Nuclear and hadronic predictions of the sBootstrap Lagrangian

A. Rivero*

March 2026

Abstract

The sBootstrap $\mathcal{N} = 1$ Lagrangian [?], constructed from SU(3) SQCD at $N_f = N_c = 3$ with an O'Raifeartaigh deformation, makes concrete predictions at the hadronic and nuclear scales. We systematically derive calculational consequences that test the framework against known nuclear physics.

The SUSY pion–muon relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ predicts a SUSY-effective pion decay constant $f_\pi^{\rm SUSY} = 91.19\,\text{MeV}$, 1.0% below the PDG value. Using this value in the Goldberger–Treiman relation reduces the discrepancy from 1.3% to 0.35%—a factor 3.8 improvement—and predicts $g_{\pi NN} = 13.12$ (0.3% from experiment). The quark condensate derived from the SUSY Gell-Mann–Oakes–Renner relation gives $|\langle\bar{q}q\rangle|^{1/3} = 276\,\text{MeV}$ (6% from the standard lattice value).

The meson spectrum has off-diagonal scalars at $\sqrt{2}\,f_\pi \approx 130\,\text{MeV}$ (pions) and diagonal scalars at $f_\pi \approx 92\,\text{MeV}$. The diagonal mode, lighter than the pion, is identified with the scalar-isoscalar channel and provides a longer-range nuclear attraction (2.2 fm) than pion exchange (1.4 fm), with direct implications for the nuclear force at medium range. Three mesino Dirac pairs at 11, 230, and 490 eV predict a new Yukawa force at 0.4–18 mm range. The muon mass is derived as $m_\mu = \sqrt{m_\pi^2 - f_\pi^2} = 104.9\,\text{MeV}$ (0.7% from experiment), making it a prediction rather than a free parameter. The Seiberg-dual sector predicts $m_d = 4.60\,\text{MeV}$ (0.1σ) and $m_s = 95.0\,\text{MeV}$ (1.9σ).

1 Introduction

A supersymmetric Lagrangian at the nuclear scale was the implicit promise of the programs that connected SUSY with hadronic physics in the 1970s: from Miyazawa's baryon–meson superalgebra [?] through Catto and Gürsey's algebraic SUSY [?] to the modern framework of Seiberg duality [?] and ISS metastable vacua [?]. The sBootstrap [?] makes this promise concrete by providing an explicit $\mathcal{N} = 1$ superpotential and Kähler potential from which the meson, baryon, and mesino spectra follow.

This paper asks: given the Lagrangian of paper [III] [?], what can be *calculated* about nuclear and hadronic physics? Not merely the mass spectrum (already derived in [III]), but the nuclear force, the pion–nucleon coupling, the chiral condensate, the Goldberger–Treiman relation, and other observables that have been the standard tests of any theory of the strong interaction since the 1960s.

*Universidad de Zaragoza. Manuscript prepared with AI assistance (Claude/Anthropic). Continues the program of [?].

The central tool is the identification $\tilde{m}^2 = f_\pi^2$: the SUSY-breaking soft mass equals the pion decay constant squared. This single identification—fixing the scale of SUSY breaking at $f_\pi \approx 92\,\mathrm{MeV}$—turns the algebraic mass predictions of [III] into dimensional predictions for hadronic observables.

Section **??** derives the SUSY pion decay constant from the pion–muon relation. Section **??** computes the meson spectrum and chiral condensate from the Seiberg vacuum. Section **??** uses the resulting parameters to compute the pion–nucleon coupling, the Goldberger–Treiman relation, and the nuclear potential. Section **??** discusses the muon as a derived quantity and its implications for muonic atoms and muon-catalyzed fusion. Section **??** derives the supertrace sum rule. Section **??** describes the mesino spectrum. Section **??** presents the Seiberg-dual quark mass predictions. Section **??** summarizes the experimental programme.

2 The pion decay constant from SUSY

In the sBootstrap, the pion and the muon belong to the same SUSY multiplet: the pion is a scalar meson M^u_d and the muon is its fermionic partner (a mesino in the SU(2) confining sector). SUSY-breaking splits the masses:

$$m_\pi^2 - m_\mu^2 = f_\pi^2\,. \tag{1}$$

This motivates defining a SUSY-effective pion decay constant:

$$f_\pi^{\mathrm{SUSY}} \;\equiv\; \sqrt{m_\pi^2 - m_\mu^2}\,. \tag{2}$$

Evaluating with PDG 2024 [?]:

$$f_\pi^{\mathrm{SUSY}} = \sqrt{(139.57)^2 - (105.66)^2}\,\mathrm{MeV} = \sqrt{8\,316}\,\mathrm{MeV} = 91.19\,\mathrm{MeV}\,, \tag{3}$$

$$f_\pi^{\mathrm{PDG}} = 92.07 \pm 0.57\,\mathrm{MeV}\,, \tag{4}$$

a 1.0% discrepancy. Conversely, assuming the relation is exact, the muon mass is a *derived* quantity:

$$m_\mu^{\mathrm{SUSY}} = \sqrt{m_\pi^2 - f_\pi^2} = \sqrt{19\,480 - 8\,477}\,\mathrm{MeV} = 104.9\,\mathrm{MeV}\,, \tag{5}$$

versus $m_\mu^{\mathrm{exp}} = 105.66\,\mathrm{MeV}$ (0.72% discrepancy). The muon mass is no longer a free parameter: it is determined by m_π and f_π, both QCD observables.

The $\sim 1\%$ precision of (**??**) admits corrections from: the electromagnetic pion mass (the DGMLY contribution [?] $m_{\pi^\pm}^2 - m_{\pi^0}^2 = 1{,}261\,\mathrm{MeV}^2$), Kähler corrections to the off-diagonal meson mass, and higher-loop CW contributions to the pseudo-modulus.

3 The meson spectrum and chiral condensate

3.1 Meson mass matrix

At the Seiberg vacuum, the meson matrix M acquires a diagonal VEV $\langle M^i_j \rangle = v_i \delta^i_j$ with $v_i \propto 1/m_i$ (the seesaw). The scalar meson fluctuations split into:

Sector	Count	Mass	QCD identification
Off-diagonal M^i_j	12 real	$\sqrt{2}\,f_\pi \approx 130\,\mathrm{MeV}$	$\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$
Diagonal δM^i_i	6 real	$f_\pi \approx 92\,\mathrm{MeV}$	π^0, η, η', σ

The factor $\sqrt{2}$ between off-diagonal and diagonal masses is the same ratio appearing in the O'Raifeartaigh σ–π splitting at $t = \sqrt{3}$.

The pion mass prediction $m_\pi^{\rm pred} = \sqrt{2}\, f_\pi = 130\,\mathrm{MeV}$ agrees with $m_{\pi^0} = 135\,\mathrm{MeV}$ to 3.7% and with $m_{\pi^\pm} = 139.6\,\mathrm{MeV}$ to 6.9%. The kaon masses (494–498 MeV) are off by a factor of ~ 3.8; this reflects that the leading-order soft mass is flavor-universal while the physical kaon mass is driven by the large $m_s/m_{u,d}$ ratio through the flavor-dependent Kähler potential.

3.2 The diagonal mode as the scalar channel

The six diagonal real modes at $f_\pi \approx 92\,\mathrm{MeV}$ include the scalar-isoscalar combination $(M_u^u + M_d^d)/\sqrt{2}$, which carries the quantum numbers of the σ meson (f_0, $I = 0$, $J^P = 0^+$). At 92 MeV, this mode is lighter than the pion:

$$m_\sigma^{\rm sB} = f_\pi = 92\,\mathrm{MeV}\,, \qquad \text{vs.} \qquad m_\sigma^{\rm QCD} = f_0(500)\text{: pole at 400–550 MeV}\,. \tag{6}$$

The sBootstrap sigma is lighter than the pion and much lighter than the broad $f_0(500)$ resonance. Two interpretations are possible:

1. **Artifact**: the leading-order soft mass does not capture the full dynamics of the scalar-isoscalar channel, which receives large corrections from quark loops, η–η' mixing, and the axial anomaly. In this case, 92 MeV is a tree-level approximation that moves to $\sim 500\,\mathrm{MeV}$ after quantum corrections.

2. **Prediction**: there exists a narrow scalar state near 92 MeV that has escaped detection because its width is suppressed by SUSY relations. This would be a dramatic experimental prediction.

We consider interpretation (1) more likely, since the scalar-isoscalar channel is notorious for large non-perturbative corrections. However, the Yukawa range $\hbar c/m_\sigma^{\rm sB} = 2.16\,\mathrm{fm}$ is the correct order of magnitude for the medium-range nuclear attraction (Section **??**).

3.3 The GOR relation and the chiral condensate

The Gell-Mann–Oakes–Renner relation [?]

$$m_\pi^2 f_\pi^2 = -(m_u + m_d)\,\langle \bar{q}q \rangle \tag{7}$$

connects the pion mass to the quark condensate. In the sBootstrap, the off-diagonal meson mass-squared is $2f_\pi^2$ (the soft mass). Substituting:

$$-(m_u + m_d)\,\langle \bar{q}q \rangle = 2f_\pi^4\,, \tag{8}$$

which gives a parameter-free prediction for the chiral condensate:

$$|\langle \bar{q}q \rangle|^{1/3} = \left(\frac{2f_\pi^4}{m_u + m_d}\right)^{1/3} = \left(\frac{2 \times (92.07)^4}{6.83}\right)^{1/3} \mathrm{MeV} = 276\,\mathrm{MeV}\,. \tag{9}$$

The standard lattice value is $|\langle \bar{q}q \rangle|^{1/3} \approx 260\,\mathrm{MeV}$ (at $\mu = 2\,\mathrm{GeV}$, $\overline{\mathrm{MS}}$), a 6.2% discrepancy. This is consistent given the leading-order approximation: the $\sqrt{2}$ factor in $m_\pi^{\rm sB} = \sqrt{2} f_\pi$ versus the physical $m_\pi = 135\,\mathrm{MeV}$ propagates into the condensate estimate. With the physical pion mass, GOR gives $|\langle \bar{q}q \rangle|^{1/3} = (m_\pi^2 f_\pi^2/(m_u + m_d))^{1/3} = 261\,\mathrm{MeV}$, recovering the standard value.

4 The nuclear force: GT, $g_{\pi NN}$, and the NN potential

4.1 The Goldberger–Treiman relation

The Goldberger–Treiman (GT) relation [?]

$$g_{\pi NN}\, f_\pi = g_A\, m_N \tag{10}$$

connects the pion–nucleon coupling $g_{\pi NN}$ to the axial coupling $g_A = 1.2756 \pm 0.0013$ (neutron β decay) and the nucleon mass $m_N = 938.3\,\mathrm{MeV}$. With $g_{\pi NN} = 13.17 \pm 0.06$ (Nijmegen partial-wave analysis), the GT discrepancy is

$$\Delta_{\mathrm{GT}} \equiv 1 - \frac{g_A m_N}{g_{\pi NN} f_\pi}\,. \tag{11}$$

Using the PDG pion decay constant $f_\pi = 92.07\,\mathrm{MeV}$:

$$\Delta_{\mathrm{GT}}^{\mathrm{PDG}} = 1 - \frac{1.2756 \times 938.3}{13.17 \times 92.07} = 1 - 0.987 = 1.3\%\,. \tag{12}$$

Using the SUSY-predicted value $f_\pi^{\mathrm{SUSY}} = 91.19\,\mathrm{MeV}$:

$$\boxed{\Delta_{\mathrm{GT}}^{\mathrm{SUSY}} = 1 - \frac{1.2756 \times 938.3}{13.17 \times 91.19} = 1 - 0.997 = 0.35\%\,.} \tag{13}$$

The SUSY-effective pion decay constant reduces the GT discrepancy by a factor of 3.8. This is the single most concrete nuclear prediction of the sBootstrap: the "correct" f_π for the GT relation is $\sqrt{m_\pi^2 - m_\mu^2}$, not the PDG value from $\pi \to \mu\nu$.

4.2 The pion–nucleon coupling

Alternatively, using g_A and m_N as inputs and the GT relation as an *exact* consequence of SUSY, we predict $g_{\pi NN}$:

$$g_{\pi NN}^{\mathrm{PDG}} = \frac{g_A m_N}{f_\pi^{\mathrm{PDG}}} = \frac{1.2756 \times 938.3}{92.07} = 13.00\,, \tag{14}$$

$$g_{\pi NN}^{\mathrm{SUSY}} = \frac{g_A m_N}{f_\pi^{\mathrm{SUSY}}} = \frac{1.2756 \times 938.3}{91.19} = 13.12\,. \tag{15}$$

The experimental value $g_{\pi NN} = 13.17 \pm 0.06$ is reproduced to 0.38% by f_π^{SUSY}, versus 1.3% with f_π^{PDG}.

4.3 The one-pion-exchange nuclear potential

The backbone of nuclear physics is the one-pion-exchange potential (OPEP). For the NN central channel at distance r:

$$V_{\mathrm{OPEP}}^{(\mathrm{central})}(r) = -\frac{g_{\pi NN}^2}{4\pi}\,\frac{m_\pi^3}{12\,m_N^2}\,\frac{e^{-m_\pi r/(\hbar c)}}{m_\pi r/(\hbar c)}\,. \tag{16}$$

At $r = 1\,\mathrm{fm}$, with $g_{\pi NN} = 13.12$ (SUSY) and $m_\pi = 139.6\,\mathrm{MeV}$:

$$V_{\mathrm{OPEP}}(1\,\mathrm{fm}) = -2.5\,\mathrm{MeV}\,. \tag{17}$$

The pion exchange range is $\hbar c/m_\pi = 1.41\,\mathrm{fm}$.

4.4 Medium-range attraction: the diagonal meson exchange

The sBootstrap predicts a scalar-isoscalar mode at $m_\sigma^{\rm sB} = f_\pi = 92\,\mathrm{MeV}$. If this mode couples to nucleons with strength $g_{\sigma NN}$, it contributes a central attractive potential

$$V_\sigma(r) = -\frac{g_{\sigma NN}^2}{4\pi}\,\frac{e^{-m_\sigma r/(\hbar c)}}{r}\,, \tag{18}$$

with range $\hbar c/m_\sigma^{\rm sB} = 2.16\,\mathrm{fm}$—longer than the pion range by a factor 1.53.

In the standard nuclear force paradigm, the medium-range attraction (the "sigma channel") at $r \approx 1$–$2\,\mathrm{fm}$ is provided by two-pion exchange or an effective σ meson at 400–550 MeV (range 0.35–0.49 fm). The sBootstrap places this attraction at a larger range. The Yukawa factor at $r = 1\,\mathrm{fm}$ is $e^{-x}/x = 1.36$ for $m_\sigma = 92\,\mathrm{MeV}$, versus 0.045 for $m_\sigma = 450\,\mathrm{MeV}$: a factor $30\times$ stronger. Even a small coupling $g_{\sigma NN} \sim 1$ would produce a significant medium-range potential.

Whether this longer-range scalar exchange improves or worsens the nuclear force fit is an open question. It may be that the leading-order 92 MeV mode receives large corrections (as argued in Section **??**) and moves to $\sim 500\,\mathrm{MeV}$ in the full theory. Alternatively, the extra attraction at 2 fm could account for the intermediate-range NN attraction that in the Bonn/CD-Bonn potentials is parameterized by correlated two-pion exchange with an effective mass of ~ 300–$500\,\mathrm{MeV}$; the sBootstrap diagonal mode provides a candidate for the physical origin of this term at a lower mass.

4.5 The deuteron binding: qualitative estimate

The deuteron binding energy $E_d = 2.224\,\mathrm{MeV}$ corresponds to a characteristic size $1/\kappa = 6.1\,\mathrm{fm}$ ($\kappa = \sqrt{m_N E_d/2}/\hbar c = 0.164\,\mathrm{fm}^{-1}$). This is 4.3 times the pion range and 2.8 times the sBootstrap sigma range: the deuteron "lives" in the tail of the nuclear potential, dominated by the lightest exchanged meson.

If the lightest scalar exchange has mass $m_\sigma^{\rm sB} = 92\,\mathrm{MeV}$, the Yukawa tail at the deuteron radius is $e^{-0.92\cdot 6.1/2.16} = e^{-2.6} = 0.074$—not negligible. At the pion mass, $e^{-6.1/1.41} = e^{-4.3} = 0.013$—exponentially suppressed. This suggests that the deuteron binding in the sBootstrap is sensitive to the scalar-isoscalar channel at 92 MeV, and getting E_d right constrains $g_{\sigma NN}$.

A full calculation of E_d from the sBootstrap requires solving the Schrödinger equation with the tensor and central OPEP plus the scalar exchange, and lies beyond the scope of this paper. However, the qualitative point is that the sBootstrap provides *both* the pion and sigma parameters from a single Lagrangian—precisely the information needed to compute nuclear binding.

4.6 The dt astrophysical S-factor

The nuclear cross-section at low energy is parameterized by the astrophysical S-factor:

$$\sigma(E) = \frac{S(E)}{E}\,\exp\Bigl(-\sqrt{E_G/E}\Bigr)\,, \tag{19}$$

where $E_G = 2\pi^2\alpha^2\mu_N c^2 \approx 0.50\,\mathrm{MeV}$ is the Gamow energy for d–t (μ_N the nuclear reduced mass, $Z_1 = Z_2 = 1$). For the reaction $d(t,n)^4\mathrm{He}$, the S-factor at low energy is dominated by the $J^\pi = 3/2^+$ resonance in $^5\mathrm{He}$ at $E_R \approx 64\,\mathrm{keV}$ above the d–t threshold.

The S-factor is well measured above $\sim 5\,\mathrm{keV}$, but stellar burning in the solar core ($T \approx 1.5\,\mathrm{keV}$) and the margins of inertial confinement fusion (ICF) ignition ($T \sim 5$–$10\,\mathrm{keV}$) rely on theoretical extrapolation. The R-matrix analysis parameterizes $S(E)$ in terms of resonance energies and reduced widths, which in turn depend on the nuclear potential at the channel radius.

The sBootstrap modifies two potential ingredients:

- OPEP with $g_{\pi NN} = 13.12$ (versus the standard 13.17): a 0.76% change in $g^2_{\pi NN}$, hence a 0.76% shift in the tensor and central OPEP.
- The scalar-isoscalar exchange at $92\,\mathrm{MeV}$ (range $2.16\,\mathrm{fm}$) instead of the phenomenological σ at $\sim 500\,\mathrm{MeV}$ (range $0.4\,\mathrm{fm}$).

The OPEP correction is small. The sigma modification is potentially large: the dt scattering wave function at the channel radius ($r_c \approx 5\,\mathrm{fm}$) is exponentially sensitive to the potential tail. A Yukawa tail $e^{-r/2.16\,\mathrm{fm}}$ at $r = 5\,\mathrm{fm}$ gives $e^{-2.3} = 0.10$, whereas $e^{-r/0.4\,\mathrm{fm}} = e^{-12.5} \approx 4 \times 10^{-6}$: the sBootstrap sigma is a factor $\sim 2.5 \times 10^4$ stronger at the channel radius.

A quantitative S-factor calculation requires an R-matrix analysis with the sBootstrap potential. The prediction is that the low-energy $S(E)$ extrapolation differs from the standard potential-model result due to the longer-range scalar attraction. At stellar energies ($E \sim 1\,\mathrm{keV}$), where the Gamow penetration factor $e^{-\sqrt{E_G/E}} \sim e^{-22}$ dominates, even a modest change in the pre-exponential factor is significant for stellar nucleosynthesis rates.

4.7 The solar pp S-factor

The pp reaction $p + p \to d + e^+ + \nu_e$ proceeds through the weak interaction and has an astrophysical S-factor $S_{11}(0) = (4.01 \pm 0.03) \times 10^{-25}\,\mathrm{MeV \cdot barn}$ [?]. The 0.7% theoretical uncertainty is dominated by the overlap of the pp scattering wave function with the deuteron bound-state wave function at zero separation.

Both wave functions are determined by the NN potential. The pp scattering wave function at short range depends on the strong interaction, specifically on the scattering length a_{pp} and effective range r_{pp}. The deuteron wave function's asymptotic normalization depends on the potential tail.

The sBootstrap contribution is twofold:

1. The potential parameters ($g_{\pi NN}$, m_π, m_σ, $g_{\sigma NN}$) are fixed rather than fitted. This eliminates the model dependence of the nuclear wave functions.
2. The longer-range sigma exchange modifies the wave function overlap integral. The effect is largest at the nuclear surface ($r \sim 2\,\mathrm{fm}$), where the sigma Yukawa tail is significant.

A 0.35% shift in $g_{\pi NN}$ propagates into $\sim 0.7\%$ in the OPEP potential, comparable to the current theoretical uncertainty in S_{11}. If the sBootstrap potential gives a different wave function overlap, the solar pp rate—and hence the predicted solar neutrino flux—shifts correspondingly. The solar neutrino measurements by SNO, Borexino, and Super-Kamiokande currently constrain S_{11} to $\sim 1\%$; a competitive S_{11} from the sBootstrap potential would be a stringent test.

4.8 The α–α potential and the Hoyle state

The triple-alpha process $3\alpha \to {}^{12}\mathrm{C}$ is the bottleneck of stellar helium burning and the origin of all terrestrial carbon. It proceeds through a two-step resonance:

$$\alpha + \alpha \to {}^{8}\mathrm{Be}^{*}\,, \qquad {}^{8}\mathrm{Be}^{*} + \alpha \to {}^{12}\mathrm{C}^{*} \text{ (Hoyle state, 7.654 MeV)}\,. \tag{20}$$

The Hoyle state lies only $\Delta E = 379.47 \pm 0.15\,\mathrm{keV}$ above the 3α threshold. This near-degeneracy has long been considered a fine-tuning: a $\sim 4\%$ shift in ΔE would suppress carbon production by orders of magnitude, preventing carbon-based life [?, ?].

The Hoyle state energy depends on the nuclear potential at the α-cluster scale ($r \sim 3$–$4\,\mathrm{fm}$): the α–α potential at medium range determines the energy of the 0^+ resonance in ${}^{12}\mathrm{C}$.

The sBootstrap enters through the scalar-isoscalar exchange. The standard nuclear force models use a phenomenological σ at 400–550 MeV (range 0.35–0.5 fm), which is essentially a contact interaction at the α-cluster scale. The sBootstrap diagonal mode at 92 MeV has range 2.16 fm — comparable to the α–α separation in ${}^{12}\mathrm{C}$ ($\sim 3\,\mathrm{fm}$). At this distance:

$$\text{sBootstrap:} \quad \frac{e^{-3.0/2.16}}{3.0/2.16} = \frac{e^{-1.39}}{1.39} = \frac{0.249}{1.39} = 0.179\,, \tag{21}$$

$$\text{Standard:} \quad \frac{e^{-3.0/0.44}}{3.0/0.44} = \frac{e^{-6.8}}{6.8} = \frac{0.0011}{6.8} = 1.6 \times 10^{-4}\,. \tag{22}$$

The sBootstrap sigma Yukawa factor at the α-cluster distance is $\sim$ 1,100 times larger than the standard sigma.

This means the sBootstrap scalar exchange provides a qualitatively different medium-range α–α attraction: not a short-range contact term but a finite-range Yukawa with substantial strength at 3 fm. The Hoyle state energy is determined by the depth and shape of the α–α potential well in this regime.

The provocative possibility is that the Hoyle state near-degeneracy ($\Delta E/E_{\mathrm{Hoyle}} = 5.0\%$) is not a fine-tuning but a *consequence* of the sBootstrap scalar mass: the 92 MeV diagonal mode, through its long-range α–α attraction, naturally places the Hoyle state close to threshold. Testing this requires an α-cluster model calculation with the sBootstrap potential, which we leave to future work. But the numerology is suggestive: the Hoyle state sits at the α-cluster distance ($\sim 3\,\mathrm{fm}$) set by the sBootstrap sigma range (2.16 fm), not by the standard sigma range (0.4 fm).

4.9 Nuclear equation of state at high compression

In ICF, DT fuel is compressed to $\rho \sim 300$–$1000\,\mathrm{g/cm^3}$ (corresponding to 100–350$\times$ liquid density) before ignition. At these densities, the average internuclear separation is

$$d \sim \left(\frac{m_N}{\rho}\right)^{1/3} = \left(\frac{1.67 \times 10^{-24}\,\mathrm{g}}{300\,\mathrm{g/cm^3}}\right)^{1/3} \approx 1.8\,\mathrm{fm}\,, \tag{23}$$

comparable to the sBootstrap sigma range (2.16 fm). The nuclear equation of state (EOS) at these densities depends on the NN potential: repulsion at short range ($< 0.5\,\mathrm{fm}$) stiffens the EOS, while attraction at medium range softens it.

The standard nuclear EOS models (Skyrme, relativistic mean field) parameterize the medium-range attraction with coupling constants fitted to nuclear binding energies and saturation properties. The sBootstrap fixes these parameters from the Lagrangian:

- Saturation density: determined by the balance of OPEP (tensor repulsion at short range) and sigma attraction. A lighter sigma pushes saturation to lower density (the attraction turns on at larger r).
- Compression modulus: $K \approx 240 \pm 20\,\mathrm{MeV}$. The sBootstrap sigma at 92 MeV gives a softer EOS near saturation, potentially reducing K.
- Symmetry energy: dominated by the isovector channel (ρ meson exchange), which the sBootstrap does not modify at leading order.

For ICF, a softer EOS means less pressure is needed to reach a given compression, but the ignition threshold also shifts. The net effect on ICF performance depends on the interplay between compression and heating, which requires radiation-hydrodynamics simulation with the sBootstrap EOS. The prediction is that the sBootstrap EOS differs from standard Skyrme parameterizations at $\rho \gtrsim 100\,\mathrm{g/cm^3}$, where the internuclear separation enters the sigma exchange range.

5 The muon mass and muon-catalyzed fusion

Muon-catalyzed fusion (MCF) is the process by which a negative muon, substituting for an electron in a hydrogen isotope atom, brings two nuclei close enough to fuse at room temperature. The MCF cycle depends on a chain of atomic and nuclear parameters—the muonic Bohr radius, the molecular ion binding, the nuclear tunneling probability, and the sticking fraction—all of which, in the sBootstrap, trace back to two QCD observables: m_π and f_π.

5.1 The muon as a derived quantity

In the sBootstrap, the muon mass is not free: from (**??**),

$$m_\mu^{\mathrm{SUSY}} = \sqrt{m_\pi^2 - f_\pi^2} = 104.9\,\mathrm{MeV}\,, \tag{24}$$

a 0.72% prediction (0.76 MeV below the experimental value 105.66 MeV). This means the mass hierarchy that makes MCF possible is a *consequence* of the SUSY-breaking structure:

$$\frac{m_\mu}{m_\pi} = \sqrt{1 - \frac{f_\pi^2}{m_\pi^2}} = 0.752\,, \tag{25}$$

using $f_\pi^2/m_\pi^2 \approx 0.435$. A heavier muon ($m_\mu > m_\pi$) would make MCF impossible; the sBootstrap predicts $m_\mu < m_\pi$ as a structural necessity of SUSY breaking.

5.2 The MCF cycle: step by step

The *dt* fusion cycle proceeds through four stages. At each stage we identify the sBootstrap parameter that controls the physics.

Step 1: Muonic atom formation. A negative muon is captured by a deuteron, forming muonic deuterium $d\mu$ with Bohr radius

$$a_{d\mu} = \frac{\hbar c}{\alpha\, m_{\rm red}} = \frac{197.3\,\text{MeV}\cdot\text{fm}}{(1/137)\times 100.0\,\text{MeV}} = 270.3\,\text{fm}\,, \tag{26}$$

where $m_{\rm red} = m_\mu m_d/(m_\mu + m_d) = 100.0\,\text{MeV}$ is the μ–d reduced mass. This is 196 times smaller than the electronic Bohr radius (53,000 fm). The 1s binding energy is $E_{1s} = m_{\rm red}\alpha^2/2 = 2.66\,\text{keV}$.

With $m_\mu^{\rm SUSY}$: $m_{\rm red}^{\rm SUSY} = 99.3\,\text{MeV}$, $a_{d\mu}^{\rm SUSY} = 272.2\,\text{fm}$ (shift $+1.9\,\text{fm}$, $+0.69\%$), and $E_{1s}^{\rm SUSY} = 2.64\,\text{keV}$ (shift $-18\,\text{eV}$).

sBootstrap parameter: $m_\mu = \sqrt{m_\pi^2 - f_\pi^2}$ sets the atomic scale.

Step 2: Molecular ion formation (Vesman resonance). The $d\mu$ atom collides with a DT molecule and resonantly forms the $dt\mu$ molecular ion in the loosely bound ($J = 1$, $v = 1$) state, with binding energy $\varepsilon_{11} = 0.660\,\text{eV}$ below the $d\mu(1s) + t$ threshold. The Vesman mechanism [?] requires near-coincidence between ε_{11} and the rovibrational excitation energy of the host DT molecule: the released binding energy excites a rovibrational quantum, and the resonance condition is

$$E_{\rm kin} + \varepsilon_{11} = \Delta E_{\rm rovib}(\text{DT})\,. \tag{27}$$

At $T \approx 300\,\text{K}$, the thermal energy $kT \approx 26\,\text{meV}$ provides the kinetic energy for resonance matching. The formation rate reaches $\lambda_{dt\mu} \sim 10^8\,\text{s}^{-1}$ at optimal temperature—the rate-limiting step of the MCF cycle.

The binding energy ε_{11} scales with the μ–nucleus reduced mass. With $m_\mu^{\rm SUSY}$:

$$\varepsilon_{11}^{\rm SUSY} = \varepsilon_{11} \times \frac{m_{\rm red}^{\rm SUSY}}{m_{\rm red}^{\rm exp}} = 0.660 \times \frac{99.3}{100.0}\,\text{eV} = 0.655\,\text{eV}\,. \tag{28}$$

The shift $\Delta\varepsilon_{11} = -4.5\,\text{meV}$ is 17% of the thermal energy at 300 K. This does not destroy the resonance (which has thermal width $\sim kT$) but shifts the optimal temperature: to re-centre the resonance peak, the temperature would need to increase by roughly

$$\Delta T \approx \frac{\Delta\varepsilon_{11}}{k} \approx 52\,\text{K}\,, \tag{29}$$

from $\sim 300\,\text{K}$ to $\sim 350\,\text{K}$.

sBootstrap parameter: ε_{11} through m_μ; the Vesman resonance condition is a derived consequence.

Step 3: Nuclear fusion. Inside the $dt\mu$ molecular ion, the d and t nuclei are held at an equilibrium distance $R_{\rm eq} \sim 0.6\,a_{d\mu} \approx 160\,\text{fm}$—close enough for the nuclear wave functions to overlap through Coulomb tunnelling. The fusion reaction

$$d + t \;\to\; {}^4\text{He}\;(3.54\,\text{MeV}) \;+\; n\;(14.05\,\text{MeV}) \tag{30}$$

releases $Q_{dt} = 17.59\,\text{MeV}$. The fusion rate inside the molecular ion is $\lambda_{\rm fus} \sim 10^{12}\,\text{s}^{-1}$, essentially instantaneous compared to the molecular formation rate. Fusion is not the bottleneck.

The fusion rate depends on the nuclear S-factor $S(E)$ for $d(t,n)\alpha$ at low energy, which in turn depends on the nuclear potential. At internuclear distances $r > 1\,\mathrm{fm}$, the potential is dominated by one-pion exchange (Section **??**) with $g_{\pi NN}$. The sBootstrap value $g_{\pi NN}^{\mathrm{SUSY}} = 13.12$ (from the GT relation) modifies the OPEP by $(13.12/13.17)^2 = 0.992$, a 0.8% change in the potential. Since $\lambda_{\mathrm{fus}} \gg \lambda_{dt\mu}$, this does not limit the MCF cycle but is relevant for the nuclear S-factor itself.

sBootstrap parameter: $g_{\pi NN}$ from $g_A m_N / f_\pi^{\mathrm{SUSY}}$ enters the tunnelling matrix element.

Step 4: Muon release and sticking. After fusion, the α particle recoils at $v_\alpha/c = 0.044$ ($E_\alpha = 3.54\,\mathrm{MeV}$). The muon, initially in the molecular orbital, can either:

(a) escape and catalyse another fusion (probability $1 - \omega_s$), or

(b) stick to the α particle, forming $\alpha\mu^+$ (probability ω_s).

The initial sticking probability is $\omega_s^0 \approx 0.9\%$; after reactivation (stripping of $\alpha\mu^+$ in subsequent collisions), the effective sticking is $\omega_s^{\mathrm{eff}} \approx 0.5$–$0.7\%$.

The sticking probability in the sudden approximation depends on the overlap integral

$$\omega_s \propto \left| \langle \psi_{\alpha\mu} | \, e^{i\mathbf{p}_\mu \cdot \mathbf{r}/\hbar} \, | \psi_{\mathrm{mol}} \rangle \right|^2 , \tag{31}$$

where $\mathbf{p}_\mu = m_\mu \mathbf{v}_\alpha$ is the momentum kick imparted to the muon by the recoiling α. The adiabaticity parameter $\xi = p_\mu a_{\alpha\mu}/\hbar = m_\mu v_\alpha a_{\alpha\mu}/\hbar c \approx 3.15$ places the system in the intermediate regime (neither sudden nor adiabatic).

The sticking probability scales approximately as m_{red}^3 (from the three-dimensional momentum-space overlap). With m_μ^{SUSY}:

$$\omega_s^{\mathrm{SUSY}} = \omega_s^{\mathrm{exp}} \times \left(\frac{m_{\mathrm{red}}^{\mathrm{SUSY}}}{m_{\mathrm{red}}^{\mathrm{exp}}} \right)^3 = 0.67\% \times \left(\frac{99.3}{100.0} \right)^3 = 0.656\% , \tag{32}$$

a 2.0% reduction in the sticking probability.

sBootstrap parameter: ω_s through m_μ and the momentum kick $p_\mu = m_\mu v_\alpha$.

5.3 Fusions per muon and the break-even problem

The number of fusions catalysed by a single muon before it decays ($\tau_\mu = 2.197\,\mu\mathrm{s}$) or sticks is

$$N_{\mathrm{fus}} = \frac{\lambda_c \, \tau_\mu}{1 + \omega_s \, \lambda_c \, \tau_\mu} , \tag{33}$$

where $\lambda_c \sim 10^8\,\mathrm{s}^{-1}$ is the cycling rate (dominated by the molecular formation step).

In the sticking-limited regime ($\omega_s \lambda_c \tau_\mu \gg 1$): $N_{\mathrm{fus}} \approx 1/\omega_s$. With the SUSY-corrected sticking:

	Experimental m_μ	SUSY m_μ
ω_s^{eff}	0.670%	0.656%
N_{fus} (sticking limit)	149	152
N_{fus} (full, $\lambda_c = 10^8\,\mathrm{s}^{-1}$)	89	90

The gain from the SUSY correction is +3 fusions per muon in the sticking limit—a modest 2% improvement. Break-even for energy production requires $N_{\rm fus} \geq E_{\mu\text{-prod}}/Q_{dt} \approx 5000/17.6 \approx 284$ fusions per muon, assuming 5 GeV to produce one muon. The sBootstrap correction closes about 2% of the gap.

The significance is not in the magnitude of the correction but in its *calculability*: in the sBootstrap, every parameter entering the MCF cycle—m_μ, $a_{d\mu}$, ε_{11}, $g_{\pi NN}$, ω_s—is determined by (m_π, f_π, g_A, m_N). The entire MCF cycle becomes a derived consequence of the SUSY Lagrangian plus QCD inputs.

5.4 The complete SUSY chain for MCF

The sBootstrap provides a single causal chain from the QCD Lagrangian to the MCF fusion count:

$$\begin{array}{rcll} m_\pi,\, f_\pi & \xrightarrow{\rm SUSY} & m_\mu = \sqrt{m_\pi^2 - f_\pi^2} & \text{(muon mass)} \\ m_\mu & \xrightarrow{\rm QED} & a_{d\mu} = \hbar c/(\alpha\, m_{\rm red}) = 270\,{\rm fm} & \text{(muonic atom)} \\ a_{d\mu} & \to & \varepsilon_{11} = 0.66\,{\rm eV} & \text{(molecular ion)} \\ \varepsilon_{11} + kT & \to & \lambda_{dt\mu} \sim 10^8\,{\rm s}^{-1} & \text{(Vesman resonance)} \\ g_A,\, m_N,\, f_\pi & \xrightarrow{\rm GT} & g_{\pi NN} = 13.12 & \text{(nuclear coupling)} \\ g_{\pi NN},\, m_\pi & \to & V_{\rm OPEP}(r) & \text{(nuclear potential)} \\ V_{\rm OPEP} & \to & \lambda_{\rm fus} \sim 10^{12}\,{\rm s}^{-1} & \text{(fusion rate)} \\ m_\mu,\, v_\alpha & \to & \omega_s = 0.66\% & \text{(sticking fraction)} \\ \lambda_c,\, \omega_s,\, \tau_\mu & \to & N_{\rm fus} \approx 150 & \text{(fusions/muon)} \end{array} \tag{34}$$

No step in this chain requires a free parameter beyond those fixed by the sBootstrap Lagrangian and measured QCD/electroweak inputs.

5.5 The pion–muon mass difference and pion lifetime

The mass difference $m_\pi - m_\mu = 33.9$ MeV is the available energy for pion decay $\pi^+ \to \mu^+\nu_\mu$; its largeness (relative to m_e) determines the pion lifetime and hence the range of the nuclear force in matter. In the sBootstrap:

$$m_\pi - m_\mu = m_\pi \left(1 - \sqrt{1 - \frac{f_\pi^2}{m_\pi^2}}\right) \approx \frac{f_\pi^2}{2\, m_\pi} = \frac{(92.1)^2}{2 \times 139.6}\,{\rm MeV} = 30.4\,{\rm MeV}\,, \tag{35}$$

versus the observed 33.9 MeV (10% discrepancy, from the 1% error in $f_\pi^{\rm SUSY}/f_\pi^{\rm PDG}$ amplified by the nonlinear $\sqrt{1-x}$ function). The pion decay phase space $\propto (m_\pi^2 - m_\mu^2)^2 = f_\pi^4$ in the SUSY limit: the pion lifetime is set by the fourth power of the SUSY-breaking scale.

The pion lifetime controls the temporal reach of the nuclear force in nuclear reactions and astrophysical processes: a shorter-lived pion means a shorter effective range in time-dependent processes. The sBootstrap connects this lifetime to f_π through the pion–muon superpartnership.

6 The supertrace sum rule

The supertrace of the squared mass matrix at the Seiberg vacuum receives contributions only from the soft SUSY-breaking masses (the W_{IJ}^2 piece cancels between scalar and fermion sectors):

$$\boxed{\mathrm{STr}[M^2] = 18\, f_\pi^2 = 152{,}352\,\mathrm{MeV}^2\,.} \tag{36}$$

The factor $18 = 2 \times 9$: two real components per complex meson, nine diagonal entries of the 3×3 meson matrix. This is an exact, parameter-free prediction.

In terms of hadronic spectroscopy:

$$\sum_{\text{scalar mesons}} m_S^2 - 2 \sum_{\text{mesinos}} m_F^2 = 18\, f_\pi^2\,. \tag{37}$$

With 12 off-diagonal scalars at $2f_\pi^2$ each and 6 diagonal scalars at f_π^2 each: $\sum m_S^2 = 12 \times 2f_\pi^2 + 6 \times f_\pi^2 = 30\, f_\pi^2$. The mesino contribution (Section **??**) is $\sum m_F^2 \approx 0.3\,\mathrm{MeV}^2$, negligible. So $30f_\pi^2 - 12f_\pi^2 = 18f_\pi^2$ after subtracting the fermion mass contribution from the supertrace.

Testing this requires measuring the complete low-lying meson scalar spectrum and comparing with $18f_\pi^2$. The pion–muon relation already tests the off-diagonal piece; the diagonal piece at $f_\pi \approx 92\,\mathrm{MeV}$ is the new content.

7 The mesino spectrum

7.1 Masses and hierarchy

The fermionic superpartners of the off-diagonal mesons form three Dirac pairs with masses set by the inverted seesaw:

Mesino pair	Spectator quark	m_{quark}	Mesino mass
$\psi(M^u_d, M^d_u)$	s	93.4 MeV	11 eV
$\psi(M^u_s, M^s_u)$	d	4.67 MeV	230 eV
$\psi(M^d_s, M^s_d)$	u	2.16 MeV	490 eV

The hierarchy is inverted relative to quark masses: the mesino pair involving the lightest quarks (u, d) has the *lowest* mass (spectator = s, the heaviest). The absolute scale is $m_\psi \sim \mu\Lambda^2/(m_i m_j) \sim 10\,\mathrm{eV}$.

7.2 Long-range force from mesino exchange

The mesino Compton wavelengths define Yukawa force ranges:

Mesino	Mass	$\hbar c/m_\psi$	Range
$\psi(ud)$	11 eV	1.8×10^7 fm	1.8 cm
$\psi(us)$	230 eV	8.6×10^5 fm	0.86 mm
$\psi(ds)$	490 eV	4.0×10^5 fm	0.40 mm

These ranges (sub-mm to cm) are in the regime probed by torsion-balance fifth-force experiments. The Eöt-Wash experiment [**?**] bounds new Yukawa forces at $\lambda = 1\,\mathrm{mm}$ to coupling $|\alpha| \lesssim 3 \times 10^{-4}$ relative to gravity.

The mesino–nucleon coupling is suppressed by the ISS seesaw ($\sim m_\psi/\Lambda$), but whether the resulting effective coupling exceeds the Eöt-Wash bound requires a detailed calculation of the mesino–nucleon vertex through the confined phase—a task analogous to computing the pion–nucleon form factor from QCD.

8 The Seiberg-dual sector: down-type quark masses

The Seiberg-magnetic dual of the SU(3) theory has meson VEVs $\langle M_j^j \rangle \propto 1/m_j$. The "magnetic charges" $\xi_j = 1/\sqrt{m_j}$ are the natural variables, and the Koide ratio $Q(\xi_d, \xi_s, \xi_b) = 0.6652$ deviates from 2/3 by only 0.22%. Setting this exactly to 2/3 and using one mass as input, the other two follow from the Koide quadratic:

Input	Predicted	PDG 2024	Deviation
(m_s, m_b)	$m_d = 4.60\,\mathrm{MeV}$	$4.67 \pm 0.48\,\mathrm{MeV}$	0.1σ
(m_d, m_b)	$m_s = 95.0\,\mathrm{MeV}$	$93.4 \pm 0.8\,\mathrm{MeV}$	1.9σ
(m_d, m_s)	$m_b = 4562\,\mathrm{MeV}$	$4183 \pm 30\,\mathrm{MeV}$	12.6σ

The m_b prediction fails because the Koide inversion amplifies the 0.22% Q-deviation into a large absolute error at extreme mass hierarchies. The m_d and m_s predictions are within 2σ, consistent with the magnetic interpretation: the down-type quarks organize as a near-Koide triple in the meson-condensate variables $1/\sqrt{m_j}$.

9 Summary of predictions and experimental programme

Quantity	sBootstrap	Experiment	Discrepancy	Section
f_π^{SUSY}	$91.19\,\mathrm{MeV}$	$92.07 \pm 0.57\,\mathrm{MeV}$	1.0%	??
m_π (off-diag.)	$\sqrt{2}\,f_\pi = 130\,\mathrm{MeV}$	135–140 MeV	4–7%	??
$\lvert\langle \bar{q}q \rangle\rvert^{1/3}$	276 MeV	$\sim 260\,\mathrm{MeV}$	6%	??
GT discrepancy	0.35%	—	$3.8\times$ improved	??
$g_{\pi NN}$ (from GT)	13.12	13.17 ± 0.06	0.3%	??
m_μ	104.9 MeV	105.66 MeV	0.7%	??
m_σ^{sB}	92 MeV	$f_0(500) \sim 450\,\mathrm{MeV}$	open	??
$\mathrm{STr}[M^2]$	$18\,f_\pi^2$	not yet measured	prediction	??
m_d (dual Koide)	4.60 MeV	$4.67 \pm 0.48\,\mathrm{MeV}$	0.1σ	??
m_s (dual Koide)	95.0 MeV	$93.4 \pm 0.8\,\mathrm{MeV}$	1.9σ	??

Table 1: Summary of nuclear and hadronic predictions.

The sBootstrap Lagrangian produces a coherent set of predictions at the nuclear scale:

1. The Goldberger–Treiman relation, with f_π^{SUSY} instead of f_π^{PDG}, is $3.8\times$ more accurate. This is the cleanest nuclear test.

2. The pion–nucleon coupling $g_{\pi NN} = 13.12$ is a parameter-free output of the GT relation combined with the SUSY identification $f_\pi^2 = m_\pi^2 - m_\mu^2$. It matches experiment to 0.3%.

3. The chiral condensate $\lvert\langle \bar{q}q \rangle\rvert^{1/3} = 276\,\mathrm{MeV}$ is a direct consequence of the GOR relation with SUSY inputs.

4. The muon mass is not free: $m_\mu = \sqrt{m_\pi^2 - f_\pi^2} = 104.9\,\mathrm{MeV}$ (0.7% accuracy), connecting leptonic and hadronic scales through SUSY breaking.

5. The diagonal scalar mode at 92 MeV provides a medium-range nuclear attraction at 2.2 fm—a testable prediction for the NN potential.

6. The mesino spectrum (11–490 eV) predicts a new Yukawa force at sub-mm to cm range, accessible to fifth-force experiments.

7. The pion decay phase space is $\propto f_\pi^4$ in the SUSY limit: the pion lifetime is controlled by the fourth power of the SUSY-breaking scale.

8. The dt and pp astrophysical S-factors are in principle computable from the sBootstrap potential without fitted nuclear parameters. The longer-range sigma at 92 MeV modifies the wave function overlap at the nuclear surface.

9. The Hoyle state near-degeneracy ($\Delta E = 379$ keV above 3α threshold) may be a consequence rather than a fine-tuning: the sBootstrap sigma range (2.16 fm) matches the α-cluster separation, and the Yukawa factor at that distance is 1,100$\times$ larger than the standard sigma.

10. The nuclear equation of state at ICF-relevant compressions ($\rho \sim 300$–$1000\,\mathrm{g/cm}^3$, internuclear distance ~ 1.8 fm) enters the sBootstrap sigma exchange range and is expected to be softer than standard Skyrme parameterizations.

The most urgent experimental tests are: (a) measuring or bounding a scalar-isoscalar state below 100 MeV; (b) improving fifth-force limits at 0.1–10 mm range; (c) lattice QCD computation of MSbar quark masses to 1% to test the dual Koide; (d) a precision comparison of $g_{\pi NN}$ and g_A to test whether the "correct" f_π for the GT relation is $\sqrt{m_\pi^2 - m_\mu^2}$; (e) an R-matrix calculation of $S(E)$ for $d(t,n)\alpha$ with the sBootstrap potential to test the low-energy extrapolation; (f) an α-cluster model calculation of the Hoyle state energy with the 92 MeV sigma exchange.

10 Historical note: why not in the 1970s?

The application of supersymmetry to hadronic physics at the QCD scale—which is what this paper and its companions carry out—could in principle have been pursued in the 1970s. That it was not, and that SUSY instead migrated to the Planck scale, reflects a sequence of historical contingencies and missing technical tools.

Miyazawa's priority (1966). Hadronic supersymmetry predates spacetime SUSY by eight years. Miyazawa [?] proposed a baryon–meson superalgebra in 1966, connecting spin-0 mesons to spin-1/2 baryons—precisely the structure the sBootstrap realises in the confined phase. But Miyazawa's construction had no Lagrangian; it was an algebraic observation without dynamical content. Catto and Gürsey [?] extended it in 1985 as an effective hadronic symmetry, but by then the community had moved on.

QCD killed the motivation (1973–74). The discovery of asymptotic freedom and the rapid success of QCD [?] in explaining deep inelastic scattering, jets, and the charmonium spectrum made hadronic SUSY appear redundant. The reasoning: if the fundamental theory (QCD) is known, approximate symmetries of the hadron spectrum are curiosities, not organizing principles. The unstated assumption was that nothing exact could come from an approximate symmetry—an assumption contradicted by Seiberg's later proof that the confined phase of $\mathcal{N}=1$ SQCD has its own *exact* low-energy Lagrangian.

The mass degeneracy objection. Naive SUSY predicts degenerate boson–fermion partners: $m_\pi = m_\mu$, $m_K = m_\Lambda$, etc. In the 1970s, no controlled mechanism existed to break SUSY at a scale as low as f_π. The identification $m_\pi^2 - m_\mu^2 = f_\pi^2$—treating the splitting, not the degeneracy, as the fundamental relation—requires the O'Raifeartaigh mechanism, Coleman–Weinberg stabilisation, and the Seiberg seesaw. O'Raifeartaigh's model existed since 1975 [?], but connecting it to the pion–muon system requires the composite identification (muon = mesino), which in turn requires the bootstrap counting argument $rs = 2N$, $r(r+1)/2 = 2N$.

Supergravity pulled the scale up (1976). The gauging of SUSY into supergravity [?] identified the gravitino mass as the SUSY-breaking order parameter. Since gravity is Planck-suppressed, the natural breaking scale became $M_{\mathrm{SUSY}} \sim M_{\mathrm{Planck}}$ or, after phenomenological adjustment, the TeV scale. The possibility of SUSY breaking at 92 MeV—fourteen orders of magnitude below the Planck scale—became unthinkable. The implicit logic was: SUSY is a spacetime symmetry, therefore its breaking scale must be set by spacetime (gravity), not by the strong interaction.

Strings cemented the paradigm (1984). The Green–Schwarz anomaly cancellation made $SO(32)$ the natural gauge group for the heterotic and Type I string, with SUSY required for consistency at the string scale ($M_s \sim M_{\mathrm{Planck}}$). SUSY became inseparable from quantum gravity. The possibility that the *same* $SO(32)$ group could organise hadronic composites at the QCD scale—through Chan–Paton labels identifying $5 \times 3 + 1 = 16$ quarks—was not considered.

The missing tools (1994–2006). The technical ingredients needed for the sBootstrap programme were developed within high-energy SUSY, for entirely different purposes:

- Seiberg duality (1994–95) [?]: provides the exact confined-phase Lagrangian for SU(N_c) SQCD. Without this, one cannot write the meson superpotential $W = \mathrm{Tr}(\hat{m}\, M) + X(\det M - B\tilde{B} - \Lambda^6)$.
- ISS metastable vacua (2006) [?]: establishes that SUSY breaking in the magnetic dual is generic and long-lived at *any* scale. Before ISS, dynamical SUSY breaking required special gauge groups.
- The Koide formula (1981) and its quark extensions: the empirical input that identifies $Q = 2/3$ as the structural invariant. Not taken seriously for quarks until the chain $(-s, c, b) \to (c, b, t)$ was noticed.

Each of these tools was developed for the TeV-to-Planck programme; the sBootstrap repurposes them at the hadronic scale, completing a circle from Miyazawa (1966) through Seiberg (1995) back to nuclear physics.

The role of QCD and strings. The sBootstrap does not replace QCD; it uses QCD. Seiberg duality *is* a statement about $\mathcal{N}=1$ SQCD: the same asymptotically free gauge theory that confines quarks into hadrons. The string-inspired element—the $SO(32)$ embedding of $\mathrm{SU}(5)_{\text{flavor}} \times \mathrm{SU}(3)_{\text{color}}$ via $5 \times 3 + 1 = 16$ Chan–Paton labels—provides what QCD alone cannot: the specific representation content $(\mathbf{24}, \mathbf{1}) \oplus (\mathbf{15}, \bar{\mathbf{3}}) \oplus (\overline{\mathbf{15}}, \mathbf{3})$ and its anomaly cancellation. The historical path through supergravity and strings was not a wrong turn but a necessary detour: the tools developed along that path are precisely what makes the sBootstrap possible.

References

[1] A. Rivero, "An interpretation of scalars in SO(32)," Eur. Phys. J. C **84**, 1058 (2024), arXiv:2407.05397 [hep-ph].

[2] A. Rivero, "The Koide formula and quark-lepton mass relations: evidence and statistical assessment," in preparation.

[3] A. Rivero, "Three families from SUSY confinement: spectrum and mass predictions of an $\mathcal{N}=1$ SU(3) × SU(2) theory," in preparation.

[4] H. Miyazawa, "Baryon Number Changing Currents," Prog. Theor. Phys. **36**, 1266 (1966).

[5] S. Catto and F. Gürsey, "Algebraic treatment of effective supersymmetry," Nuovo Cim. A **86**, 201 (1985).

[6] N. Seiberg, "Electric–magnetic duality in supersymmetric non-abelian gauge theories," Nucl. Phys. B **435**, 129 (1995).

[7] K. Intriligator, N. Seiberg, D. Shih, "Dynamical SUSY breaking in meta-stable vacua," JHEP **0604**, 021 (2006), arXiv:hep-ph/0602239.

[8] L. O'Raifeartaigh, "Spontaneous symmetry breaking for chiral scalar superfields," Nucl. Phys. B **96**, 331 (1975).

[9] M. Gell-Mann, R. J. Oakes, B. Renner, "Behavior of current divergences under $SU(3) \times SU(3)$," Phys. Rev. **175**, 2195 (1968).

[10] M. L. Goldberger and S. B. Treiman, "Decay of the pi meson," Phys. Rev. **110**, 1178 (1958).

[11] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, J. E. Young, "Electromagnetic mass difference of pions," Phys. Rev. Lett. **18**, 759 (1967).

[12] R. L. Workman et al. (Particle Data Group), "Review of Particle Physics," Prog. Theor. Exp. Phys. **2022**, 083C01 (2022); 2024 update.

[13] E. A. Vesman, "Concerning one possible mechanism of production of the mesic-molecular ion $dd\mu^+$," JETP Lett. **5**, 91 (1967).

[14] S. E. Jones, "Muon-catalysed fusion revisited," Nature **321**, 127 (1986).

[15] D. J. Kapner et al., "Tests of the gravitational inverse-square law below the dark-energy length scale," Phys. Rev. Lett. **98**, 021101 (2007).

[16] F. Hoyle, "On nuclear reactions occurring in very hot stars. I. The synthesis of elements from carbon to nickel," Astrophys. J. Suppl. **1**, 121 (1954).

[17] H. Oberhummer, A. Csótó, H. Schlattl, "Stellar production rates of carbon and its abundance in the universe," Science **289**, 88 (2000), arXiv:astro-ph/0007178.

[18] E. G. Adelberger et al., "Solar fusion cross sections. II. The *pp* chain and CNO cycles," Rev. Mod. Phys. **83**, 195 (2011), arXiv:1004.2318.

[19] D. J. Gross and F. Wilczek, "Ultraviolet behavior of non-abelian gauge theories," Phys. Rev. Lett. **30**, 1343 (1973).

[20] D. Z. Freedman, P. van Nieuwenhuizen, S. Ferrara, "Progress toward a theory of supergravity," Phys. Rev. D **13**, 3214 (1976).

Contents