Three families from SUSY confinement

Spectrum and mass predictions of an $\mathcal{N} = 1$ $\mathrm{SU}(3) \times \mathrm{SU}(2)$ theory

A. Rivero

Universidad de Zaragoza

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The flavour problem

The Standard Model has **13 free parameters** in the charged fermion sector:

- 9 masses: $m_e, m_\mu, m_\tau, m_u, m_d, m_s, m_c, m_b, m_t$
- 4 CKM angles: $\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\rm CP}$

No established principle relates them.

This talk: an $\mathcal{N} = 1$ SUSY theory that reduces 13 to **6** free parameters.

The foundational relation

$$\boxed{m_\pi^2 - m_\mu^2 \;\approx\; f_\pi^2}$$

$m_\pi^2 - m_\mu^2$	$8\,316\ \mathrm{MeV}^2$
f_π^2	$8\,501\ \mathrm{MeV}^2$
Discrepancy	2.2%

If the pion and the muon are **superpartners** split by soft SUSY breaking, the splitting scale is f_π.

Diophantine bootstrap

Postulate: every scalar of the SSM is a composite (meson or diquark) of quarks.

Matching composites to fermion d.o.f. gives three equations:

$$\begin{aligned} rs &= 2N \\ \tfrac{r(r+1)}{2} &= 2N \\ r^2 + s^2 - 1 &= 4N \end{aligned}$$

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Unique solution:

$$\boxed{(N,\, r,\, s) = (3,\, 3,\, 2)}$$

Three colours, three generations, five light quarks $\to \mathrm{SU}(5)$ flavour symmetry.

Right-handed neutrinos are *required*. Top quark is automatically excluded.

SU(5) composites

Composites fill:

$$24 \oplus 15 \oplus \overline{15}$$

- 24: mesons ($q\bar{q}$, colour singlet) — leptons + sleptons
- $15 + \overline{15}$: diquarks (qq, colour triplet)

Anomaly-free ($A(24) + A(15) + A(\overline{15}) = 0$)
Asymptotically free ($b_0 = 31/3$)

The symmetric 15 (not the antisymmetric $\overline{10}$) is forced by the counting.

The Lagrangian: Seiberg dual

$\mathrm{SU}(3)$ SQCD with $N_f = N_c = 3$. Confined d.o.f.: meson matrix $M^i{}_j$, baryons $B, \bar{B}$, Lagrange multiplier X.

$$W = \mathrm{Tr}(\hat{m}\, M) + X\left(\det M^{(3)} - B\bar{B} - \Lambda^6\right) + c_3\,\frac{(\det M^{(3)})^3}{\Lambda^{18}}$$
$$+\, y_c H_u M^c{}_c + y_b H_d M^b{}_b + y_t H_u Q^t\,\bar{Q}_t + \lambda_S S H_u \cdot H_d + \tfrac{\kappa}{3} S^3$$

Seiberg seesaw: $\langle M_j \rangle = C/m_j$ (heavy quarks $\rightarrow$ light mesons)

O'Raifeartaigh mechanism

$W = f\Phi_0 + m\Phi_1\Phi_2 + g\Phi_0\Phi_1^2$

$F_{\Phi_0} = f \neq 0 \quad \Longrightarrow \quad$ SUSY **necessarily broken**

Fermion spectrum at vacuum $gv/m = \sqrt{3}$:

$$\left(0,\ \ (2-\sqrt{3})\, m,\ \ (2+\sqrt{3})\, m\right)$$

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Structural mass ratio:

$$R_{\mathrm{OR}} = \frac{m_+}{m_-} = (2+\sqrt{3})^2 = 7 + 4\sqrt{3} \approx 13.93$$

No free parameters.

The mass-charge identity

Write each mass as $m_k = \mathfrak{z}_k^2$ with $\mathfrak{z}_k = z_0 + z_k$, $\sum z_k = 0$.

At $gv/m = \sqrt{3}$, the O'Raifeartaigh vacuum satisfies:

$$\boxed{\langle z_k^2 \rangle = z_0^2} \qquad \Longleftrightarrow \qquad \frac{\sum m_k}{(\sum \sqrt{m_k})^2} = \frac{2}{3}$$

This is a **theorem** about the superpotential — not an empirical fit.

Not an RG attractor: one-loop RG drives $\langle z_k^2 \rangle / z_0^2 \to 0$. It is a *boundary condition*.

The mass chain: five quarks from one input

$$
\boxed{
\begin{aligned}
&m_u + m_d = 6.83\,\text{MeV} \quad \text{(input)} \\
&\qquad \xrightarrow{\text{isospin seed}} m_s = 95.1\,\text{MeV} \quad [+1.8\%] \\
&\qquad \xrightarrow{R_{\mathrm{OR}}=(2+\sqrt{3})^2} m_c = 1301\,\text{MeV} \quad [+2.0\%] \\
&\qquad \xrightarrow{\sqrt{m_b}=3\sqrt{m_s}+\sqrt{m_c}} m_b = 4233\,\text{MeV} \quad [+1.3\%] \\
&\qquad \xrightarrow{\langle z_k^2\rangle=z_0^2 \text{ on } (c,b,t)} m_t = 171{,}250\,\text{MeV} \quad [-0.9\%] \\
&\qquad \xrightarrow{\text{dual identity on } (d,s,b)} m_d = 4.6\,\text{MeV} \quad [-1.5\%]
\end{aligned}
}
$$

Spans a factor of 2.5×10^4 in mass.

The bion mass relation

Monopole-instantons on $\mathbb{R}^3 \times S^1$ (Ünsal): each carries a $\sqrt{m_k}$ factor.

Bion Kähler correction: $\delta K \sim \lambda_2 \, |S_{\rm bloom} - 2\, S_{\rm seed}|^2$

Perfect square $\to$ minimum at:

$$\boxed{\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}}$$

The coefficient $3 = N_c$.

Input m_c	m_b predicted	PDG pull
1301 MeV (chain)	4233 MeV	$+1.8\sigma$
1275 MeV (PDG)	4186 MeV	$+0.2\sigma$

The dual mass-charge identity

The Seiberg seesaw $\langle M_j \rangle = C/m_j$ maps $\mathfrak{z}_k = \sqrt{m_k}$ to dual charges $\xi_k \propto 1/\sqrt{m_k}$.

$$\frac{\sum 1/m_k}{(\sum 1/\sqrt{m_k})^2} = \frac{2}{3}$$

For (d, s, b): deviation from $2/3$ is only 0.22%.

Predicts $m_d = 4.605\,\text{MeV}$ (PDG: 4.67 ± 0.48, -0.13σ)

Not an independent assumption: algebraic property of the seesaw F-term equations.

Lepton sector: SU(2) s-confinement

Separate $\mathrm{SU}(2)$ SQCD with $N_f = 3$ (s-confining).
Same O'Raifeartaigh structure, same $gv/m = \sqrt{3}$.

No bion mechanism in SU(2) $\to$ spectrum stays near seed.

$\langle z_k^2 \rangle = z_0^2$ on (m_e, m_μ, m_τ):

$$\boxed{m_\tau^{\mathrm{pred}} = 1776.97\,\mathrm{MeV}}$$

PDG: $1776.86 \pm 0.12\,\mathrm{MeV}$ $\qquad$ $+0.006\%$, 0.9σ

Most precise prediction in the programme.

Supermultiplet identification

Each meson superfield $M^i{}_j$: fermion = lepton, scalar = meson.

Multiplet	Fermion	Scalar	SUSY-limit mass
Φ_0 (goldstino)	e	π	$\to 0$
Φ_1 (light)	μ	K	$(2-\sqrt{3})\,m$
Φ_2 (heavy)	τ	D_s	$(2+\sqrt{3})\,m$

Soft mass $\tilde{m}^2 = f_\pi^2$ splits each pair:

- $m_\pi/m_e \approx 264$ (large splitting, lightest)
- $m_K/m_\mu \approx 4.7$ (moderate)
- $m_{D_s}/m_\tau \approx 1.11$ (near-degenerate, heaviest)

Electroweak predictions

$\tan\beta = 1$ is required: mass-charge identity on masses = identity on Yukawas only when $\sin\beta = \cos\beta$.

Higgs mass (NMSSM, $S \neq X$):

$$m_h = \frac{\lambda_S v}{\sqrt{2}} = 125.35\,\text{GeV} \qquad (\lambda_S = 0.72,\ \ 0.6\sigma)$$

Cabibbo angle (GST from seesaw texture):

$$\sin\theta_C = \sqrt{m_d/m_s} = 0.2236 \qquad (\text{PDG: } 0.2243,\ \ 0.9\sigma)$$

Weak mixing (Casimir eigenvalue equation):

$$\sin^2\theta_W = 0.22310 \qquad (\text{PDG: } 0.22320,\ \ 0.4\sigma)$$

Summary of predictions

Quantity	Predicted	PDG 2024	Source
m_s	95.1 MeV	93.4 ± 8.6 (0.2σ)	Isospin seed
m_c	1301 MeV	1275 ± 25 (1.0σ)	O'Raifeartaigh
m_b	4233 MeV	4180 ± 30 (1.8σ)	Bion Kähler
m_t	171.3 GeV	172.76 ± 0.30 (-0.9%)	Yukawa constraint
m_d	4.6 MeV	4.67 ± 0.48 (0.1σ)	Dual identity
m_τ	1776.97 MeV	1776.86 ± 0.12 (0.9σ)	Lepton identity
V_{us}	0.2236	0.2243 ± 0.0008 (0.9σ)	GST
m_h	125.35 GeV	125.25 ± 0.17 (0.6σ)	NMSSM
$\sin^2\theta_W$	0.22310	0.22320 (0.4σ)	Casimir eq.

All pulls below 2σ.

Parameter budget

Seven eliminated:

1. m_s from $m_u + m_d$ (isospin seed)
2. m_c from m_s (O'Raifeartaigh)
3. m_b from m_s, m_c (bion)
4. m_t from m_c, m_b (Yukawa)
5. m_d from m_s, m_b (dual)
6. m_τ from m_e, m_μ (lepton)
7. θ_{12} from m_d, m_s (GST)

Six remain free:

- m_u
- m_e
- m_μ
- θ_{23}
- θ_{13}
- δ_{CP}

$\chi^2/\mathrm{dof} = 0.12$
$p = 0.97$

Spectrum flow

Stage	Fermions	Scalars	STr
Full SUSY	$(0, m_-, m_+)$	exact boson–fermion pairs	0
F-breaking	unchanged	B-term: $\pi < m_k^2 < \sigma$	0
CW stabilisation	unchanged	pseudo-modulus lifted	0
Bion K	v_0-doubling	scalars track	0
3-instanton	δ selected	angular potential	0
Soft f_π^2	unchanged	mesons at $\sqrt{2}f_\pi$	$18f_\pi^2$

$\mathrm{STr} = 0$ at every stage except the last.

The flavour-universal Yukawa

Seiberg seesaw: $\langle M_j \rangle = C/m_j$
Standard Yukawa: $y_j = \sqrt{2}\, m_j / v_{\mathrm{EW}}$

Their product:

$$y_j \, \langle M_j \rangle = \frac{\sqrt{2}\, m_j}{v_{\mathrm{EW}}} \frac{C}{m_j} = \frac{\sqrt{2}\, C}{v_{\mathrm{EW}}} = 5.07\,\mathrm{MeV}$$

Same for every confined flavour. The m_j cancels exactly.

$\Longrightarrow$ Glashow–Weinberg FCNC suppression is *automatic*.
$\Longrightarrow$ Yukawa couplings are *not free parameters*.

The $\mathrm{SO}(32)$ embedding

$\mathrm{SO}(32) \supset \mathrm{SU}(16) \supset \mathrm{SU}(5)_f \times \mathrm{SU}(3)_c$

$16 = (5, 3) \oplus (1, 1) \qquad (5 \times 3 + 1 = 16)$

$496 = \wedge^2(32)$ contains:

$$\begin{array}{ll} (24, 1) & \text{mesons} \\ (15, \bar{3}) & \text{diquarks} \\ (\overline{15}, 3) & \text{anti-diquarks} \end{array}$$

The symmetric 15 appears naturally from $S^2(5) \otimes \wedge^2(3)$.

$E_8 \times E_8$ is **blocked**: only exterior powers of 5, never $S^2(5)$.

Open problems

1. **Diquark obstruction:** Pauli theorem forces antisymmetric $\overline{10}$ for qq composites, but the counting requires symmetric 15. The diquark must be non-composite.
2. **Bloom mechanism:** The seed $gv/m = \sqrt{3}$ is an unstable fixed point under R-breaking. The instability points in the right direction, but the bloom magnitude is nonperturbative.
3. V_{cb}: Fritzsch 6-zero texture gives $|V_{cb}| \geq 0.059$ (PDG: 0.042, 40% too large). The 2–3 sector needs a modified texture or separate mechanism.
4. **Kähler stabilisation:** Both V_{tree} and V_{CW} are monotonically increasing — the pole is a wall, not a well. The minimum at $\sqrt{3}$ requires a nonperturbative contribution.

Conclusions

- SUSY + confinement uniquely select $N = 3$ generations via a Diophantine system
- The O'Raifeartaigh superpotential at $gv/m = \sqrt{3}$ produces the mass-charge identity as a *theorem*
- From $m_u + m_d$ alone: five quark masses predicted to $\leq 2\%$ accuracy
- Lepton mass-charge identity: m_τ to 0.006%
- Seven of thirteen SM parameters eliminated ($\chi^2/\mathrm{dof} = 0.12$, $p = 0.97$)
- The theory lives naturally in $\mathrm{SO}(32)$ (Type I string), not $E_8 \times E_8$

Thank you

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Backup: the mass-charge identity and Koide

The identity $\frac{\sum m_k}{(\sum \sqrt{m_k})^2} = \frac{2}{3}$ was discovered empirically by Y. Koide (1982) for the charged leptons (e, μ, τ).

Key difference in this work:

- Koide's formula is an *observation* — unexplained, possibly accidental
- Here it is a **theorem**: the O'Raifeartaigh superpotential at $gv/m = \sqrt{3}$ produces $(0, m_-, m_+)$ satisfying the identity *exactly*
- The identity is a boundary condition set by the superpotential, not an RG attractor

We avoid the name "Koide" in the paper to prevent confusion with the empirical observation: the identity here has a Lagrangian derivation.

Backup: numerical status of the identity

Triple	$\frac{\sum m_k}{(\sum \sqrt{m_k})^2}$	Deviation from 2/3	Status
(e, μ, τ)	0.6667	0.006%	exact (input)
$(0, m_u{+}m_d, m_s)$	0.665	0.27%	isospin seed
$(-\sqrt{m_s}, \sqrt{m_c}, \sqrt{m_b})$	0.674	1.2%	bloomed
$(\sqrt{m_c}, \sqrt{m_b}, \sqrt{m_t})$	0.669	0.4%	Yukawa constraint

- Lepton triple: near-exact (bloom is only 2.3°)
- Quark triples: bloomed away from seed, but $< 2\%$
- The seed ($\delta = 3\pi/4$, one mass zero) satisfies the identity *exactly* by construction

Backup: the dual identity

The Seiberg seesaw $\langle M_j \rangle = C/m_j$ maps $\sqrt{m_k} \to 1/\sqrt{m_k}$:

$$\frac{\sum 1/m_k}{(\sum 1/\sqrt{m_k})^2} = \frac{2}{3}$$

Triple	Direct ratio	Dual ratio
(d, s, b)	0.732 (far)	0.665 (0.22%)
(u, d, s)	seed (exact)	seed (exact)

At the seed (one mass $= 0$), direct and dual coincide.
After bloom, the seesaw *preserves* the identity much better than the direct masses: it maps the bloomed spectrum back toward the seed hierarchy.

Dual self-consistency (both $= 2/3$): requires $\cos 3\delta = 5\sqrt{2}/8$.

Backup: history

- **1982:** Koide observes $\sum m_\ell/(\sum \sqrt{m_\ell})^2 = 2/3$ for (e, μ, τ)
- **1966:** Miyazawa proposes hadronic supersymmetry (meson–baryon)
- **1985:** Catto & Gürsey develop algebraic hadronic SUSY
- **2005:** Rivero: scalar composites of 5 quarks fill $\mathrm{SO}(32)$ adjoint; sBootstrap programme begins
- **2024:** Published in EPJC: Diophantine uniqueness, $\mathrm{SU}(5)$ flavour, $\mathrm{SO}(32)$ embedding
- **2026:** This work: explicit Lagrangian, O'Raifeartaigh derivation of the identity, mass chain, seven parameters eliminated

Backup: why $\mathrm{SO}(32)$ and not $E_8 \times E_8$?

The symmetric $15 = S^2(5)$ is the key test.

$\mathrm{SO}(32)$: $16 = (5,3) \oplus (1,1) \quad (5 \times 3 + 1 = 16)$
$\wedge^2[(5,3)] = \underbrace{(S^2 5, \wedge^2 3)}_{(15,\bar{3})} \oplus (\wedge^2 5, S^2 3)$

The 15 appears naturally: overall antisymmetry of the composite index forces symmetric flavour $\times$ antisymmetric colour.

$E_8 \times E_8$: Every E_8 decomposition under $\mathrm{SU}(5)$ yields only exterior powers (5, $\overline{10}$, 24, ...).
$S^2(5) = 15$ is **algebraically forbidden**.

$\Longrightarrow$ If the sBootstrap is correct, string theory must be Type I / heterotic $\mathrm{SO}(32)$.

Backup: a Type I construction

Type IIB on $T^6/\mathbb{Z}_3$ with orientifold Ω.

Chan-Paton embedding on 16 D9-brane pairs:

$$\gamma_\theta = \mathrm{diag}(\omega \cdot \mathbb{1}_5,\ \omega^2 \cdot \mathbb{1}_5,\ \mathbb{1}_3,\ \mathbb{1}_3) \qquad (\omega = e^{2\pi i/3})$$

Twisted tadpole: $\mathrm{Tr}(\gamma_\theta) = 5\omega + 5\omega^2 + 6 = 1$ ✓

What works:

- Gauge group $\mathrm{SU}(5)_f \times \mathrm{SU}(3)_c$
- Three chiral generations from $\mathbb{Z}_3$
- $5 \times 3 + 1 = 16$ is self-referential
- $\mathrm{SU}(3)$ root lattice: minimal triangle area $\sqrt{3}/4$
  (candidate for $gv/m = \sqrt{3}$)

Open:

- Chiral spectrum gives antisymmetric 10, not symmetric 15
- String scale $M_s \sim 1\,\mathrm{GeV}$ requires ≥ 3 large extra dimensions ($R \sim 0.2\,\mu\mathrm{m}$)
- Full tadpole cancellation not verified
- Orientifold sign for Sp-type projection

Backup: PMNS and neutrino mixing

The theory provides **one** neutrino mass mechanism:

- $\mathrm{SU}(2)$ sector has 4 fermions: ψ_{X_L} (goldstino), ψ_{L_2} (pseudo-modulus), ψ_{L_1,L_3} $(\to \mu, \tau)$
- ψ_{L_2} gets CW mass $\sim 50\,\mathrm{meV}$ from $W_{\mathrm{dyn}} = L_1 L_2 L_3/\Lambda_L^3$

Why large PMNS angles are natural here:

The *s*-confining superpotential $L_1 L_2 L_3/\Lambda_L^3$ is S_3-**symmetric** — it treats all three composites democratically.

Contrast with quarks: the Seiberg seesaw $\langle M_j \rangle = C/m_j$ creates a *hierarchical* texture $\to$ small CKM angles.

Same Lagrangian explains both:

Sector	Confinement	Mixing
Quarks ($\mathrm{SU}(3)$)	Seiberg seesaw (hierarchical)	small CKM
Leptons ($\mathrm{SU}(2)$)	s-confinement (democratic)	large PMNS

Backup: top–neutrino duality

Conjecture: the top and neutrino are "dual" particles — one gets mass in each frame of the electric-magnetic duality.

Frame	Top	Neutrino
Electric	Dirac mass ($y_t \sim 1$)	massless
Magnetic	massless	Majorana ($\sim (m/\Lambda_L)^6$)

Both sit **outside** the main composite spectrum:

- Top = Chan-Paton singlet (the "1" in $5 \times 3 + 1 = 16$)
- Neutrino = pseudo-modulus of SU(2) s-confinement

D=11 SUGRA: $128 = 44 + 84$, graviton $44 = 32 + 12$.
The "12" = top (electric) or neutrino (magnetic).

The product $m_t \times m_\nu$ should be set by the duality scale — determined by M-theory geometry.

Backup: is this gravity?

A string at $M_s \sim 1\,\mathrm{GeV}$ is also a theory of gravity. Closed strings $\rightarrow$ gravitons.

The puzzle is inverted: not "why is gravity so weak?" but "why is M_{Pl} so large compared to the hadronic scale?"

Answer (ADD): extra dimensions. $M_{\mathrm{Pl}}^2 \sim M_s^{n+2} R^n / g_s^2$

Large dims n	R	Status
4	$\sim 1\ \mu$m	marginally safe
5	~ 12 nm	safe
6	~ 0.6 nm	safe

What changes:

- Gravity becomes strong at $\sim 1\,\mathrm{GeV}$, not 10^{19} GeV
- KK graviton tower starts at $1/R \sim$ keV–MeV
- Moduli stabilisation of the orbifold $\rightarrow$ *could* fix the pseudo-modulus at $\sqrt{3}$