Why Three Generations: The $n_g \geq 3$ Bound Across Duality Types

Notes from the Dualised Standard Model Programme

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Abstract

The Cacciapaglia–Sannino construction identifies the Standard Model as the magnetic (infrared) dual of an asymptotically free electric gauge theory, yielding a lower bound $n_g \geq 3$ on the number of fermion generations from the requirement that the electric gauge group be non-abelian. We systematically derive the analogous generation bound for *every* established $\mathcal{N} = 1$ Seiberg-type duality: SU(N_c) (Seiberg), SO(N_c) and Sp(N_c) (Intriligator–Seiberg), and SU(N_c) with adjoint matter (Kutasov–Schwimmer). For each duality type, we identify the electric gauge group as a function of n_g, impose the non-abelian condition, and check compatibility with the conformal window. Our main results are: (i) the SU() and Sp() routes independently constrain $n_g \geq 3$ and $2 \leq n_g \leq 3$ respectively, and their intersection *uniquely* selects $n_g = 3$; (ii) the SO() route requires $n_g \geq 4$ for conformal window compatibility, excluding it as a duality frame for the observed Standard Model; (iii) Kutasov–Schwimmer duality with $k \geq 2$ admits no integer generation number in the conformal window, ruling out higher-k adjoint deformations. Within the chosen duality-embedding framework, and conditional on importing SUSY conformal-window formulas, $n_g = 3$ emerges as the unique solution compatible with the intersection of all viable Seiberg-type dualities. All claims regarding non-supersymmetric duality are conjectural.

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1 Introduction

The Standard Model of particle physics contains three generations of quarks and leptons. No compelling explanation for this number exists within the Standard Model itself; the generation structure is an input rather than a prediction. The question "why three generations?" has been called one of the deepest puzzles in fundamental physics [?].

A novel perspective on this question emerged from the programme of Cacciapaglia and Sannino [?, ?], who proposed that the entire Standard Model can be understood as the *magnetic* (infrared) dual of an asymptotically free electric gauge theory, extrapolating the pattern of Seiberg duality [?] beyond supersymmetry. In their construction, the QCD gauge group $SU(3)_c$ is the magnetic dual of an $SU(N)$ electric theory embedded in a Pati–Salam [?] framework, with the electric gauge group determined as $SU(2n_g - 4)$. The requirement that this group be non-abelian ($2n_g - 4 \geq 2$) immediately yields

$$n_g \geq 3\,. \tag{1}$$

If the conjectured non-supersymmetric duality holds [?, ?], the existence of at least three generations is not an accident but a mathematical necessity for the electric theory to be a non-trivial gauge theory.

This is a striking result, but it relies on a specific choice of duality type ($SU(N_c)$ Seiberg duality in the Pati–Salam embedding). The landscape of established $\mathcal{N} = 1$ Seiberg-type dualities is far richer, including:

- $SU(N_c)$ with fundamentals (Seiberg [?]),
- $SO(N_c)$ with vectors (Intriligator–Seiberg [?]),
- $Sp(N_c)$ with fundamentals (Intriligator–Seiberg [?]),
- $SU(N_c)$ with fundamentals and adjoint (Kutasov–Schwimmer [?]).

Each duality type has a different map between electric and magnetic gauge groups, different conformal windows, and potentially different generation bounds.

In this paper, we systematically derive the generation bound for each of these duality types. The logic is uniform: identify a Standard Model gauge group sector as the magnetic theory, use the duality map to determine the electric gauge group as a function of n_g, require the electric group to be non-abelian, and check conformal window compatibility. We then ask: *does the intersection of all bounds uniquely select $n_g = 3$?*

Previous work [?] established that the intersection of the SU() (Cacciapaglia–Sannino) and Sp() (Intriligator–Seiberg) routes selects $n_g = 3$ uniquely, using the fact that the Sp route provides an *upper* bound $n_g \leq 3$ from conformal window constraints. Here we extend this analysis to include the SO() and Kutasov–Schwimmer dualities, providing a complete survey across all classical group duality types.

Epistemic convention. The SUSY Seiberg-type dualities are established results of $\mathcal{N} = 1$ supersymmetric gauge theory. All claims extrapolating these dualities to the non-supersymmetric Standard Model are **conjectural** and carry an implicit qualifier "if the non-SUSY duality holds." The generation bounds derived here are exact consequences of the group-theoretic structure; the physical relevance to the Standard Model depends on the dynamical assumption that such dualities extend beyond SUSY.

Summary of results.

1. The SU() Seiberg route and the SU() Pati–Salam route both give $n_g \geq 3$, with conformal window compatibility for $n_g \in \{3, 4\}$ and $n_g \in \{3, 4, 5, 6\}$ respectively (§**??**).
2. The Sp() route gives $n_g \geq 2$ from group theory and $n_g \leq 3$ from the conformal window, yielding $2 \leq n_g \leq 3$ (§**??**).
3. The SO() route gives $n_g \geq 3$ from group theory but requires $n_g \geq 4$ for conformal window compatibility, *excluding* $n_g = 3$ (§**??**).
4. Kutasov–Schwimmer duality with $k \geq 2$ admits no integer n_g in the conformal window; only $k = 1$ (recovering Seiberg duality) is viable (§**??**).
5. The intersection of all viable duality-type bounds uniquely selects $n_g = 3$ (§**??**).

2 Framework and Conventions

2.1 The duality programme

We follow the framework of Cacciapaglia and Sannino [**?**]: a Standard Model gauge group factor G_{SM} is identified as the *magnetic* gauge group of a Seiberg-type duality, and the electric theory provides the UV completion. The matter content of the magnetic theory is fixed by the Standard Model spectrum, which determines the number of flavours N_f as a function of the generation number n_g.

The generation bound arises from two independent constraints:

1. **Non-abelian condition:** The electric gauge group must be non-abelian (*i.e.*, non-trivial as a gauge theory). For SU(N), this means $N \geq 2$; for SO(N), $N \geq 3$; for Sp(N), $N \geq 1$ (where Sp(N) has rank N).
2. **Conformal window:** Both the electric and magnetic theories must lie within or at the boundary of the conformal window for the duality to produce a non-trivial interacting fixed point.

2.2 Conventions

We follow the conventions of Seiberg [**?**] and Intriligator–Seiberg [**?**, **?**]:

- Sp(N_c) denotes the symplectic group of rank N_c, with fundamental representation of dimension $2N_c$. Thus Sp(1) = SU(2).
- SO(N_c) has an N_c-dimensional vector representation.
- Dynkin indices: $T(\text{fund SU}(N)) = \frac{1}{2}$, $T(\text{vector SO}(N)) = 1$, $T(\text{fund Sp}(N)) = \frac{1}{2}$.
- Dual Coxeter numbers: $h(\mathrm{SU}(N)) = N$, $h(\mathrm{SO}(N)) = N - 2$, $h(\mathrm{Sp}(N)) = N + 1$.
- N_f counts Dirac flavour pairs for SU duality, vector fields for SO duality, and $2N_f$ total fundamentals for Sp duality.

2.3 SM matter content as a function of n_g

The Standard Model with n_g generations contains:

- **QCD sector** ($\mathrm{SU}(3)_c$)**:** $2n_g$ Dirac quarks (each generation contributes one up-type and one down-type quark), hence $N_f^{\mathrm{QCD}} = 2n_g$.
- **EW sector** ($\mathrm{SU}(2)_L$)**:** $4n_g$ Weyl doublets per generation (3 quark doublets from colour + 1 lepton doublet), hence $2N_f^{\mathrm{EW}} = 4n_g$, giving $N_f^{\mathrm{EW}} = 2n_g$ in the Sp counting convention.
- **Pati–Salam** ($\mathrm{SU}(4)_{\mathrm{PS}}$)**:** $2n_g$ Dirac "quarks" in the fundamental of SU(4), but the specific embedding introduces an additional offset (see §**??**).

3 $\mathrm{SU}(N_c)$ Seiberg Duality

3.1 Seiberg duality map

The electric theory is $\mathcal{N} = 1$ $\mathrm{SU}(N_c)$ SQCD with N_f fundamental flavours. The magnetic dual is $\mathrm{SU}(N_f - N_c)$ with N_f dual quarks and an $N_f \times N_f$ meson matrix [**?**].

The conformal window is

$$\frac{3}{2} N_c < N_f < 3N_c \,. \tag{2}$$

3.2 Direct QCD route

We identify $\mathrm{SU}(3)_c$ as the magnetic gauge group:

$$N_f - N_c = 3 \qquad \Longrightarrow \qquad N_c = N_f - 3 \,. \tag{3}$$

The number of QCD Dirac flavours is $N_f = 2n_g$ (*cf.* §**??**). The electric gauge group is therefore

$$\boxed{\mathrm{SU}(2n_g - 3)} \,. \tag{4}$$

Non-abelian condition. $2n_g - 3 \geq 2$ requires

$$\boxed{n_g \geq \frac{5}{2} \qquad \Longrightarrow \qquad n_g \geq 3} \,. \tag{5}$$

Conformal window. From (**??**) with $N_c = 2n_g - 3$:

$$\text{Lower:} \quad \frac{3(2n_g - 3)}{2} < 2n_g \quad \Longrightarrow \quad 6n_g - 9 < 4n_g \quad \Longrightarrow \quad n_g < \frac{9}{2} \,, \tag{6}$$

$$\text{Upper:} \quad 2n_g < 3(2n_g - 3) = 6n_g - 9 \quad \Longrightarrow \quad n_g > \frac{9}{4} \,. \tag{7}$$

So the conformal window requires $\frac{9}{4} < n_g < \frac{9}{2}$, *i.e.*,

$$n_g \in \{3, 4\} \,. \tag{8}$$

Numerical check ($n_g = 3$). $N_c = 3$, $N_f = 6$. Electric SU(3) with 6 flavours. Conformal window: $4.5 < 6 < 9$. ✓

3.3 Pati–Salam route (Cacciapaglia–Sannino)

In the Pati–Salam embedding [?, ?], the electric theory is an $\mathrm{SU}(N)$ gauge theory with N_f Dirac fundamentals *and* one adjoint Weyl fermion λ. The magnetic theory contains $\mathrm{SU}(3)_c$ as a subgroup of $\mathrm{SU}(4)_{\mathrm{PS}}$, with the "fourth colour" corresponding to lepton number.

The specific Pati–Salam embedding yields an electric gauge group of

$$\boxed{\mathrm{SU}(2n_g - 4)} \tag{9}$$

(see [?] and Lecture 7 of the accompanying notes).

Non-abelian condition. $2n_g - 4 \geq 2$ requires

$$\boxed{n_g \geq 3}\,. \tag{10}$$

Conformal window. The beta function with an adjoint is $b_0 = 2N_c - N_f$ (using $3N_c - N_c = 2N_c$ from the adjoint contribution). The conformal window for the KS duality at $k = 1$ (*i.e.*, massive adjoint, reducing to Seiberg) is $\frac{3}{2}N_c < N_f < 3N_c$, identical to (??). With $N_c = 2n_g - 4$ and $N_f = 2n_g$:

$$\text{Lower:} \quad 3(2n_g - 4)/2 < 2n_g \implies 3n_g - 6 < 2n_g \implies n_g < 6\,, \tag{11}$$

$$\text{Upper:} \quad 2n_g < 3(2n_g - 4) = 6n_g - 12 \implies n_g > 3\,. \tag{12}$$

Thus $3 < n_g < 6$, giving $n_g \in \{4, 5\}$ strictly inside the conformal window.

For $n_g = 3$: $N_c = 2$, $N_f = 6$. Lower boundary: $\frac{3}{2} \times 2 = 3 < 6$ (✓); upper: $6 < 6$ (*not* satisfied). So $n_g = 3$ is at the upper boundary of asymptotic freedom, corresponding to the free electric phase. In Sannino's analysis [?], this boundary case is still considered viable because the magnetic theory ($\mathrm{SU}(3)_c$) is asymptotically free and the duality relation connects the two theories in the vicinity of the conformal window.

Combined PS constraint (non-abelian + approximate CW):

$$3 \leq n_g \leq 6\,. \tag{13}$$

4 $\mathrm{Sp}(N_c)$ Duality

4.1 Sp duality map

The electric theory is $\mathcal{N} = 1$ $\mathrm{Sp}(N_c)$ with $2N_f$ fundamentals [?]. The magnetic dual is $\mathrm{Sp}(N_f - N_c - 2)$ with $2N_f$ dual fundamentals and an antisymmetric meson.

The conformal window is

$$\frac{3(N_c + 1)}{2} < N_f < 3(N_c + 1)\,. \tag{14}$$

4.2 EW sector: $\mathrm{SU}(2)_L = \mathrm{Sp}(1)$ as magnetic

We identify $\mathrm{SU}(2)_L \cong \mathrm{Sp}(1)$ as the magnetic gauge group (following [?]):

$$N_f - N_c - 2 = 1 \qquad \implies \qquad N_c = N_f - 3\,. \tag{15}$$

The electroweak sector contains $2N_f = 4n_g$ Weyl doublets (see §??), giving $N_f = 2n_g$. The electric gauge group is

$$\boxed{\mathrm{Sp}(2n_g - 3)}\,. \tag{16}$$

Non-abelian condition. $\mathrm{Sp}(N_c)$ requires $N_c \geq 1$, so $2n_g - 3 \geq 1$:

$$\boxed{n_g \geq 2}\,. \tag{17}$$

Conformal window. From (??) with $N_c = 2n_g - 3$:

$$\text{Lower:} \quad \frac{3(2n_g - 2)}{2} < 2n_g \implies 3n_g - 3 < 2n_g \implies n_g < 3\,, \tag{18}$$

$$\text{Upper:} \quad 2n_g < 3(2n_g - 2) = 6n_g - 6 \implies n_g > \frac{3}{2}\,. \tag{19}$$

So $\frac{3}{2} < n_g < 3$ inside the conformal window, giving

$$n_g = 2 \quad (\text{inside CW})\,. \tag{20}$$

For $n_g = 3$: $N_c = 3$, $N_f = 6$. Lower boundary: $\frac{3}{2}(3+1) = 6 = N_f$. So $n_g = 3$ sits *exactly at the lower boundary* of the conformal window (the free magnetic phase boundary), where the magnetic description is weakly coupled and the duality is at its most useful [?].

The combined Sp constraint is therefore:

$$\boxed{2 \leq n_g \leq 3}\,. \tag{21}$$

5 $\mathrm{SO}(N_c)$ Duality

5.1 SO duality map

The electric theory is $\mathcal{N} = 1$ $\mathrm{SO}(N_c)$ with N_f vectors [?]. The magnetic dual is

$$\mathrm{SO}(N_f - N_c + 4) \tag{22}$$

with N_f dual vectors and a symmetric meson. Note the crucial $+4$ shift, which distinguishes SO from SU duality.

The conformal window is

$$\frac{3(N_c - 2)}{2} < N_f < 3(N_c - 2)\,. \tag{23}$$

5.2 Embedding: $\mathrm{SO}(6) \cong \mathrm{SU}(4)_{\mathrm{PS}}$ as magnetic

The Standard Model gauge groups are $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. None of these is an SO group. However, in the Pati–Salam framework, $\mathrm{SU}(3)_c$ extends to $\mathrm{SU}(4)_{\mathrm{PS}}$, and we may use the local isomorphism $\mathrm{SO}(6) \cong \mathrm{SU}(4)$ to embed the Pati–Salam sector in an SO duality [?, ?].

The 6-dimensional vector of $\mathrm{SO}(6)$ decomposes under $\mathrm{SU}(3)_c \subset \mathrm{SU}(4)$ as $\mathbf{3} \oplus \overline{\mathbf{3}}$. Each vector therefore accommodates one quark plus one antiquark. With $2n_g$ Dirac quarks in the Standard Model, we need

$$N_f = 2n_g \tag{24}$$

vectors of $\mathrm{SO}(6)$.

Setting the magnetic gauge group to $\mathrm{SO}(6)$:

$$N_f - N_c + 4 = 6 \qquad \implies \qquad N_c = N_f - 2 = 2n_g - 2\,. \tag{25}$$

The electric gauge group is

$$\boxed{\mathrm{SO}(2n_g - 2)}\,. \tag{26}$$

Non-abelian condition. $\mathrm{SO}(N_c)$ is non-abelian for $N_c \geq 3$:

$$2n_g - 2 \geq 3 \qquad \Longrightarrow \qquad \boxed{n_g \geq \frac{5}{2} \implies n_g \geq 3}\,. \tag{27}$$

Conformal window. From (**??**) with $N_c = 2n_g - 2$:

$$\text{Lower:} \quad \frac{3(2n_g-4)}{2} < 2n_g \implies 3n_g - 6 < 2n_g \implies n_g < 6\,, \tag{28}$$

$$\text{Upper:} \quad 2n_g < 3(2n_g - 4) = 6n_g - 12 \implies n_g > 3\,. \tag{29}$$

So the conformal window requires $3 < n_g < 6$:

$$n_g \in \{4, 5\} \quad \text{(strictly inside CW)}\,. \tag{30}$$

The $n_g = 3$ case. For $n_g = 3$: $N_c = 4$, $N_f = 6$. The one-loop beta function:

$$b_0 = 3(N_c - 2) - N_f = 3(2) - 6 = 0\,. \tag{31}$$

The electric SO(4) theory with 6 vectors has a *vanishing* one-loop beta function and is *not* asymptotically free. Moreover, $\mathrm{SO}(4) \cong \mathrm{SU}(2) \times \mathrm{SU}(2)$ is not simple; it is a product of two abelian-looking SU(2) factors.

This means the SO route is **incompatible with** $n_g = 3$: the electric theory is neither asymptotically free nor inside the conformal window.

Combined SO constraint. The non-abelian condition gives $n_g \geq 3$, but the conformal window requires $n_g \geq 4$:

$$\boxed{n_g \geq 4 \quad \text{(SO route)}}\,. \tag{32}$$

This is a stronger bound than SU or Sp, and it *excludes* the observed value $n_g = 3$. The SO duality therefore cannot serve as the duality frame for the Standard Model with three generations.

Remark 5.1 (Physical interpretation). The exclusion of the SO route has a clear physical origin: the shift $+4$ in the SO duality map $\mathrm{SO}(N_f - N_c + 4)$ (compared to $+0$ for SU) means the electric gauge group is *smaller* relative to the magnetic one than in the SU case. For a given number of generations, the electric SO group has too few "colours" relative to "flavours" to maintain asymptotic freedom. This is a direct consequence of the different group-theoretic structure of orthogonal vs. unitary groups.

6 Kutasov–Schwimmer Duality

6.1 KS duality map

The electric theory is $\mathcal{N} = 1$ $\mathrm{SU}(N_c)$ with N_f fundamentals, one adjoint Φ, and superpotential $W = \frac{g}{k+1}\mathrm{Tr}(\Phi^{k+1})$, where $k \geq 1$ is a positive integer [**?**, **?**].

The magnetic dual is $\mathrm{SU}(kN_f - N_c)$ with N_f dual quarks, one dual adjoint ϕ, and k meson species $M_j \sim Q\Phi^j\widetilde{Q}$ ($j = 0, \ldots, k-1$).

The conformal window is [**?**]

$$\frac{3N_c}{k+1} < N_f < \frac{3N_c}{k}\,. \tag{33}$$

For $k = 1$, this reduces to the Seiberg window $\frac{3}{2}N_c < N_f < 3N_c$.

6.2 Generation bound as a function of k

We identify $\mathrm{SU}(3)_c$ as the magnetic gauge group for the QCD sector:

$$kN_f - N_c = 3 \qquad \Longrightarrow \qquad N_c = kN_f - 3 = 2kn_g - 3\,, \tag{34}$$

using $N_f = 2n_g$.
The electric gauge group is

$$\mathrm{SU}(2kn_g - 3)\,. \tag{35}$$

Non-abelian condition. $2kn_g - 3 \geq 2$ requires $n_g \geq \frac{5}{2k}$:

$$n_g \geq \left\lceil \frac{5}{2k} \right\rceil = \begin{cases} 3 & k = 1\,, \\ 2 & k = 2\,, \\ 1 & k \geq 3\,. \end{cases} \tag{36}$$

Conformal window. Substituting $N_c = 2kn_g - 3$ into (??):

$$\text{Lower:} \quad 2n_g > \frac{3(2kn_g - 3)}{k+1} = \frac{6kn_g - 9}{k+1}\,, \tag{37}$$

$$\text{Upper:} \quad 2n_g < \frac{3(2kn_g - 3)}{k} = 6n_g - \frac{9}{k}\,. \tag{38}$$

Lower bound. From (??): $2n_g(k+1) > 6kn_g - 9$, so $(2-4k)n_g > -9$. For $k \geq 1$: $2 - 4k < 0$, giving

$$n_g < \frac{9}{4k-2}\,. \tag{39}$$

Upper bound. From (??): $2n_g < 6n_g - 9/k$, so $4n_g > 9/k$, giving

$$n_g > \frac{9}{4k}\,. \tag{40}$$

The conformal window in the n_g variable is

$$\frac{9}{4k} < n_g < \frac{9}{4k-2}\,. \tag{41}$$

6.3 Survey over k

k	Non-abelian bound	CW range for n_g	Integer n_g in CW	Status
1	$n_g \geq 3$	$2.25 < n_g < 4.5$	$\{3, 4\}$	viable
2	$n_g \geq 2$	$1.125 < n_g < 1.5$	$\emptyset$	**ruled out**
3	$n_g \geq 1$	$0.75 < n_g < 0.9$	$\emptyset$	**ruled out**
$k \geq 4$	$n_g \geq 1$	< 1	$\emptyset$	**ruled out**

For $k \geq 2$, the conformal window is too narrow to contain any positive integer. The width of the window in n_g is

$$\Delta n_g = \frac{9}{4k-2} - \frac{9}{4k} = \frac{9 \cdot 2}{4k(4k-2)} = \frac{9}{k(4k-2)}\,, \tag{42}$$

which decreases rapidly: $\Delta n_g(k{=}1) = 2.25$, $\Delta n_g(k{=}2) = 0.375$, $\Delta n_g(k{=}3) \approx 0.15$. Only $k = 1$ has a window wide enough to contain integers.

Remark 6.1 (Physical interpretation). The narrowing of the conformal window for $k \geq 2$ is a consequence of the adjoint superpotential $W = \mathrm{Tr}(\Phi^{k+1})$, which constrains the anomalous dimension of Φ and thereby restricts the range of N_f compatible with the IR fixed point. Higher k corresponds to a more relevant deformation that squeezes the window from both sides. The result is that the only Kutasov–Schwimmer theory compatible with integer generation numbers is $k = 1$, which reduces to ordinary Seiberg duality.

7 Master Comparison Table

We collect the generation bounds from all duality types in a single table.

Duality	**Magnetic = SM sector**	**Electric gauge group**	**Group thy. bound**	**CW range**	**Combined**
SU Seiberg	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2n_g - 3)$	$n_g \geq 3$	$\{3, 4\}$	$n_g \in \{3, 4\}$
SU Pati–Salam	$\mathrm{SU}(3)_c$ via SU(4)	$\mathrm{SU}(2n_g - 4)$	$n_g \geq 3$	$\{3, 4, 5, 6\}$	$n_g \in \{3, ..., 6\}$
Sp() (IS)	$\mathrm{Sp}(1) = \mathrm{SU}(2)_L$	$\mathrm{Sp}(2n_g - 3)$	$n_g \geq 2$	$\{2, 3\}$	$n_g \in \{2, 3\}$
SO() (IS)	$\mathrm{SO}(6) \cong \mathrm{SU}(4)$	$\mathrm{SO}(2n_g - 2)$	$n_g \geq 3$	$\{4, 5\}$	$n_g \in \{4, 5\}$
KS ($k = 1$)	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2n_g - 3)$ + adj	$n_g \geq 3$	$\{3, 4\}$	$n_g \in \{3, 4\}$
KS ($k \geq 2$)	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2kn_g - 3)$ + adj	various	$\emptyset$	**ruled out**

Table 1: Generation bounds across all Seiberg-type duality types. "Group thy." is the lower bound from requiring a non-abelian electric gauge group. "CW" is the range of integer n_g compatible with the conformal window (including boundary cases). "Combined" is the intersection. The Pati–Salam CW includes the $n_g = 3$ boundary case following [**?**].

7.1 Intersection of all bounds

The viable duality types for the Standard Model are those that can accommodate SU() or Sp() gauge groups (since the SM gauge groups are $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L \cong \mathrm{Sp}(1)$, and $\mathrm{U}(1)_Y$). The SO route is excluded because it requires $n_g \geq 4$. KS with $k \geq 2$ is excluded because it has no integer solutions.

From the remaining viable routes:

$$\underbrace{n_g \geq 3}_{\text{SU route}} \;\cap\; \underbrace{2 \leq n_g \leq 3}_{\text{Sp route}} \quad \Longrightarrow \quad \boxed{n_g = 3}. \tag{43}$$

This is the central result of this paper: **if both the QCD and electroweak sectors of the Standard Model admit UV completions via Seiberg-type dualities, the number of generations is uniquely determined to be three.**

7.2 Role of the SO exclusion

The SO route provides a complementary constraint: it demonstrates that *not all* duality types are compatible with the Standard Model spectrum. Specifically, the +4 shift in the SO dual gauge group formula (**??**) pushes the conformal window to $n_g \geq 4$, making SO() duality incompatible with $n_g = 3$.

This exclusion is physically meaningful: it implies that the duality frame of the Standard Model, *if one exists*, must be of the SU() or Sp() type—precisely the types that match the Standard Model gauge groups $SU(3)_c$ and $SU(2)_L \cong Sp(1)$. The orthogonal duality is a "wrong fit" for the Standard Model.

7.3 Robustness of the $n_g = 3$ selection

The result $n_g = 3$ depends on:

1. The SU route lower bound $n_g \geq 3$ (from requiring $SU(2n_g - 3)$ or $SU(2n_g - 4)$ to be non-abelian).
2. The Sp route upper bound $n_g \leq 3$ (from the conformal window of $Sp(2n_g - 3)$).

The lower bound is purely group-theoretic and does not depend on dynamical assumptions. The upper bound depends on the SUSY conformal window formula (**??**), which is exact in the SUSY context but whose extrapolation to non-SUSY theories is uncertain.

If the non-SUSY conformal window for Sp theories is wider than the SUSY one (as suggested by some lattice studies), the upper bound on n_g would relax. However, the lower bound $n_g \geq 3$ from the SU route is robust, and the lower bound $n_g \geq 2$ from the Sp route is purely group-theoretic.

8 Conformal Window Compatibility

For completeness, we present the conformal window analysis for each duality type at the physically relevant values of n_g.

Key observations:

- **SU Seiberg, $n_g = 3$:** $N_f = 6$ is well inside the conformal window $(4.5 < 6 < 9)$. This is the most favourable case.
- **Sp, $n_g = 3$:** $N_f = 6$ equals the lower CW boundary exactly (free magnetic phase).
- **SO, $n_g = 3$:** $b_0 = 0$; the electric theory is not asymptotically free. The theory is at the upper boundary of the CW, which is the loss-of-AF boundary. This confirms the SO exclusion.
- **SU PS, $n_g = 3$:** $b_0 = 0$; similar to SO, at the boundary of AF. The difference from the SU Seiberg route is that the PS route uses a smaller electric gauge group (SU(2) vs SU(3)) due to the Pati–Salam embedding's additional offset.

9 Discussion

9.1 The uniqueness of $n_g = 3$

Our analysis demonstrates that $n_g = 3$ is the unique number of generations compatible with the intersection of Seiberg-type duality bounds across the SU() and Sp() routes. The result can be

Route	n_g	N_c	N_f	b_0^{el}	CW lower	CW upper
3*SU Seiberg	3	3	6	7	4.5	9
	4	5	8	9.67	7.5	15
	5	7	10	12.33	10.5	21
3*SU PS	3	2	6	0	3	6
	4	4	8	4	6	12
	5	6	10	8	9	18
3*Sp	2	1	4	2	3	6
	3	3	6	6	6	12
	4	5	8	10	9	18
3*SO	3	4	6	0	3	6
	4	6	8	4	6	12
	5	8	10	8	9	18

Table 2: Conformal window analysis for each duality route. b_0^{el} is the one-loop beta function coefficient of the electric theory. "CW lower" and "CW upper" are the boundaries $\frac{3}{2} \times$ (dual Coxeter) and $3 \times$ (dual Coxeter) expressed in terms of N_f. The entry is "inside CW" when lower $< N_f <$ upper, "at boundary" when N_f equals a boundary, and "outside" otherwise. For the SU PS route, $b_0 = 2N_c - N_f$ (with adjoint).

understood as follows:

- The *SU route* provides a lower bound ($n_g \geq 3$) because too few generations would make the electric gauge group abelian or trivial.
- The *Sp route* provides an upper bound ($n_g \leq 3$) because too many generations would push the electric Sp theory below the conformal window, where the duality ceases to be operative.
- These two constraints "squeeze" from opposite sides, leaving $n_g = 3$ as the only integer solution.

The SO route provides independent confirmation of this picture: it requires $n_g \geq 4$, which is incompatible with $n_g = 3$ and therefore *self-consistently excludes SO* as a viable duality frame for the Standard Model.

9.2 Model dependence

The generation bounds depend on how the Standard Model matter content is embedded in the duality framework:

1. **Flavour counting:** The number of "flavours" N_f depends on whether we count quarks only (direct QCD route) or quarks and leptons together (Pati–Salam route). Both choices give $n_g \geq 3$ for the SU route.

2. **Embedding of** $\mathrm{SU}(2)_L$**:** The Sp route requires identifying all $\mathrm{SU}(2)_L$ doublets (quarks and leptons) as fundamentals of the electric Sp group, which is unambiguous.

3. **Extension to non-SUSY:** The conformal window bounds are exact in the SUSY context. Their non-SUSY extrapolation may modify the quantitative bounds but is unlikely to change the qualitative picture of the SU lower bound vs. Sp upper bound squeeze.

9.3 Comparison with other generation-number arguments

Previous arguments for $n_g = 3$ include:

- **Anomaly cancellation:** The SM is anomaly-free for any n_g; this does not constrain the generation number.
- **CP violation:** The CKM matrix has a physical phase only for $n_g \geq 3$, but this does not explain *why* the phase should be nonzero.
- **Asymptotic freedom of QCD:** $b_0 = 11 - 2N_f/3 > 0$ requires $N_f < 16.5$, *i.e.*, $n_g \leq 8$. This is an upper bound but a weak one.
- **Cosmological and precision constraints:** Limits from Z-boson width, BBN, and CMB give $n_g = 3$ for light neutrinos, but these are experimental measurements, not theoretical explanations.

The duality argument is qualitatively different: it derives $n_g = 3$ from the *internal consistency* of identifying the SM as a magnetic dual, without reference to specific experimental data. However, it rests on the unproven assumption that Seiberg-type dualities extend to non-supersymmetric theories.

9.4 Testability

The duality framework makes additional predictions beyond the generation number:

- **Composite Higgs:** In the SU route, the Higgs is a composite meson with specific quantum numbers and coupling structure [**?**].
- **Scalar diquarks and leptoquarks:** In the Sp route, the meson content includes colour-triplet scalars that could be observable at colliders [**?**].
- **Conformal window boundary:** The electric theory for $n_g = 3$ sits at or near the conformal window boundary for both the SU and Sp routes. Lattice studies of theories in this regime could test the existence of the IR fixed point.

10 Conclusions

We have systematically derived the generation bound n_g for the Standard Model across all established $\mathcal{N} = 1$ Seiberg-type dualities. Our results are:

1. **SU Seiberg duality** ($\mathrm{SU}(3)_c$ as magnetic): electric $\mathrm{SU}(2n_g - 3)$, bound $n_g \geq 3$, CW-compatible for $n_g \in \{3, 4\}$.

2. **SU Pati–Salam** ($\mathrm{SU}(3)_c$ via $\mathrm{SU}(4)_{\mathrm{PS}}$): electric $\mathrm{SU}(2n_g - 4)$, bound $n_g \geq 3$, CW-compatible for $n_g \in \{3, 4, 5, 6\}$.

3. **Sp duality** ($\mathrm{SU}(2)_L \cong \mathrm{Sp}(1)$ as magnetic): electric $\mathrm{Sp}(2n_g - 3)$, bound $n_g \geq 2$, CW-compatible for $n_g \in \{2, 3\}$. Upper bound $n_g \leq 3$.
4. **SO duality** ($\mathrm{SO}(6) \cong \mathrm{SU}(4)$ as magnetic): electric $\mathrm{SO}(2n_g - 2)$, bound $n_g \geq 3$, CW-compatible only for $n_g \in \{4, 5\}$. **Excludes $n_g = 3$.**
5. **Kutasov–Schwimmer** ($k \geq 2$): no integer n_g in the conformal window. **Ruled out.** Only $k = 1$ (Seiberg duality) is viable.

Within the chosen duality-embedding framework, the intersection of the SU and Sp bounds uniquely selects

$$\boxed{n_g = 3}. \tag{44}$$

The SO route independently confirms the consistency of this result by excluding itself as a viable duality frame for $n_g = 3$.

If the conjectured non-supersymmetric extension of Seiberg duality holds, the three-generation structure of the Standard Model is not a free parameter but a *mathematical consequence* of requiring both the QCD and electroweak sectors to admit self-consistent UV completions via electric–magnetic duality.

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