (they have different parity and are not related by a SUSY transformation). Thus:

$$\frac{m_\sigma}{m_\pi} \neq 1 \qquad \text{(non-SUSY conformal, generically)}\,. \tag{16}$$

Emergent SUSY: At a superconformal fixed point, the scalar σ and pseudoscalar π mesons arise as the real and imaginary parts of the lowest component of the meson chiral superfield M. Within the superconformal multiplet, both have the same scaling dimension. Upon deforming away from the fixed point (by a relevant perturbation that gives the theory a mass gap), the mass degeneracy is preserved up to corrections proportional to the deformation parameter:

$$\boxed{\frac{m_\sigma}{m_\pi} = 1 + \mathcal{O}\left(\frac{m}{m_*}\right) \qquad \text{(emergent SUSY)}\,,} \tag{17}$$

where m is the fermion mass (the deformation) and m_* is the characteristic scale of the fixed point.

Lattice measurement: Compute the ratio m_σ/m_π as a function of the bare fermion mass m_q, and extrapolate toward the chiral limit $m_q \to 0$. If the ratio approaches unity in the chiral limit, this is evidence for emergent SUSY. If it diverges, the theory confines conventionally.

Existing data: The LatKMI collaboration [16] reported $m_\sigma/m_\pi \approx 1.1$–$1.5$ for SU(3) with $N_f = 8$ at the lightest available fermion masses. This is strikingly close to the emergent-SUSY prediction of unity, and is in tension with the ordinary confining prediction $m_\sigma/m_\pi \gg 1$. However, the light scalar could also be interpreted as a dilaton (pseudo-Goldstone boson of spontaneously broken scale invariance) rather than a SUSY partner of the pion.

4.2 Observable 2: glueball–fermion-bilinear mass relation

In a theory containing an adjoint Weyl fermion λ in addition to fundamental fermions, the glueball (0^{++}) and the adjoint-fermion bilinear ($\lambda\lambda$ bound state) should become degenerate in the emergent-SUSY scenario.

For a non-SUSY SU(3) theory with N_f fundamental flavours only (no adjoint fermion), this observable is not directly available. However, one can construct a proxy: if emergent SUSY holds, the glueball mass should satisfy relations inherited from SUSY. In $\mathcal{N} = 1$ SYM, the gluino condensate sets the scale, and the glueball mass satisfies [25]

$$m_{0^{++}} \sim |\langle \lambda\lambda \rangle|^{1/3} \, . \tag{18}$$

Confining: The glueball mass and the fermion-bilinear condensate are set by the confinement scale Λ, but there is no prediction relating $m_{0^{++}}$ to m_σ. Generically:

$$\frac{m_{0^{++}}}{m_\sigma} \sim \mathcal{O}(1)\text{–}\mathcal{O}(10) \qquad \text{(confining, no specific prediction)} \, . \tag{19}$$

Emergent SUSY: With an adjoint fermion, the glueball and gluinoball become degenerate (Eq. (14)). Without the adjoint, the prediction is weaker: the glueball mass should be related to the fermion-bilinear condensate via (18), and the glueball should enter the same tower as the meson states with calculable mass ratios.

Lattice measurement: Extract the glueball mass from Wilson-loop or gluonic correlator measurements. Compare with the meson spectrum. A clear separation $m_{0^{++}} \gg m_\sigma$ would disfavour emergent SUSY; approximate degeneracy would support it.

4.3 Observable 3: mass anomalous dimension γ_m

The mass anomalous dimension γ_m is the most precisely measurable quantity at or near a conformal fixed point. The predictions are given in Table 2 and repeated here for emphasis:

Emergent SUSY:

$$\boxed{\gamma_m = \frac{3N_c}{N_f} = \frac{9}{N_f} \quad \text{(SU(3))}} \tag{20}$$

Non-SUSY conformal (Banks–Zaks): The two-loop estimate gives values significantly smaller than the SUSY prediction for all N_f (see Table 2).

Schwinger–Dyson: The onset of chiral symmetry breaking occurs at $\gamma_m = 1$.

The discriminating power depends on the location of N_f^*. If $N_f^* \approx 10$, the SUSY value $\gamma_m = 0.9$ is close to the SD value of 1, requiring $\sim 10\%$ precision. If $N_f^* \approx 12$, the SUSY value is $\gamma_m = 0.75$ while the SD value remains 1—a 25% difference that is accessible to current lattice methods.

Lattice measurement: Extract γ_m from the scaling of the Dirac mode number $\nu(\lambda) \sim \lambda^{3/(1+\gamma_m)}$ [22], from the mass dependence of the pseudoscalar decay constant $f_\pi \sim m_q^{1/(1+\gamma_m)}$, or from finite-size scaling [23].

Existing data: For $N_f = 12$: LatKMI reports $\gamma_m \approx 0.4$–0.5 [21]. The SUSY prediction is 0.75. The discrepancy could indicate that either (a) emergent SUSY does not hold at $N_f = 12$, or (b) the lattice measurements have not yet reached the deep IR regime where the fixed-point value is attained (finite-volume and finite-mass effects can suppress the effective γ_m), or (c) the theory is not at a fixed point at all.

4.4 Observable 4: SUSY Ward identity violation

Even without fundamental SUSY, if a theory flows to a superconformal fixed point in the IR, the SUSY Ward identities should be approximately satisfied for low-energy correlation functions. We propose measuring the violation of a specific SUSY Ward identity on the lattice.

4.4.1 Construction of the Ward identity

In $\mathcal{N} = 1$ SUSY, the supercurrent S_μ^α satisfies

$$\partial^\mu S_\mu^\alpha = 0 \qquad \text{(on-shell, in the SUSY limit)}\,. \tag{21}$$

When SUSY is broken (or absent), $\partial^\mu S_\mu \neq 0$. In a lattice theory, one can construct a discretised version of the supercurrent and measure its divergence. This approach has been successfully applied in lattice studies of $\mathcal{N} = 1$ super Yang–Mills [26, 27].

Define the Ward identity violation parameter:

$$\boxed{\Delta_{\text{SUSY}}(L) \;\equiv\; \frac{\left\langle |\partial^\mu S_\mu(x)|^2 \right\rangle}{\left\langle |S_\mu(x)|^2 \right\rangle}}\,, \tag{22}$$

where L is the lattice extent and the expectation values are computed in the path integral.

Confining: SUSY is maximally broken. $\Delta_{\text{SUSY}} = \mathcal{O}(1)$ and does not decrease with increasing L.

Conformal without emergent SUSY: The supercurrent is not a conserved current. $\Delta_{\text{SUSY}} = \mathcal{O}(1)$ at the fixed point.

Emergent SUSY: As the theory flows to the superconformal fixed point, $\Delta_{\text{SUSY}} \to 0$ in the deep IR. On a finite lattice:

$$\Delta_{\text{SUSY}}(L) \sim L^{-\Delta} \qquad \text{(emergent SUSY)}\,, \tag{23}$$

where $\Delta > 0$ is related to the dimension of the leading SUSY-breaking operator in the fixed-point theory.

Lattice measurement: The challenge is constructing the supercurrent S_μ from the lattice fields. For a theory with fundamental fermions and scalars (as in the magnetic dual with mesons), the supercurrent can be constructed from the fermion and scalar fields [26]. For a theory with fundamental fermions only (the electric theory), one must work with composite operators. We propose using the lattice definition of [27], adapted to include fundamental matter, and measuring Δ_{SUSY} as a function of the lattice extent L and the bare fermion mass m_q. A power-law decrease of Δ_{SUSY} with L (or with $1/m_q$ toward the chiral limit) would be strong evidence for emergent SUSY.

4.5 Observable 5: effective potential shape and flat directions

SUSY theories possess moduli spaces of degenerate vacua—flat directions in the scalar potential that are not lifted by quantum corrections (within the conformal window). If emergent SUSY holds, the effective potential for the chiral condensate should develop approximate flat directions near N_f^*.

Define the chiral condensate as an order parameter:

$$\Sigma \;\equiv\; \langle \bar{\psi}\psi \rangle\,. \tag{24}$$

In the deep IR of a theory near the fixed point, the effective potential $V_{\text{eff}}(\Sigma)$ encodes the dynamics of chiral symmetry breaking.

Confining: The effective potential has a "Mexican hat" shape: $V_{\text{eff}}(\Sigma)$ has a minimum at $\Sigma \neq 0$, corresponding to spontaneous chiral symmetry breaking. The curvature at the minimum sets the sigma mass: $m_\sigma^2 \sim V''_{\text{eff}}(\Sigma_0)$.

Conformal without emergent SUSY: The effective potential has a minimum at $\Sigma = 0$ (no chiral symmetry breaking), with $V_{\text{eff}}(\Sigma) \sim |\Sigma|^{2/(3-\gamma_m)}$ for small Σ. The curvature depends on γ_m.

Emergent SUSY: The effective potential develops approximate flat directions corresponding to the SUSY moduli space. Near N_f^*, the potential should be approximately flat for a range of Σ:

$$V_{\text{eff}}(\Sigma) \approx V_0 + \epsilon\, f(\Sigma) + \mathcal{O}(\epsilon^2)\,, \tag{25}$$

where ϵ parameterises the deviation from the superconformal fixed point and V_0 is a constant. The flatness of the potential implies a parametrically light scalar excitation—the pseudo-modulus—whose mass scales as

$$\boxed{m^2_{\text{pseudo-mod}} \sim \epsilon\, \Lambda^2 \qquad \text{(emergent SUSY)}\,,} \tag{26}$$

where Λ is the dynamical scale. In the exact SUSY limit $\epsilon \to 0$, the pseudo-modulus becomes massless.

Lattice measurement: The shape of $V_{\text{eff}}(\Sigma)$ can be reconstructed from the probability distribution of the chiral condensate on the lattice using constraint effective potential methods [28]. A flat region in V_{eff} would signal emergent flat directions. Alternatively, measure the sigma mass as a function of the fermion mass and check whether m_σ^2 scales linearly with m_q (as expected for a pseudo-modulus) rather than remaining of order Λ^2 (as in ordinary confinement).

5 Expected signals for SU(3) with $N_f = 8$, 10, 12

We now assemble the predictions of the five observables for the three most-studied cases. The predictions depend on whether each N_f value lies inside the conformal window, outside it (confining), or at the boundary where emergent SUSY is expected.

5.1 Scenario assignments

The emergent-SUSY hypothesis predicts that SUSY-like behaviour appears precisely at $N_f = N_f^*$. Depending on the (unknown) value of N_f^*, the three cases are:

Table 3: Scenario assignments for each N_f value, depending on the location of N_f^*.

N_f^*	$N_f = 8$	$N_f = 10$	$N_f = 12$
~ 8	Boundary (SUSY?)	Conformal	Conformal
~ 10	Confining	Boundary (SUSY?)	Conformal
~ 12	Confining	Confining/walking	Boundary (SUSY?)

5.2 Prediction table

Table 4 gives the predicted value of each observable in each scenario. The SUSY values apply only *if* emergent SUSY holds *and* N_f is at the boundary.

Table 4: Predicted values of the five discriminating observables for SU(3) with $N_f = 8$, 10, 12. "Conf." = confining, "CW" = inside conformal window, "SUSY" = emergent SUSY at boundary. All SUSY values are *conjectural*.

Observable	$N_f = 8$	$N_f = 10$	$N_f = 12$
1. Scalar-to-pseudoscalar mass ratio m_σ/m_π			
Conf.	$\gg 1$	$\gg 1$	—
CW (no SUSY)	—	$\neq 1$ (generic)	$\neq 1$ (generic)
SUSY boundary	$\to 1$	$\to 1$	$\to 1$
2. Glueball–meson mass relation			
Conf.	$m_{0^{++}} \sim$ few $\times\, m_\sigma$	same	—
CW (no SUSY)	—	unrelated	unrelated
SUSY boundary	$m_{0^{++}} \approx m_\sigma$	same	same
3. Mass anomalous dimension γ_m			
Conf.	N/A (no FP)	N/A	—
CW (no SUSY, 2-loop)	—	0.26	0.13
SUSY boundary	1.125	0.900	0.750
4. SUSY Ward identity violation Δ_{SUSY}			
Conf.	$\mathcal{O}(1)$	$\mathcal{O}(1)$	—
CW (no SUSY)	—	$\mathcal{O}(1)$	$\mathcal{O}(1)$
SUSY boundary	$\sim L^{-\Delta}$	same	same
5. Effective potential shape			
Conf.	Mexican hat	same	—
CW (no SUSY)	—	Single min. at 0	Single min. at 0
SUSY boundary	Flat direction	same	same

6 Comparison with existing lattice data

We now compare the predictions of Section 5 with available lattice results. We focus on $N_f = 8$ and $N_f = 12$, where substantial data exist.

6.1 $N_f = 8$: the light scalar

The most striking existing result is the light scalar (σ) state observed by the LatKMI [16] and LSD [17, 18] collaborations in SU(3) with $N_f = 8$ fundamental flavours.

Observable 1 (m_σ/m_π): The LatKMI data show $m_\sigma \approx m_\pi$ at the lightest fermion masses studied, with $m_\sigma/m_\pi \approx 1.0$–$1.5$ depending on the fermion mass. The LSD collaboration also finds a light scalar, though with somewhat larger ratios. This is qualitatively consistent with the emergent-SUSY prediction (17), but is also consistent with the dilaton hypothesis (a light scalar arising from spontaneously broken approximate scale invariance).

Discrimination between the dilaton and SUSY hypotheses requires additional observables: a dilaton is a scalar singlet, while a SUSY partner of the pion must carry the same flavour quantum numbers and have a specific relation to the pseudoscalar decay constant.

Observable 3 (γ_m): The mass anomalous dimension for $N_f = 8$ has been estimated at $\gamma_m \approx 0.4$–1.0. The SUSY prediction is $\gamma_m = 1.125$. The upper end of the lattice range ($\gamma_m \sim 1$) is not far from the SUSY value, but the uncertainties are large. Higher precision is needed. If $N_f = 8$ is below the conformal window (as the majority view holds), then γ_m is not a fixed-point value but an effective exponent characterising the walking regime, and the comparison with the SUSY prediction is less direct.

6.2 $N_f = 12$: the conformal case

Observable 3 (γ_m): The LatKMI collaboration reports $\gamma_m \approx 0.4$–0.5 for $N_f = 12$ [21]. The SUSY prediction is $\gamma_m = 0.75$. The discrepancy is $\sim 50\%$, which is significant. If $N_f = 12$ is well inside the conformal window (far above N_f^*), then the emergent-SUSY mechanism—which is predicted to operate specifically at N_f^*—need not apply. The measured value could reflect ordinary (non-SUSY) conformal dynamics.

However, if N_f^* is as high as 12, the SUSY prediction $\gamma_m = 0.75$ should be compared directly. The discrepancy with the lattice value of 0.4–0.5 would then constitute evidence against emergent SUSY at $N_f^* = 12$.

6.3 $N_f = 10$: the missing data

This is the most critical case. If $N_f^* \approx 10$, the emergent-SUSY predictions are sharpest here: $\gamma_m = 0.9$, $m_\sigma/m_\pi \to 1$, and $\Delta_{\mathrm{SUSY}} \to 0$. Unfortunately, very little lattice data exist for $N_f = 10$. We identify this as the highest priority for future lattice simulations.

7 Proposed measurement programme

Based on the analysis above, we propose the following prioritised measurement programme:

7.1 Priority 1: SU(3) with $N_f = 10$

This is the most likely location of N_f^* and the most under-studied case. We recommend:

(a) Full spectrum calculation: pseudoscalar, scalar, vector, and axial-vector mesons. Extract m_σ/m_π in the chiral limit.

(b) Mass anomalous dimension γ_m from the Dirac mode number, f_π scaling, and finite-size scaling. Compare with $\gamma_m^{\mathrm{SUSY}} = 0.9$.

(c) Glueball spectrum: extract $m_{0^{++}}$ and compare with m_σ.

(d) Multiple fermion masses to enable chiral extrapolation and to test whether $m_\sigma^2 \sim m_q$ (pseudo-modulus scaling) or $m_\sigma^2 \sim m_q^{2/(1+\gamma_m)}$ (conformal scaling).

7.2 Priority 2: SU(3) with $N_f = 8$ (deeper IR)

Existing data already hint at a light scalar. Higher-precision measurements are needed:

(a) Lighter fermion masses to approach the chiral limit and determine whether $m_\sigma/m_\pi \to 1$ or diverges.

(b) Improved determination of γ_m with reduced systematic uncertainties. The SUSY prediction $\gamma_m = 1.125$ is distinct from the SD value of 1.

(c) Constraint effective potential measurement to probe the shape of $V_{\text{eff}}(\Sigma)$ and search for approximate flat directions.

7.3 Priority 3: SUSY Ward identity measurement (any N_f)

The SUSY Ward identity violation (22) is technically challenging but provides the most direct test of emergent SUSY. We recommend a pilot study:

(a) Construct a lattice discretisation of the supercurrent for SU(3) with N_f fundamental flavours. The discretisation of [26, 27] for pure SYM must be extended to include fundamental matter.

(b) Measure Δ_{SUSY} at multiple lattice volumes for $N_f = 8$ or $N_f = 10$.

(c) Check for power-law scaling $\Delta_{\text{SUSY}} \sim L^{-\Delta}$.

7.4 Priority 4: comparison across N_f values

A definitive test of the emergent-SUSY hypothesis requires comparing all five observables across $N_f = 8$, 10, and 12 (and ideally $N_f = 6$ as a control). The emergent-SUSY predictions apply only at N_f^*; away from N_f^*, the ordinary confining or conformal predictions should hold. Observing a sharp change in m_σ/m_π, γ_m, or Δ_{SUSY} at a specific value of N_f would be strong evidence for a qualitative change in the IR dynamics at the conformal window boundary.

7.5 Summary of discriminating signatures

Table 5: Summary of the three scenarios and their distinguishing signatures. A single observable suffices to distinguish confining from conformal; discriminating emergent SUSY from ordinary conformal requires at least two observables measured at the same N_f.

Observable	**Confining**	**Conformal**	**Emergent SUSY**
m_σ/m_π	$\gg 1$	$\neq 1$ (generic)	$\to 1$
$m_{0^{++}}/m_\sigma$	$\mathcal{O}(1)$–$\mathcal{O}(10)$	unrelated	$\to 1$
γ_m	N/A	$\ll 3N_c/N_f$	$= 3N_c/N_f$
Δ_{SUSY}	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\sim L^{-\Delta}$
V_{eff} shape	Mexican hat	Min. at $\Sigma = 0$	Flat direction

8 Conclusions

We have identified five lattice-measurable observables that can discriminate between emergent supersymmetry, ordinary confinement, and non-SUSY conformal behaviour at the lower boundary of the conformal window for SU(3) gauge theories with fundamental fermions.

The emergent-SUSY hypothesis of Antipin, Mojaza, Pica, and Sannino [7] makes sharp, quantitative predictions: a mass anomalous dimension $\gamma_m = 9/N_f$, a scalar-to-pseudoscalar mass ratio approaching unity, power-law vanishing of SUSY Ward identity violations, and the emergence of approximate flat directions in the effective potential. These predictions are qualitatively different from both ordinary confining behaviour and non-SUSY conformal dynamics.

Existing lattice data provide intriguing but inconclusive evidence. The light scalar observed at $N_f = 8$ is consistent with emergent SUSY but also admits a dilaton interpretation. The mass anomalous dimension at $N_f = 12$ falls below the SUSY prediction, but this may reflect $N_f = 12$ being well above N_f^*. The critical case $N_f = 10$ remains largely unexplored on the lattice and should be the highest priority for future simulations.

The most direct test of emergent SUSY—the measurement of a SUSY Ward identity violation and its scaling with lattice volume—is technically challenging but feasible with current lattice technology. A pilot study at $N_f = 8$ or $N_f = 10$ would provide qualitatively new information not available from spectral measurements alone.

We emphasise that the emergent-SUSY hypothesis remains a conjecture. The predictions in this paper are the sharpest available consequences of that conjecture, and the proposed measurement programme is designed to test it decisively. A positive result would be extraordinary: it would demonstrate that supersymmetry can emerge dynamically in a non-supersymmetric theory, with profound implications for our understanding of strongly coupled gauge dynamics and the naturalness of the Standard Model.

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