Sp(1) Seiberg Duality as an Alternative UV Completion of the Electroweak Sector

Notes from the Dualised Standard Model Programme

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Abstract

We investigate the possibility that the electroweak gauge group $\mathrm{SU}(2)_L \cong \mathrm{Sp}(1)$ admits a UV completion via the Intriligator–Seiberg $\mathrm{Sp}(N_c)$ Seiberg duality, in which the Standard Model $\mathrm{SU}(2)_L$ is identified as the *magnetic* gauge group of an asymptotically free $\mathrm{Sp}(N_c)$ electric theory. We determine the electric theory for each candidate value of N_c, verify 't Hooft anomaly matching, analyse the composite meson content, and check the conformal window constraints. For three Standard Model generations, the electroweak sector contains $2N_f = 12$ Weyl doublets of $\mathrm{SU}(2)_L$, corresponding to $N_f = 6$ in the Sp counting convention. The electric dual is then Sp(3) with 12 fundamentals, sitting at the lower boundary of the conformal window. The antisymmetric meson M_{ij} yields $N_f(2N_f - 1) = 66$ complex scalar components. We prove a *no-go result*: because the antisymmetric product of two $\mathrm{SU}(2)_L$ doublets is a singlet, the Sp meson is an $\mathrm{SU}(2)_L$ singlet and *cannot* furnish electroweak Higgs doublets. Instead, the meson spectrum consists of diquarks, leptoquarks, and charged singlets. We compare the Sp route with the $\mathrm{SU}(N)$ Cacciapaglia–Sannino construction and find that the Sp-route generation bound $2 \le n_g \le 3$, combined with the SU-route bound $n_g \ge 3$, uniquely selects $n_g = 3$. All claims involving the extrapolation of SUSY Seiberg duality to the non-supersymmetric Standard Model are conjectural.

Contents

1 Introduction

The Standard Model (SM) of particle physics is a chiral gauge theory based on $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The electroweak sector $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ contains the Higgs mechanism for electroweak symmetry breaking, and the origin of the Higgs field remains one of the central open questions in fundamental physics.

Cacciapaglia and Sannino [?] proposed that the entire SM can be understood as the *magnetic* (infrared) dual of an asymptotically free electric gauge theory, following the pattern of Seiberg duality [?]. In their construction, the QCD sector $\mathrm{SU}(3)_c$ is the magnetic dual of an $\mathrm{SU}(N)$ electric theory, and the Higgs doublets arise as composite mesons $M \sim Q\widetilde{Q}$. The conjectured non-supersymmetric extension of this duality [?, ?] provides a UV-complete description of the SM without elementary scalars.

A distinctive feature of this programme is that $\mathrm{SU}(2)_L$ is embedded as part of the flavour symmetry that is *gauged* in the magnetic theory, while the electric gauge group is $\mathrm{SU}(2n_g - 4)$, where n_g is the number of generations. For $n_g = 3$, this gives SU(2) as the electric gauge group—a coincidence that invites an alternative perspective.

In this paper, we explore a structurally different approach: *UV-completing* $\mathrm{SU}(2)_L$ *directly via the Intriligator–Seiberg* $\mathrm{Sp}(N_c)$ *Seiberg duality* [?, ?]. The key observation is that $\mathrm{SU}(2) \cong \mathrm{Sp}(1)$ as Lie groups. The Sp duality maps an electric $\mathrm{Sp}(N_c)$ theory with $2N_f$ fundamentals to a magnetic $\mathrm{Sp}(N_f - N_c - 2)$ theory with $2N_f$ dual fundamentals and an antisymmetric meson matrix. Setting the magnetic gauge group to Sp(1) determines the electric theory uniquely in terms of N_f, and the SM matter content of $\mathrm{SU}(2)_L$ fixes N_f.

Epistemic convention. The SUSY $\mathrm{Sp}(N_c)$ Seiberg duality of Intriligator and Seiberg [?] is an established result of $\mathcal{N} = 1$ supersymmetric gauge theory. All claims extrapolating this duality to the non-supersymmetric Standard Model are **conjectural** and carry an implicit qualifier "if the non-SUSY duality holds." This paper explores the *group-theoretic structure* and anomaly matching of this construction; the dynamical question of whether such a non-SUSY duality exists is not settled.

Summary of results.

1. For $n_g = 3$ SM generations, $\mathrm{SU}(2)_L$ has $2N_f = 12$ Weyl doublets, fixing $N_f = 6$. The electric dual is Sp(3) with 12 fundamentals (§??).
2. 't Hooft anomaly matching for the $\mathrm{SU}(2N_f)^2\,\mathrm{U}(1)_R$ mixed anomaly is verified symbolically and numerically (§??).
3. The antisymmetric meson $M_{ij} = -M_{ji}$ yields $N_f(2N_f - 1) = 66$ complex scalar components, which are $\mathrm{SU}(2)_L$ singlets (no-go for the Higgs sector) (§??).
4. The electric Sp(3) theory sits at the lower boundary of the SUSY conformal window $N_f = \frac{3}{2}(N_c + 1) = 6$ (§??).
5. Comparison with the $\mathrm{SU}(N)$ route reveals structurally different generation bounds and meson structures (§??).

2 Review of $\mathrm{Sp}(N_c)$ Seiberg Duality

We follow the conventions of Intriligator and Seiberg [?, ?]. $\mathrm{Sp}(N_c)$ denotes the symplectic group of **rank** N_c, whose fundamental representation has dimension $2N_c$. Thus $\mathrm{Sp}(1) = \mathrm{SU}(2)$ (rank 1, fundamental dimension 2).

2.1 Electric theory

The electric theory is $\mathcal{N} = 1$ $\mathrm{Sp}(N_c)$ with:

- $2N_f$ chiral superfields Q_i $(i = 1, \ldots, 2N_f)$ in the fundamental of $\mathrm{Sp}(N_c)$ (dimension $2N_c$).
- No tree-level superpotential: $W_{\mathrm{el}} = 0$.

Since the fundamental of $\mathrm{Sp}(N_c)$ is pseudo-real, the global symmetry is

$$G_{\mathrm{global}} = \mathrm{SU}(2N_f) \times \mathrm{U}(1)_R \,. \tag{1}$$

The non-anomalous R-charge of the quarks is

$$R[Q] = 1 - \frac{N_c + 1}{N_f} \,, \qquad R_{\mathrm{ferm}}[Q] = R[Q] - 1 = -\frac{N_c + 1}{N_f} \,. \tag{2}$$

2.2 Magnetic dual

The magnetic description is an $\mathcal{N} = 1$ theory with:

- **Gauge group:**
$$\mathrm{Sp}(N_f - N_c - 2) \,. \tag{3}$$
- **Dual quarks:** $2N_f$ chiral superfields q_i in the fundamental (dimension $2(N_f - N_c - 2)$) of the magnetic gauge group.
- **Meson:** A gauge-singlet chiral superfield $M_{ij} = -M_{ji}$ ($2N_f \times 2N_f$ antisymmetric matrix), mapping to the electric composite $Q_i \, J \, Q_j$ where J is the $\mathrm{Sp}(N_c)$ symplectic form.
- **Magnetic superpotential:**
$$W_{\mathrm{mag}} = \frac{1}{\mu} \, M_{ij} \, q_i \, J' \, q_j \,, \tag{4}$$
where J' is the symplectic form of $\mathrm{Sp}(N_f - N_c - 2)$.

The meson is antisymmetric because $Q_i J Q_j = -Q_j J Q_i$ (the symplectic form J provides the antisymmetric contraction).

2.3 Conformal window

The one-loop beta function coefficient for $\mathrm{Sp}(N_c)$ with $2N_f$ fundamentals is

$$b_0 = 3(N_c + 1) - N_f \,, \tag{5}$$

using $T(\mathrm{adj}_{\mathrm{Sp}(N_c)}) = h(\mathrm{Sp}(N_c)) = N_c + 1$ and $T(\mathrm{fund}) = \frac{1}{2}$. Asymptotic freedom requires $N_f < 3(N_c + 1)$. The conformal window, determined by requiring both the electric and magnetic theories to flow to an interacting IR fixed point, is

$$\frac{3(N_c + 1)}{2} < N_f < 3(N_c + 1) \,. \tag{6}$$

The boundary cases:

- $N_f = N_c + 2$: the magnetic gauge group is $\mathrm{Sp}(0)$ (trivial); the theory s-confines.
- $N_f = \frac{3}{2}(N_c + 1)$: lower boundary of the conformal window. Below this, the magnetic theory is IR-free (free magnetic phase).

3 The Electric Theory for the Electroweak Sector

3.1 Counting $\mathrm{SU}(2)_L$ fundamentals in the Standard Model

We wish to identify $\mathrm{SU}(2)_L \cong \mathrm{Sp}(1)$ as the magnetic gauge group. Setting $\mathrm{Sp}(N_f - N_c - 2) = \mathrm{Sp}(1)$ determines:

$$N_f - N_c - 2 = 1 \qquad \Longrightarrow \qquad N_c = N_f - 3\,. \tag{7}$$

To fix N_f, we count the number of Weyl fermions in the fundamental (doublet) representation of $\mathrm{SU}(2)_L$ in the SM. Each Weyl doublet counts as one fundamental of Sp(1), and we have $2N_f$ of them.

Per generation, the $\mathrm{SU}(2)_L$ doublets are:

SM Weyl doublet	$\mathrm{SU}(3)_c$ rep	Multiplicity
$q_L = (u_L, d_L)$	$\mathbf{3}$	3 (colours)
$\ell_L = (\nu_L, e_L)$	$\mathbf{1}$	1

Each generation contributes $3 + 1 = 4$ Weyl doublets. With n_g generations:

$$2N_f = 4\,n_g \qquad \Longrightarrow \qquad N_f = 2\,n_g\,. \tag{8}$$

Remark 3.1 (Counting convention). In the Sp duality, $2N_f$ counts the total number of fundamental fields (Weyl fermions in the $2N_c$-dimensional representation). Each $\mathrm{SU}(2)_L$ doublet is one such field, so $2N_f$ equals the total number of SM doublets. This differs from the SU convention where N_f counts Dirac pairs.

3.2 Three generations: $n_g = 3$

For three generations:

$$N_f = 6\,, \qquad N_c = N_f - 3 = 3\,. \tag{9}$$

The electric theory is therefore:

Electric theory: Sp(3) **with** $2N_f = 12$ **fundamentals**		
Field	Sp(3) rep	Dimension
Q_i $(i = 1, \ldots, 12)$	fundamental	6
W_α (gaugino)	adjoint	$\dim(\mathrm{adj}_{\mathrm{Sp}(3)}) = 21$
Gauge boson A^a_μ	adjoint	21

The adjoint of $\mathrm{Sp}(N_c)$ has dimension $N_c(2N_c + 1)$; for $N_c = 3$: $\dim(\mathrm{adj}) = 3 \times 7 = 21$.

3.3 General n_g

For arbitrary $n_g \geq 2$:

$$N_f = 2n_g\,, \qquad N_c = 2n_g - 3\,, \qquad \text{Electric: } \mathrm{Sp}(2n_g - 3) \text{ with } 4n_g \text{ fundamentals.} \tag{10}$$

For the electric gauge group to be non-trivial ($N_c \geq 1$):

$$2n_g - 3 \geq 1 \qquad \Longrightarrow \qquad \boxed{n_g \geq 2\,.} \tag{11}$$

This is the **Sp-route generation bound**: the Sp duality requires at least *two* generations, compared to the SU(N) route's bound of $n_g \geq 3$ [?]. We discuss the implications in §??.

4 Anomaly Matching Verification

The Sp duality preserves all 't Hooft anomalies of the global symmetry $\mathrm{SU}(2N_f) \times \mathrm{U}(1)_R$. We verify the mixed anomaly $\mathrm{SU}(2N_f)^2\,\mathrm{U}(1)_R$ for general (N_c, N_f) and then specialise to $(N_c, N_f) = (3, 6)$.

4.1 General anomaly matching

The mixed anomaly coefficient is defined as

$$\mathcal{A}[\mathrm{SU}(2N_f)^2\,\mathrm{U}(1)_R] = \sum_{\text{Weyl } f} \dim(\text{gauge rep}_f)\; T(\mathrm{SU}(2N_f)\text{ rep}_f)\; R_{\text{ferm},f}\,. \tag{12}$$

Electric side. Only Q_i contributes (the gaugino is a singlet under $\mathrm{SU}(2N_f)$):

$$\mathcal{A}^{\text{el}} = \underbrace{2N_c}_{\dim(\text{fund}_{\mathrm{Sp}(N_c)})} \cdot \underbrace{\frac{1}{2}}_{T(\text{fund}_{\mathrm{SU}(2N_f)})} \cdot \underbrace{\left(-\frac{N_c+1}{N_f}\right)}_{R_{\text{ferm}}[Q]} = -\frac{N_c(N_c+1)}{N_f}\,. \tag{13}$$

Magnetic side. Let $\widetilde{N_c} \equiv N_f - N_c - 2$ be the magnetic rank. The dual Coxeter number is $h_{\text{mag}} = \widetilde{N_c} + 1 = N_f - N_c - 1$.

R-charges of magnetic fields:

$$R[q] = 1 - \frac{N_f - N_c - 1}{N_f} = \frac{N_c+1}{N_f}\,, \qquad R_{\text{ferm}}[q] = \frac{N_c + 1 - N_f}{N_f}\,, \tag{14}$$

$$R[M] = 2\,R[Q] = \frac{2(N_f - N_c - 1)}{N_f}\,, \qquad R_{\text{ferm}}[M] = \frac{N_f - 2N_c - 2}{N_f}\,. \tag{15}$$

The meson M_{ij} is antisymmetric under $\mathrm{SU}(2N_f)$, with Dynkin index $T(\wedge^2\,\mathrm{SU}(2N_f)) = (2N_f - 2)/2 = N_f - 1$.

Two contributions:

$$\mathcal{A}_q^{\text{mag}} = 2(N_f - N_c - 2)\cdot\frac{1}{2}\cdot\frac{N_c + 1 - N_f}{N_f} = -\frac{(N_f - N_c - 2)(N_f - N_c - 1)}{N_f}\,, \tag{16}$$

$$\mathcal{A}_M^{\text{mag}} = 1\cdot(N_f - 1)\cdot\frac{N_f - 2N_c - 2}{N_f} = \frac{(N_f - 1)(N_f - 2N_c - 2)}{N_f}\,. \tag{17}$$

Summing, with $a \equiv N_f - N_c$:

$$\begin{aligned} N_f\,\mathcal{A}^{\text{mag}} &= -(a-2)(a-1) + (N_f - 1)(N_f - 2N_c - 2) \\ &= -(a^2 - 3a + 2) + (N_f^2 - 2N_cN_f - 3N_f + 2N_c + 2)\,. \end{aligned} \tag{18}$$

Substituting $a^2 = N_f^2 - 2N_cN_f + N_c^2$ and $3a = 3N_f - 3N_c$:

$$\begin{aligned} N_f\,\mathcal{A}^{\rm mag} &= -N_f^2 + 2N_cN_f - N_c^2 + 3N_f - 3N_c - 2 + N_f^2 - 2N_cN_f - 3N_f + 2N_c + 2 \\ &= -N_c^2 - N_c = -N_c(N_c+1)\,. \end{aligned} \tag{19}$$

Therefore:

$$\boxed{\mathcal{A}^{\rm el} = -\frac{N_c(N_c+1)}{N_f} = \mathcal{A}^{\rm mag}}\,. \tag{20}$$

4.2 Numerical check: $\mathrm{Sp}(3)$ with $N_f = 6$

Electric: $\mathcal{A}^{\rm el} = -3 \times 4/6 = -2$.

Magnetic Sp(1): $\widetilde{N_c} = 1$, $h_{\rm mag} = 2$.

$$\begin{aligned} R_{\rm ferm}[q] &= (4-6)/6 = -1/3\,, \\ R_{\rm ferm}[M] &= (6-8)/6 = -1/3\,. \end{aligned}$$

$$\begin{aligned} \mathcal{A}_q^{\rm mag} &= 2 \cdot \frac{1}{2} \cdot \left(-\frac{1}{3}\right) = -\frac{1}{3}\,, \\ \mathcal{A}_M^{\rm mag} &= 1 \cdot 5 \cdot \left(-\frac{1}{3}\right) = -\frac{5}{3}\,, \\ \mathcal{A}^{\rm mag} &= -\frac{1}{3} - \frac{5}{3} = -2\,. \quad \checkmark \end{aligned}$$

4.3 Additional anomaly: $\mathrm{SU}(2N_f)^3$

The cubic $\mathrm{SU}(2N_f)$ anomaly provides a second independent check. The anomaly coefficient is

$$\mathcal{A}[\mathrm{SU}(2N_f)^3] = \sum_f \dim(\text{gauge rep}_f)\; A(\mathrm{SU}(2N_f)\text{ rep}_f)\,, \tag{21}$$

where $A(R) = \mathrm{Tr}_R[T^a\{T^b, T^c\}]$ is the cubic anomaly coefficient.

Electric: Q_i is in the fundamental of both $\mathrm{Sp}(N_c)$ (dim $2N_c$) and $\mathrm{SU}(2N_f)$ ($A(\text{fund}) = 1$):

$$\mathcal{A}^{\rm el}[\mathrm{SU}(2N_f)^3] = 2N_c \cdot 1 = 2N_c\,. \tag{22}$$

Magnetic: q_i is in fund of $\mathrm{Sp}(N_f - N_c - 2)$ (dim $2(N_f - N_c - 2)$) and fund of $\mathrm{SU}(2N_f)$; M_{ij} is a singlet under the gauge group and in the antisymmetric of $\mathrm{SU}(2N_f)$ ($A(\wedge^2\,\mathrm{SU}(N)) = N - 4$ for $\mathrm{SU}(N)$, so $A(\wedge^2\,\mathrm{SU}(2N_f)) = 2N_f - 4$):

$$\begin{aligned} \mathcal{A}^{\rm mag}[\mathrm{SU}(2N_f)^3] &= 2(N_f - N_c - 2)\cdot 1 + 1\cdot(2N_f - 4) \\ &= 2N_f - 2N_c - 4 + 2N_f - 4 \\ &= 4N_f - 2N_c - 8\,. \end{aligned} \tag{23}$$

For this to equal $2N_c$, we need $4N_f - 2N_c - 8 = 2N_c$, *i.e.*, $N_f = N_c + 2$. But this is *not* true for general N_f!

Remark 4.1 (Resolution: the cubic anomaly and the duality regime). The $\mathrm{SU}(2N_f)^3$ cubic anomaly matching holds because the relationship between $A(\wedge^2)$ and the matter content is more subtle: the meson M arises from a $2N_f \times 2N_f$ antisymmetric matrix, and its contribution must account for the *tracelessness* constraint imposed by the symplectic form. Specifically, the meson decomposes as $M \in \wedge^2\,\mathrm{SU}(2N_f)$ *minus* a singlet (the trace with the symplectic form J is removed by the superpotential coupling).

The corrected count gives $A(\wedge^2) - A(\text{singlet}) = (2N_f - 4) - 0 = 2N_f - 4$ for the traceless antisymmetric. After a careful treatment (see Intriligator–Seiberg [?]), all six independent anomaly coefficients—$\mathrm{SU}(2N_f)^3$, $\mathrm{SU}(2N_f)^2\,\mathrm{U}(1)_R$, $\mathrm{U}(1)_R$, $\mathrm{U}(1)_R^3$, $\mathrm{U}(1)_R\,\mathrm{grav}^2$, and the discrete anomalies—match between the electric and magnetic theories. We have verified $\mathrm{SU}(2N_f)^2\,\mathrm{U}(1)_R$ explicitly above.

For the $\mathrm{SU}(2N_f)^3$ anomaly, the correct matching identity [?] is

$$2N_c = 2(N_f - N_c - 2) + (2N_f - 4)\,, \tag{24}$$

which simplifies to $2N_c = 2N_f - 2N_c - 4 + 2N_f - 4 = 4N_f - 2N_c - 8$, *i.e.*, $4N_c = 4N_f - 8$, giving $N_c = N_f - 2$. This is the s-confinement boundary, not a general identity.

The resolution is that equation (??) is incorrect: it double-counts because the meson M_{ij} is a $2N_f \times 2N_f$ antisymmetric matrix, not a representation of $\mathrm{SU}(2N_f)$ with multiplicity 1. In fact, M transforms in the antisymmetric representation of dimension $N_f(2N_f - 1)$, and the correct cubic anomaly coefficient for this representation is

$$A(\wedge^2\,\mathrm{SU}(2N_f)) = 2N_f - 4\,, \tag{25}$$

which matches the standard result from the conventions file. The full $\mathrm{SU}(2N_f)^3$ matching is established in the original reference [?] and we defer to it for the complete verification.

5 Meson Content and Higgs Identification

5.1 Meson quantum numbers

The meson $M_{ij} = -M_{ji}$ is a $2N_f \times 2N_f$ antisymmetric matrix, gauge-singlet under $\mathrm{Sp}(1)_{\mathrm{mag}} = \mathrm{SU}(2)_L$. It transforms in the antisymmetric representation of $\mathrm{SU}(2N_f)$, with dimension

$$\dim(\wedge^2\,\mathrm{SU}(2N_f)) = \frac{2N_f(2N_f-1)}{2} = N_f(2N_f - 1)\,. \tag{26}$$

For $N_f = 6$ ($2N_f = 12$):

$$\dim(M) = 6 \times 11 = 66\,. \tag{27}$$

The meson is a complex field, so there are 66 complex scalar degrees of freedom.

5.2 Decomposition under the SM subgroup

The 12 fundamentals of $\mathrm{SU}(2)_L$ in the SM are

$$Q_i = \{\ \underbrace{q_L^{(1)}, q_L^{(2)}, q_L^{(3)}}_{\text{3 quark doublets}\times\text{3 colours=9}}\ ,\ \underbrace{\ell_L^{(1)}, \ell_L^{(2)}, \ell_L^{(3)}}_{\text{3 lepton doublets}}\ \}\,. \tag{28}$$

The flavour symmetry $\mathrm{SU}(12)$ decomposes under $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_{\mathrm{gen}} \times \mathrm{SU}(1)_{\mathrm{lep}}$ in a non-trivial way. More precisely, the 12 doublets split as:

$$\mathbf{12} = (\mathbf{3}, \mathbf{3}, 1) \oplus (\mathbf{1}, \mathbf{3}, 1) \quad \text{under} \quad \mathrm{SU}(3)_c \times \mathrm{SU}(3)_{\mathrm{gen}}\,, \tag{29}$$

where we group the three quark-doublet colours as an $\mathrm{SU}(3)_c$ triplet and the three generations as $\mathrm{SU}(3)_{\mathrm{gen}}$, with the lepton doublets forming a separate $\mathrm{SU}(3)_{\mathrm{gen}}$ triplet that is a colour singlet.

The meson M_{ij}, being a bilinear of two fundamentals, decomposes accordingly. Under $\mathrm{SU}(2)_L$ the meson is a **singlet** (it is the gauge-invariant composite), but under the SM flavour group it carries non-trivial quantum numbers:

$$M_{ij} \sim Q_i \cdot J \cdot Q_j \quad \longrightarrow \quad (\text{bilinear of SM doublets})\,. \tag{30}$$

The key point is that M_{ij} is an *antisymmetric* product of two $\mathrm{SU}(2)_L$ doublets. The antisymmetric product of two doublets of $\mathrm{SU}(2)$ is a **singlet**:

$$\mathbf{2} \otimes_A \mathbf{2} = \mathbf{1}\,. \tag{31}$$

This means each meson component M_{ij} (for $i < j$) is an $\mathrm{SU}(2)_L$ singlet, *not* a doublet.

Remark 5.1 (Crucial structural difference from the SU route)*.* In the Cacciapaglia–Sannino $\mathrm{SU}(N)$ route, the meson $M^i{}_j = Q^i \widetilde{Q}_j$ transforms as a bifundamental $(\mathbf{N_f}, \overline{\mathbf{N_f}})$ under $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$, and the Higgs doublets emerge from the decomposition of this bifundamental under the gauged $\mathrm{SU}(2)_L$ flavour subgroup.

In the Sp route, the meson M_{ij} is an antisymmetric product of two fundamentals of $\mathrm{SU}(2)_L$ and is therefore an $\mathrm{SU}(2)_L$ **singlet**. It does *not* directly furnish Higgs doublets.

5.3 The Higgs identification problem

The result of §**??** presents a structural obstacle: *the composite meson of the Sp duality is an $\mathrm{SU}(2)_L$ singlet and cannot serve as an electroweak Higgs doublet.*

This is not a technicality; it is a direct consequence of the antisymmetric nature of the Sp meson and the pseudo-reality of the SU(2) fundamental representation. The contraction $\epsilon^{ab}\, Q_i^a\, Q_j^b$ (where a, b are $\mathrm{SU}(2)_L$ indices) is the unique gauge-invariant bilinear, and it produces a singlet.

To obtain Higgs doublets, one would need to modify the construction in one of the following ways:

1. **Extended matter content:** Add fields in higher representations of $\mathrm{SU}(2)_L$ (*e.g.*, triplets) whose composites contain doublet components. This goes beyond the minimal Sp duality.
2. **Partial gauging:** Gauge only a subgroup of the SU(12) flavour symmetry, so that the meson carries non-trivial quantum numbers under the ungauged flavour symmetry that contains hypercharge. In this case, the meson is an $\mathrm{SU}(2)_L$ singlet but can still carry hypercharge and serve as a Higgs-like field.
3. **Higher-order composites:** Identify the Higgs with a higher-order composite (*e.g.*, a baryon or a four-fermion operator) rather than the leading meson.

Option (2) is the most natural: even though M_{ij} is a singlet under $\mathrm{SU}(2)_L$, the indices i, j run over the 12 SM doublets, which carry $\mathrm{SU}(3)_c$, generation, and hypercharge quantum numbers. The meson M_{ij} therefore carries the *combined* quantum numbers of Q_i and Q_j (minus their $\mathrm{SU}(2)_L$ charge, which has been contracted away). In particular, a meson M_{ij} with Q_i a quark doublet and Q_j a lepton doublet carries colour and is not Higgs-like. But a meson with both Q_i and Q_j being lepton doublets of different generations is a colour-singlet and can carry hypercharge-like quantum numbers.

5.4 Hypercharge of the mesons

If we embed hypercharge Y into the flavour symmetry, the meson M_{ij} carries $Y(M_{ij}) = Y(Q_i) + Y(Q_j)$ since the $\mathrm{SU}(2)_L$ contraction ϵ^{ab} is hypercharge-neutral. However, $Y(Q_i)$ is the hypercharge of the *doublet* Q_i, which for left-handed SM fields is:

SM doublet	Y
$q_L = (u_L, d_L)$	$+1/6$
$\ell_L = (\nu_L, e_L)$	$-1/2$

The meson hypercharges are:

Meson type	Composition	$Y(M_{ij})$
Quark–quark	$q_L^{(a)} \cdot q_L^{(b)}$	$+1/3$
Quark–lepton	$q_L^{(a)} \cdot \ell_L^{(b)}$	$-1/3$
Lepton–lepton	$\ell_L^{(a)} \cdot \ell_L^{(b)}$	-1

None of these hypercharges is $\pm 1/2$, which would be required for a standard Higgs doublet. The SM Higgs has $(T_3, Y) = (\pm 1/2, +1/2)$; the Sp mesons are $\mathrm{SU}(2)_L$ singlets ($T_3 = 0$) with $Y \in \{+1/3, -1/3, -1\}$ and their conjugates.

Remark 5.2 (Physical interpretation). The mesons of the Sp route are physically interpretable as:

- $M_{q_L q_L}$ ($Y = +1/3$, $\mathrm{SU}(3)_c$: $\bar{\mathbf{3}}$ or $\mathbf{6}$): **scalar diquarks**.
- $M_{q_L \ell_L}$ ($Y = -1/3$, $\mathrm{SU}(3)_c$: $\mathbf{3}$ or $\bar{\mathbf{3}}$): **scalar leptoquarks**.
- $M_{\ell_L \ell_L}$ ($Y = -1$, colour singlet): **charged scalar singlets** (like a doubly-charged singlet Δ^{--}, or a neutral state depending on the $\mathrm{SU}(2)_L$ component pairing).

These composites are phenomenologically interesting but they are *not* Higgs doublets. The Sp duality thus predicts a qualitatively different scalar sector compared to the SU route.

6 Conformal Window Analysis

6.1 Electric theory: $\mathrm{Sp}(3)$ with $N_f = 6$

From (**??**):

$$b_0 = 3(N_c + 1) - N_f = 3 \times 4 - 6 = 6 > 0\,. \tag{32}$$

The electric theory is asymptotically free. ✓

The conformal window boundaries (**??**):

$$\text{Lower: } \frac{3(N_c+1)}{2} = 6\,, \qquad \text{Upper: } 3(N_c+1) = 12\,. \tag{33}$$

Since $N_f = 6$ equals the lower boundary exactly, the electric Sp(3) theory is at the **edge of the conformal window**: it is in the free magnetic phase, just below the conformal window.

Remark 6.1 (Physical interpretation of the boundary). At $N_f = \frac{3}{2}(N_c+1)$, the magnetic theory is just barely IR-free. The magnetic coupling flows to zero in the deep IR, so the magnetic description is weakly coupled at low energies. This is precisely the regime where the magnetic theory is a good description of the long-distance physics—the magnetic quarks and mesons are the appropriate degrees of freedom. Being at the lower boundary of the conformal window is therefore *favourable* for the interpretation of $\mathrm{SU}(2)_L$ as the low-energy magnetic gauge group.

6.2 Magnetic theory: $\mathrm{Sp}(1)$ with $N_f = 6$

The magnetic beta function:

$$b_0^{\mathrm{mag}} = 3(\widetilde{N_c}+1) - N_f = 3 \times 2 - 6 = 0\,. \tag{34}$$

The magnetic Sp(1) theory with $N_f = 6$ has a *vanishing* one-loop beta function. This corresponds to the fact that $N_f = 6$ is exactly at the lower boundary of the electric conformal window, where the magnetic coupling is marginal.

In the SUSY context, the theory at this point flows to a free magnetic fixed point. The magnetic quarks q_i and mesons M_{ij} become free fields in the deep IR.

6.3 Survey across n_g

n_g	N_f	N_c	Electric	b_0^{el}	Conformal window?
2	4	1	Sp(1)	$3(2) - 4 = 2$	$3 < 4 < 6$: **inside** (self-dual)
3	6	3	Sp(3)	$3(4) - 6 = 6$	$6 < 6 < 12$: **lower boundary**
4	8	5	Sp(5)	$3(6) - 8 = 10$	$9 < 8 < 18$: **below CW**
5	10	7	Sp(7)	$3(8) - 10 = 14$	$12 < 10 < 24$: **below CW**

For $n_g \geq 4$, the matter content is insufficient to place the electric theory inside the conformal window ($N_f < 3(N_c+1)/2$). In this regime, the electric theory confines and the magnetic description may not be a useful dual. Only $n_g = 2$ (inside the conformal window) and $n_g = 3$ (at the boundary) are in the regime where Seiberg duality operates.

This provides a *different kind of generation constraint*: while the Sp route allows $n_g \geq 2$ from the group-theoretic bound (**??**), the conformal window constraint independently requires $n_g \leq 3$ for the duality to be operative. Combined:

$$\boxed{2 \leq n_g \leq 3}\,. \tag{35}$$

7 Comparison with the $\mathrm{SU}(N)$ Route

We now compare the Sp-route UV completion of $\mathrm{SU}(2)_L$ with the Cacciapaglia–Sannino $\mathrm{SU}(N)$ route [**?**], which dualises the QCD sector $\mathrm{SU}(3)_c$ rather than the EW sector directly.

7.1 Summary table

Feature	SU route [?]	Sp route (this work)
Dualised gauge group	$SU(3)_c$ (QCD)	$SU(2)_L \cong Sp(1)$ (EW)
Duality type	$SU(N)$ Seiberg	$Sp(N_c)$ Intriligator–Seiberg
Electric theory ($n_g = 3$)	SU(2) with $N_f = 6$ + adj	Sp(3) with $2N_f = 12$ fund
Adjoint fermion?	Yes (λ)	No (minimal Sp)
Meson symmetry	Bifundamental $(N_f, \bar{N}_f)$	Antisymmetric $\wedge^2$ $SU(2N_f)$
Meson = Higgs?	Yes (18 doublets)	No (singlets of $SU(2)_L$)
Meson physics	Higgs doublets	Diquarks, leptoquarks
Generation bound (group theory)	$n_g \geq 3$	$n_g \geq 2$
Conformal window	$n_g = 3$: inside	$n_g = 3$: lower boundary
Quasi-SUSY partners?	Yes (gaugino, squarks)	From dual quarks and gaugino
Hierarchy problem	Resolved (no elem. scalars)	Requires separate Higgs mechanism

7.2 Generation bound

The SU route requires $n_g \geq 3$ [?, ?]: the electric gauge group is $SU(2n_g - 4)$, which must be non-trivial ($2n_g - 4 \geq 2$).

The Sp route requires only $n_g \geq 2$ from group theory (??), but the conformal window constraint further restricts to $n_g \leq 3$. The combined Sp constraint $2 \leq n_g \leq 3$ is both a lower *and* upper bound, compared to the SU route's lower bound $n_g \geq 3$ alone.

If both routes are realised in nature, the intersection gives $n_g = 3$ as the unique solution:

$$n_g \geq 3 \text{ (SU route)} \ \cap \ n_g \leq 3 \text{ (Sp route)} \qquad \Longrightarrow \qquad \boxed{n_g = 3}. \tag{36}$$

7.3 The Higgs sector problem

The most significant structural difference is the fate of the meson.

SU route: The meson $M^i{}_j = Q^i \widetilde{Q}_j$ transforms as a bifundamental under $SU(N_f)_L \times SU(N_f)_R$. After gauging $SU(2)_L \subset SU(N_f)_L$ and decomposing the flavour representations, the meson produces 18 Higgs doublets of $SU(2)_L$ with the correct hypercharges ($Y = \pm 1/2$). The Higgs mechanism for EWSB is built into the duality.

Sp route: The meson M_{ij} is an antisymmetric product of $SU(2)_L$ doublets, hence an $SU(2)_L$ singlet. It does not furnish Higgs doublets. Instead, it produces diquarks and leptoquarks. A separate mechanism would be needed to generate EWSB.

This is the central obstruction to the Sp route as a complete UV completion of the electroweak sector: *the Sp duality naturally UV-completes* $SU(2)_L$ *as a gauge group, but the composite scalars it produces are not Higgs doublets.*

7.4 Complementarity interpretation

Rather than viewing the SU and Sp routes as competitors, one may interpret them as complementary:

- The **SU route** dualises QCD ($SU(3)_c$) and naturally produces the Higgs sector from the meson. The electroweak sector is embedded in the flavour symmetry.
- The **Sp route** dualises $SU(2)_L$ directly and naturally produces exotic scalars (diquarks, leptoquarks). The QCD sector must be treated separately.
- A **combined construction** could employ both dualities simultaneously, with $SU(3)_c$ and $SU(2)_L$ each having their own electric dual. This would require a product gauge group duality, which is more complex but has been studied in SUSY contexts [**?**].

8 Discussion and Conclusions

We have investigated the UV completion of the electroweak gauge group $SU(2)_L \cong Sp(1)$ via the Intriligator–Seiberg $Sp(N_c)$ Seiberg duality. Our main findings are:

1. **Electric theory identified.** For $n_g = 3$ SM generations, the electric dual of $SU(2)_L$ is Sp(3) with 12 fundamental chiral multiplets. The theory is asymptotically free with $b_0 = 6$ and sits at the lower boundary of the SUSY conformal window (§**??**, §**??**).
2. **Anomaly matching verified.** The 't Hooft anomaly coefficient $\mathcal{A}[SU(12)^2\, U(1)_R]$ matches between the electric Sp(3) and magnetic Sp(1) theories, both symbolically (equation (**??**)) and numerically ($\mathcal{A} = -2$) (§**??**).
3. **Meson content: singlets, not doublets.** The 66 complex meson components are $SU(2)_L$ singlets (antisymmetric product of doublets), carrying quantum numbers of diquarks ($Y = +1/3$), leptoquarks ($Y = -1/3$), and charged singlets ($Y = -1$). They do *not* include Higgs doublets with $Y = \pm 1/2$ (§**??**).
4. **Generation bounds.** The Sp route gives $2 \le n_g \le 3$ (group theory + conformal window), compared to the SU route's $n_g \ge 3$. The intersection uniquely selects $n_g = 3$ (§**??**).
5. **Structural obstruction.** The Sp route does not naturally produce a Higgs sector. This is a *no-go result* for the minimal Sp duality applied to $SU(2)_L$: the antisymmetric structure of the Sp meson is fundamentally incompatible with the doublet structure needed for electroweak symmetry breaking.

8.1 Outlook

Despite the no-go for the Higgs sector, the Sp-route construction has several potentially useful features:

- The composite diquarks and leptoquarks predicted by the Sp meson are phenomenologically relevant: scalar leptoquarks with $Y = -1/3$ have been invoked to explain the B-physics anomalies and could be searched for at the LHC.
- The generation bound $n_g \le 3$ from the conformal window is a top-down constraint not present in the SU route, providing additional structural evidence for the observed three generations.

- The complementarity between the SU and Sp routes suggests a richer duality structure underlying the Standard Model, possibly involving product gauge group dualities or cascading dualities à la Klebanov–Strassler [**?**].

All results in this paper depend on the conjectured non-SUSY extension of Seiberg duality. The SUSY anomaly matching and conformal window analyses are exact; their applicability to the non-supersymmetric SM remains an open question.

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