From mesinos and quarkinos to the Standard Model: the sBootstrap as a structural EFT from Seiberg confinement, charge–mass duality, and M-theory

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Abstract

We develop a unified framework in which Standard Model fermions are identified with composite fermions of a confining $\mathrm{SU}(3)_F$ family gauge theory: leptons are mesinos (fermionic partners of the meson superfield $M^i{}_j = Q^i\tilde{Q}_j$), and quarks are quarkinos arising through partial compositeness in the complementary Higgs phase. The confined spectrum naturally produces a doublet $(M_i, \mathrm{adj}(M)_i)$ per family with the mathematical structure of an $\mathcal{N}$=2 hypermultiplet, emergent from confinement plus isospin. Breaking this doublet structure to $\mathcal{N}$=1—by giving a mass to the auxiliary adjoint—generates electroweak isospin: the meson doublet $M^{1,2}$ becomes H_d and the adjugate doublet $\mathrm{adj}(M)^{1,2}$ becomes H_u, producing two Higgs doublets, Yukawa couplings, and the up/down mass splitting from a single composite field.

Having established the identification, we observe that the mass spectrum of the extended meson sector of the Standard Model exhibits a precise algebraic regularity. The signed mass charge $\mathfrak{z}_k = z_0 + z_k$ (with $m_k = \mathfrak{z}_k^2$, $\sum z_k = 0$, and $\langle z_k^2\rangle = z_0^2$) satisfies the energy-balance condition $K = 2/3$ as a trace identity of the U(3) charge operator, independent of the Cartan direction. This regularity appears in the pseudoscalar mesons: the seed triple $(0, \pi, D_s)$ to 0.18% and the bloomed triple $(-\pi, D_s, B)$ to 0.10%; in the charged leptons (e, μ, τ) to 0.001%; in the inverse down quarks $K^{-1}(d, s, b)$ to 0.22%; and in the quark chain (t, b, c)–$(b, c, -s)$–$(c, s, 0)$–$(s, 0, m_u{+}m_d)$. The O'Raifeartaigh model produces the seed spectrum exactly when the coupling ratio satisfies $gv/m = \sqrt{3}$.

The electroweak gauge bosons are accommodated through a Casimir eigenvalue equation $x^2 + s(s{+}1)\,x - s(s{+}1) = 0$ that predicts $\sin^2\theta_W = 0.22310$ (0.4σ from PDG) and $M_W = 80.374\,\mathrm{GeV}$ (0.39σ). A Diophantine system on the scalar composite content uniquely selects $N = 3$ generations, with the composites filling $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$ of $\mathrm{SU}(5)_f$—the representation content naturally contained in the SO(32) adjoint but blocked by $E_8 \times E_8$. In the M-theory lift, mesons are M2-branes (C_3 sector) and baryons are M5-branes (C_6 sector), with the moduli-space involution $M_i \leftrightarrow \Lambda^4/M_i$ realised as electromagnetic duality of the C-field. The membrane dimensionality $p = 2$ is the unique value giving $K = 2/3$, connecting the charge-squared mass law to the geometry of M-theory.

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Contents

1 Introduction: the sBootstrap philosophy

The Standard Model contains 13 free parameters in the flavour sector—six quark masses, three charged-lepton masses, three CKM angles, and a CP phase—with no known relations among them. This paper develops a programme that addresses this problem not by postulating new symmetries on the Yukawa matrices, but by *identifying the observed fermions with composite fermions* of a confining gauge theory, in the spirit of nuclear democracy [?] and hadronic supersymmetry [?, ?].

The foundational observation is the *sBootstrap* relation [?]:

$$m_\pi^2 - m_\mu^2 \ \approx\ f_\pi^2\,, \tag{1}$$

where $f_\pi \approx 92.2\,\mathrm{MeV}$ is the pion decay constant. Numerically, $m_\pi^2 - m_\mu^2 = 8\,316\,\mathrm{MeV}^2$ versus $f_\pi^2 = 8\,501\,\mathrm{MeV}^2$ (a 2.2% discrepancy). This is the Volkov–Akulov mass formula for a pseudo-Goldstino—the muon—whose scalar partner—the pion—is a pseudo-Goldstone boson. If taken seriously, Eq. (**??**) implies that the pion and muon sit in the same $\mathcal{N}{=}1$ supermultiplet, broken at scale f_π.

This identification has a precise realisation in $\mathcal{N}{=}1$ SU(3) SQCD with $N_f = N_c = 3$ flavours. The confined meson superfield $M^i{}_j = Q^i\tilde{Q}_j$ contains both a complex scalar (the meson) and a Weyl fermion (the *mesino*). The sBootstrap postulate is:

> *Mesinos are leptons. The nine fermionic components of the* $3{\times}3$ *meson matrix furnish the SM lepton sector:* $3L_i + 3e_i^c$ *(three doublets plus three singlets).*

The scalar components of the same superfield provide the Higgs doublets and sleptons.

The quark sector arises through a complementary mechanism. In the Higgs phase of the same theory—smoothly connected to the confining phase by the Fradkin–Shenker theorem [?]—the "dressed quarks" $\psi_M^{ij} \sim \tilde{v}_j\psi_Q^i + v_i\psi_{\tilde{Q}}^j$ are the same fermions as the mesinos. Elementary quarks acquire mass through partial compositeness [?]: they mix linearly with composite operators carrying the same quantum numbers. The diquark partners needed to complete the SUSY multiplets fill the symmetric $\mathbf{15}+\overline{\mathbf{15}}$ of $\mathrm{SU}(5)_f$—a representation forbidden for point-like qq composites by a kinematic theorem (the Pauli obstruction), but naturally present in the SO(32) string adjoint.

The paper is organised to follow the logical flow of the programme:

1. **Sections ??–??**: The composite identification (quarks as quarkinos, leptons as mesinos) and the exact Seiberg dynamics that governs the confined spectrum.

2. **Section ??**: The emergent $\mathcal{N}{=}2$ doublet structure and its breaking to $\mathcal{N}{=}1$, which generates electroweak isospin, the two Higgs doublets, and the Yukawa structure.

3. **Sections ??–??**: The charge-squared mass law $m = \mathfrak{z}^2$, the energy-balance condition, and the observation that extended SM mesons satisfy the same algebraic regularity.

4. **Section ??**: The Seiberg seesaw, inverse Koide, and the full chain of direct, inverse, and waterfall formulae.

5. **Section ??**: The role of the W and Z bosons: the Casimir eigenvalue equation and $\sin^2\theta_W$.

6. **Sections ??–??**: Generation uniqueness, the SO(32) embedding, and the M-theory lift.

Throughout, the mass of a particle is the *square* of a signed charge $\mathfrak{z}$: $m = \mathfrak{z}^2$. The charge $\mathfrak{z} = z_0 + z_k$ can be positive or negative; the mass is always non-negative. This is not a notational convention but a physical statement: the fundamental variable is the charge, and the mass is its norm-square—the field energy stored in the gauge flux of a confined composite state.

2 Quarks as quarkinos, leptons as mesinos

2.1 The Miyazawa superalgebra

The idea that mesons and baryons might be related by a fermionic symmetry predates conventional supersymmetry. Miyazawa [?] proposed in 1966 a graded Lie algebra connecting mesons ($q\bar{q}$, bosons) to baryons (qqq, fermions), with the supercharge carrying colour and flavour indices. Catto and Gürsey [?] developed this into an algebraic framework where the diquark $[qq]$ serves as the SUSY partner of the antiquark $\bar{q}$, so that the baryon $q[qq]$ is the fermionic partner of the meson $q\bar{q}$.

In this picture, every quark has a diquark partner, and every meson has a baryon partner. The sBootstrap [?] sharpens this: if the SUSY-breaking scale is f_π rather than the TeV scale, then the meson–lepton pairing is *literal*: the pion and muon are superpartners, split by chiral symmetry breaking.

2.2 The gauge-theoretic realisation

The natural arena for this identification is $\mathcal{N}$=1 $SU(3)_F$ SQCD with $N_f = N_c = 3$ flavours of chiral superfields Q^i, $\tilde{Q}_j$ $(i, j = 1, 2, 3)$. Above the confinement scale Λ_F, the UV gauge group is

$$G_{\mathrm{UV}} = \mathrm{SU}(3)_F \times \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y\,, \tag{2}$$

where $\mathrm{SU}(3)_F$ is the family (preon-colour) gauge group that confines at Λ_F, while the SM gauge factors are weakly gauged spectators.

The preons carry both family colour and electroweak quantum numbers. Embedding $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(3)_L$ via the branching rule $\mathbf{3}_L \to \mathbf{2}_{-1/2} \oplus \mathbf{1}_{+1}$, the preon field content is:

Field	$\mathrm{SU}(3)_F$	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$
Q^A_a $(A = 1, 2)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
Q^3_a	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$+1$
$\tilde{Q}^a{}_i$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	0

After $\mathrm{SU}(3)_F$ confinement, the composite meson superfield decomposes as:

$$M^\alpha{}_i = \underbrace{M^{1,2}{}_i}_{(\mathbf{2}_{-1/2})} \oplus \underbrace{M^3{}_i}_{(\mathbf{1}_{+1})}\,. \tag{3}$$

The central sBootstrap identification is then:

$$\boxed{\psi_{M^{1,2}{}_i} \equiv L_i\,, \qquad \psi_{M^3{}_i} \equiv e^c_i\,.} \tag{4}$$

The nine mesino fermions furnish the SM lepton sector: three lepton doublets plus three charged singlets, with anomaly cancellation guaranteed by 't Hooft matching [**?**].

The scalar components of the same superfields provide:

SQCD object	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	sBootstrap role
${M^A}_i$	$(\mathbf{2}, -1/2)$	L_i mesinos, H_d-type scalars
${M^3}_i$	$(\mathbf{1}, +1)$	e_i^c mesinos, charged scalars
$\mathrm{adj}(M)_A{}^i$	$(\mathbf{2}, +1/2)$	H_u-type composite operator
$B, \tilde{B}$	$(\mathbf{1}, 0)$	singlet baryonic superfields

2.3 The quark sector: diquarkinos and complementarity

Quarks are *not* mesinos—they carry colour. In the sBootstrap, quarks are identified with effective coloured multiplets that arise through Higgs–confinement complementarity [**?**]. For SU(3) with $N_f = 3$ fundamentals, the centre $\mathbb{Z}_3$ is fully screened, and the Higgs and confinement regimes are analytically connected: no local order parameter distinguishes them.

In the Higgs phase, giving the squarks a diagonal VEV breaks all 8 generators of $\mathrm{SU}(3)_F$. After Higgsing, $18 - 8 = 10$ light chiral multiplets remain, matching exactly the confined description: 9 mesons + 2 baryons − 1 constraint = 10. The "mesino" in the confined description is the *same fermion* as the "dressed quark" in the Higgs phase.

The diquark partners $D \sim \wedge^2 Q$ needed to complete the SUSY multiplets encounter a **bare diquarkino obstruction**: requiring the meson doublet to furnish (L_i, e_i^c) forces the diquarkino singlet to carry hypercharge $Y = -4/3$ rather than $+1/3$ or $-2/3$. The resolution [**?**] completes the flavour sector into $\mathbf{15} \oplus \overline{\mathbf{15}} \oplus \mathbf{24}$ of $\mathrm{SU}(5)_f$, with quarkino fields as effective coloured multiplets of this completion.

A kinematic theorem shows that *no* $J = 0$, colour-$\bar{3}$ diquark can have symmetric flavour—regardless of orbital angular momentum—so the sBootstrap diquark in the **15** cannot be a *qq* composite. It must be a fundamentally string-theoretic state, as we discuss in Section **??**.

3 Seiberg confinement and the moduli space

3.1 The quantum-modified constraint

For $N_f = N_c = 3$, the theory confines with a quantum-modified moduli space. The infrared degrees of freedom—mesons ${M^i}_j$, baryon B, antibaryon $\tilde{B}$—satisfy the exact quantum constraint

$$\det M - B\tilde{B} = \Lambda^6 \,, \tag{5}$$

enforced by a Lagrange multiplier X through the superpotential

$$W = X\big(\det M - B\tilde{B} - \Lambda^6\big) + \sum_i m_i \, {M^i}_i \,. \tag{6}$$

The mass deformations m_i break the flavour symmetry. The F-term equations on the mesonic branch ($B = \tilde{B} = 0$) give the central result:

$$\langle M_i \rangle = \frac{\Lambda^2 P^{1/3}}{m_i} \,, \qquad P = m_1 m_2 m_3 \,. \tag{7}$$

Heavy preons produce light mesons, and vice versa. This is the Seiberg seesaw—the inverse relation that will connect the charged-lepton and down-quark sectors.

3.2 The adjugate mechanism

The adjugate of the meson matrix, $\mathrm{adj}(M)_i = \prod_{j\neq i} M_j$, is a *polynomial* (quadratic) in the meson entries—holomorphic, no division required. On the constraint surface $\det M = \Lambda^6$:

$$\mathrm{adj}(M) = \Lambda^6\, M^{-1}\,, \tag{8}$$

so the adjugate provides a *holomorphic realisation of the matrix inverse.* From the F-term equations:

$$\mathrm{adj}(M)_i = -\Lambda^3\, m_i\,. \tag{9}$$

Thus the adjugate is proportional to the UV mass matrix, while $M \propto 1/m$. This gives two natural Yukawa operators at the confinement scale:

$$W_{\mathrm{down}} \supset y_d\, q_L\, d^c\, \frac{M^{1,2}}{\Lambda}\,, \qquad [H_d\text{-type}]\,, \tag{10}$$

$$W_{\mathrm{up}} \supset y_u\, q_L\, u^c\, \frac{\mathrm{adj}(M)^{1,2}}{\Lambda^3}\,, \qquad [H_u\text{-type}]\,. \tag{11}$$

Species coupling through M acquire masses $m_{\mathrm{phys}} \propto 1/m_i$ (inverse hierarchy); species coupling through $\mathrm{adj}(M)$ acquire $m_{\mathrm{phys}} \propto m_i$ (direct hierarchy). There are no independent Yukawa parameters: the mass hierarchy *is* the family charge hierarchy.

4 The emergent $\mathcal{N}$=2 doublet and electroweak isospin

4.1 The composite doublet

The confined spectrum contains two composite operators per family: M_i (meson) and $\mathrm{adj}(M)_i$ (adjugate). Their mass eigenvalues have opposite charge dependence:

$$\langle M_i\rangle \propto 1/m_i\,, \qquad \mathrm{adj}(M)_i \propto m_i\,. \tag{12}$$

This pairing has the mathematical structure of an $\mathcal{N}$=2 hypermultiplet: the two components are rotated by an $\mathrm{SU}(2)_R$ that, in the $\mathcal{N}$=2 language, exchanges the scalar and its dual. Crucially, this doublet structure is *emergent*—it arises from confinement plus isospin, not from a UV $\mathcal{N}$=2 theory that is subsequently broken.

4.2 From $\mathcal{N}$=2 to $\mathcal{N}$=1: the old trick

The "old trick" of getting electroweak isospin from extended SUSY works as follows. Introduce an auxiliary adjoint chiral field Φ with mass μ_Φ and the standard $\mathcal{N}$=2 coupling $W \supset \sqrt{2}\, g\, \tilde{Q}\, \Phi\, Q$. Integrating out Φ via its F-term gives an effective $\mathcal{N}$=1 superpotential with a quartic remnant:

$$W_{\mathrm{eff}} = -\frac{g^2}{\mu_\Phi}\left[\mathrm{Tr}(Q\tilde{Q})^2 - \frac{1}{3}\big(\mathrm{Tr}Q\tilde{Q}\big)^2\right] + m_i\, Q^i \tilde{Q}_i\,. \tag{13}$$

As $\mu_\Phi \to \infty$ (decoupling), the quartic vanishes and one recovers pure $\mathcal{N}$=1 SQCD. At finite μ_Φ, the quartic couples the direct and inverse branches.

The key structural consequence: the breaking $\mathcal{N}=2 \to \mathcal{N}=1$ splits the hypermultiplet $(M_i, \mathrm{adj}(M)_i)$ into two separate chiral superfields with *different* electroweak quantum numbers:

- ${M^{1,2}}_i \sim (\mathbf{2}, -1/2)$: the down-type Higgs doublet H_d.
- ${\mathrm{adj}(M)^{1,2}}_i \sim (\mathbf{2}, +1/2)$: the up-type Higgs doublet H_u.

Both MSSM Higgs doublets emerge from a single meson matrix. The $\mathrm{adj}(M)$ is not an independent superfield but a holomorphic polynomial in M, constrained by the quantum constraint $\det M = \Lambda^6$. At the superpotential level, this constrains $\tan\beta$: $\langle \mathrm{adj}(M)_i \rangle / \langle M_i \rangle = \Lambda^6 / \langle M_i \rangle^2$ is fixed by the quantum constraint (subject to Kähler corrections).

The pairing of a direct mass ($\mathrm{adj}(M)_i \propto m_i$) with an inverse mass ($M_i \propto 1/m_i$) within each family is the composite manifestation of **electroweak isospin**: up-type and down-type quarks in each generation acquire masses from different faces of the same meson matrix.

4.3 The CST alternative

An alternative construction, due to Csaki, Shirman, and Terning [**?**], derives $\mathrm{SU}(2)_L$ dynamically as the magnetic dual gauge group of a confining theory. Consider an electric theory $\mathrm{SU}(N_e)$ with N_f flavours; Seiberg duality gives a magnetic theory with gauge group $\mathrm{SU}(N_f - N_e)$. Setting $N_f - N_e = 2$, the magnetic gauge group is SU(2), identified with $\mathrm{SU}(2)_L$. The magnetic superpotential $W_{\mathrm{mag}} = q\,\mathcal{M}\,\tilde{q}/\mu$ generates Yukawa couplings between magnetic quarks and the meson Higgs. This costs two extra parameters (Λ_{CST} and the electric gauge coupling) that do not enter any numerical prediction; the branching-rule embedding of the main text achieves the same Higgs identification with zero additional parameters.

5 The charge-squared mass law

5.1 Charges from U(3) normalisation

Let the three generations transform as the fundamental $\mathbf{3}$ of $\mathrm{U}(3)_F = \mathrm{SU}(3)_F \times \mathrm{U}(1)$. The Cartan charges satisfy

$$\sum_{i=1}^{3} z_i = 0\,, \qquad \sum_{i=1}^{3} z_i^2 = \frac{1}{2}\,, \tag{14}$$

with the U(1) charge fixed by canonical normalisation: $z_0 = 1/\sqrt{6}$. The total signed charge of the i-th species is

$$\mathfrak{z}_i \;\equiv\; z_0 + z_i\,, \tag{15}$$

and the physical mass is

$$\boxed{m_i \;=\; \mu\,\mathfrak{z}_i^2\,,} \tag{16}$$

with μ a generation-independent scale. The charge can be positive or negative; the mass is always non-negative.

5.2 Why the square: four routes

The quadratic dependence $m \propto \mathfrak{z}^2$ has multiple independent motivations:

(i) Field energy. By Gauss's law, the chromoelectric field of a static preon with family charge $\mathfrak{z}_i$ is $|\mathbf{E}_i| \propto \mathfrak{z}_i$. Since $\mathcal{L} \supset -(1/4g^2)F_{\mu\nu}^2$, the stored field energy is $U_i = (2g^2)^{-1}\int d^3x\, \mathbf{E}_i^2 \propto \mathfrak{z}_i^2$—quadratic in charge, like a capacitor $E = Q^2/(2C)$.

(ii) Bilinear composites. The meson $M^i{}_i = Q^i\tilde{Q}_i$ is a bilinear in the preon fields. If each preon contributes a factor proportional to $\mathfrak{z}_i$, the composite VEV scales as $\mathfrak{z}_i^2$. This is the mechanism identified by Koide [?]: any mass arising through *two* insertions of a VEV (a seesaw or radiative loop) gives $m \propto v^2$, which is the charge-squared structure.

(iii) Membrane area. In the M-theory lift, the meson is an M2-brane stretched between D6-branes. If both worldvolume directions scale linearly with $\mathfrak{z}_i$, the BPS mass is $m \propto T_2 \times \text{Area} \propto \mathfrak{z}_i^2$. This is the unique brane dimensionality that produces $K = 2/3$ (Section **??**).

(iv) Uniqueness theorem. For monomial mass laws $m_i = c\,(z_0 + z_i)^n$, the generalised ratio $K_n = \sum(z_0+z_i)^n/(\sum(z_0+z_i)^{n/2})^2$ is independent of the Cartan direction α if and only if $n = 2$ (besides the trivial $n = 0$). For $n \geq 3$, the cubic Casimir $\sum z_i^3 \propto \cos 3\alpha$ introduces direction-dependent terms.

5.3 The energy-balance condition

For three charges $\mathfrak{z}_k = z_0 + z_k$ with $\sum z_k = 0$:

$$K \;=\; \frac{\sum \mathfrak{z}_k^2}{(\sum \mathfrak{z}_k)^2} \;=\; \frac{3z_0^2 + \sum z_k^2}{9z_0^2} \;=\; \frac{1}{3} + \frac{\langle z_k^2\rangle}{3z_0^2}\,. \tag{17}$$

Therefore:

$$\boxed{K = \frac{2}{3} \quad \Longleftrightarrow \quad \langle z_k^2\rangle = z_0^2\,.} \tag{18}$$

The energy balance condition says: the mean energy of the perturbations equals the energy of the unperturbed charge. This is neither perturbative ($\sigma \ll \bar{\mathfrak{z}}$, $K \to 1/3$, all masses equal) nor dominant ($\sigma \gg \bar{\mathfrak{z}}$, $K \to 1$, one mass dominates). The value $K = 2/3$ is the equipartition point.

One mass can vanish naturally: if $\langle z_k^2\rangle = z_0^2$, a perturbation z_k can reach $|z_k| \sim z_0$, making $\mathfrak{z}_k = z_0 + z_k \approx 0$. A vanishing mass is the natural extreme of energy balance, not fine-tuning.

5.4 The shift parameter

Promoting the fixed offset $z_0 = 1/\sqrt{6}$ to a free shift parameter s, the Koide ratio on the direct branch becomes

$$K(s) = \frac{1}{3} + \frac{1}{18s^2}\,, \tag{19}$$

independent of the Cartan angle α. The condition $K = 2/3$ selects $s = 1/\sqrt{6}$ uniquely. Systematic fitting of all known tuples reveals that s is approximately *universal*: $s \in$

$[0.33, 0.49]$ with a mean of $0.40 \approx z_0$, while the Cartan angle α spans 4°–37° across sectors. The universality of s—not α—is the structural content of the regularity.

6 The extended meson spectrum

6.1 Seed triples

For a triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ with one massless member, the energy balance condition requires:

$$\frac{\mathfrak{z}_a}{\mathfrak{z}_b} = 2 - \sqrt{3} \approx 0.2680\,, \tag{20}$$

a *universal* prediction: any charge triple with one zero member has the same charge ratio between the two massive members, regardless of their absolute scale.

The quark seed: $(0, s, c)$**.** The strange and charm quark charges give $\mathfrak{z}_s/\mathfrak{z}_c = 9.67/35.64 = 0.271$, deviating from the seed ratio by 1.2%. The Koide diagnostic gives $K = 0.664$ (0.35% from 2/3).

The meson seed: $(0, \pi, D_s)$**.** The pion and D_s meson charges give $\mathfrak{z}_\pi/\mathfrak{z}_{D_s} = 11.81/44.37 = 0.266$, deviating by 0.6%. The Koide diagnostic gives $K = 0.668$ (0.18% from 2/3). This is the **meson counterpart** of the quark seed—both involve the strange-charm sector. The two seeds are nearly parallel in charge space (0.3°): the meson seed is the composite image of the quark seed.

The lepton triple as "almost-seed." The charged lepton triple (e, μ, τ) has $\mathfrak{z}_e = 0.715\,\mathrm{MeV}^{1/2}$, which is small but nonzero. The leptons are not an exact seed—the electron, while very light, is not massless. But the triple is the same family: the near-critical charge puts one member close to zero.

6.2 The meson triple $(-\pi, D_s, B)$

With the pion charge *negative*, the triple satisfies:

$$K(-\pi, D_s, B) = \frac{m_\pi + m_{D_s} + m_B}{(-\mathfrak{z}_\pi + \mathfrak{z}_{D_s} + \mathfrak{z}_B)^2} = 0.6674\,, \tag{21}$$

deviating from 2/3 by 0.10%—the second most precise charge triple known (after the leptons). The pion's negative charge has physical meaning: as a pseudo-Goldstone boson, the pion approaches $\mathfrak{z} = 0$ from below as $m_q \to 0$. The sign encodes Goldstone nature.

Among $\binom{11}{3} \times 4 = 660$ combinations of the eleven pseudoscalar mesons with sign patterns, only $(-\pi, D_s, B)$ and the closely related $(-\pi, D_s, B_s)$ lie within 0.2% of $K = 2/3$. All six closest hits share the sign assignment $(-\pi, D_{(s)}, B_{(s,c)})$: a negative pion paired with charm- and bottom-containing mesons—the combination predicted by the $\mathrm{SU}(5)_f$ flavour structure.

6.3 The O'Raifeartaigh mechanism

The seed-to-bloom transition has a precise SUSY-breaking realisation. The O'Raifeartaigh model with three chiral superfields and superpotential $W = f\Phi_0 + m\Phi_1\Phi_2 + g\Phi_0\Phi_1^2$ produces a fermionic mass spectrum $(0, m_-, m_+)$ with $m_\pm = (\sqrt{g^2v^2 + m^2} \pm gv)$. The charge ratio $\mathfrak{z}_-/\mathfrak{z}_+ = 2 - \sqrt{3}$ (the seed condition) requires:

$$\boxed{\frac{gv}{m} = \sqrt{3}\,,} \tag{22}$$

a *single algebraic constraint* on the superpotential parameters. The Koide condition on the seed is therefore one equation relating the cubic coupling, the pseudo-modulus VEV, and the bilinear mass parameter.

The bloom from seed to full triple is a phase rotation $\delta : 3\pi/4 \to \delta_{\text{phys}}$ in the Koide parametrisation, at fixed scale—mass redistributes among the three eigenvalues without changing their sum. The O'Raifeartaigh seed is an *unstable fixed point* of the R-breaking perturbation: any nonzero R-breaking pushes K away from 2/3. The bloom must therefore be a nonperturbative rearrangement.

7 The Seiberg seesaw and the inverse sector

7.1 The dual Koide on (d, s, b)

At the Seiberg vacuum, $\langle M^j{}_j \rangle \propto 1/m_j$: heavy quarks produce light mesons and vice versa. This prompts the question: does the inverse-mass spectrum $\{1/m_i\}$ satisfy the energy-balance condition? Evaluating for the pure down-type quarks:

$$K(1/m_d,\, 1/m_s,\, 1/m_b) = 0.6652\,, \tag{23}$$

a 0.22% deviation from 2/3—better than any direct quark triple. No other quark triple comes close under the Seiberg seesaw (the next best, (d, s, c), deviates by 4.2%).

This "dual Koide" connects the UV mass hierarchy to the IR meson condensates: the Seiberg constraint maps the down-type quark masses into a near-energy-balance spectrum of meson VEVs. The dual charges $\xi_j = 1/\mathfrak{z}_j$ are the charges of the Seiberg meson VEVs—the magnetic Koide is an energy balance on the meson condensate, not on quark masses.

A simple algebraic identity connects the seed to the seesaw: $K(a, b) = K(1/a, 1/b)$ identically for any two masses. The seed $(0, m_s, m_c)$ is thus the unique self-dual fixed point of the seesaw: the electric and magnetic energy-balance conditions coincide only when one mass vanishes.

7.2 The interleaving theorem and the waterfall

The two branches—direct ($a_i = \mu\,\mathfrak{z}_i^2$) and inverse ($d_i = \nu/\mathfrak{z}_i^2$)—have opposite orderings: $a_1 < a_2 < a_3$ while $d_1 > d_2 > d_3$. This is the algebraic origin of the quark mass waterfall.

Theorem 1 (Interleaving)**.** *The six unmixed masses form the alternating ladder* $d_1 > a_3 > d_2 > a_2 > d_3 > a_1$ *if and only if* $\max(\mathfrak{z}_1^2\mathfrak{z}_3^2, \mathfrak{z}_2^4) < \nu/\mu < \mathfrak{z}_2^2\mathfrak{z}_3^2$.

The SM quark masses satisfy this window condition. The resulting 6-rung ladder alternates strictly between inverse and direct entries, producing consecutive windows that mix both branches. The sign flip in $K(b,c,-s)$ signals the branch mismatch in the mixed window.

The geometric mean $\sqrt{a_i \cdot d_i} = \sqrt{\mu\nu}$ is family-independent—a direct consequence of $\det M = \Lambda^6$.

7.3 The chain structure

The quark predictions arise from a chain of overlapping triples:

$$(t,b,c) - (b,c,s)^{\pm} - (c,s,0) - (s,0,m_u{+}m_d)\,, \tag{24}$$

split into two regimes. On the **Fermi side** (above the SUSY-breaking scale): $K(m_t,m_b,m_c) \approx 2/3$ (standard, 0.39%). On the **chiral side** (below the breaking scale): $m_c/m_s = (2{+}\sqrt{3})^2 \approx 13.93$ and $m_s/(m_u{+}m_d) = (2{+}\sqrt{3})^2$. The bridge tuple (b,c,s) with a signed charge flip connects the two sides.

The v_0-doubling relation $\mathfrak{z}_b = 3\mathfrak{z}_s + \mathfrak{z}_c$ matches PDG to 0.07% ($m_b = 4177$ vs. 4180 MeV). This arises from a bion Kähler correction $K_{\text{bion}} \propto |\sum s_k \mathfrak{z}_k|^2$ from monopole-instantons on $\mathbb{R}^3 \times S^1$.

7.4 CKM predictions

The self-dual charge spectrum has $R = (5+\sqrt{21})/2$, the fundamental unit of $\mathbb{Z}[(1+\sqrt{21})/2]$. The CKM elements:

Observable	Formula	Predicted	PDG 2024	Status
$\lvert V_{us}\rvert$	$\sqrt{m_d/m_s}$	0.2236	0.2243 ± 0.0008	0.9σ
$\lvert V_{cb}\rvert$	$1/(5R)$	0.0417	0.0422 ± 0.0008	0.6σ
$\lvert V_{ub}\rvert$	$2\sqrt{2}/(7R^3)$	0.00367	0.00368 ± 0.00011	0.1σ
δ_{CP}	$\arctan\sqrt{R}$	65.4°	$65.7 \pm 3.4^\circ$	0.1σ
J	closed form	3.04×10^{-5}	3.08×10^{-5}	0.3σ

The CP phase is equivalent to $\cos^2(2\delta) = N/(N{+}4) = N/(2N{+}1)$, which holds only for $N = 3$—the same condition as discriminant matching. The integers 5 and 7 are empirical pattern identifications; derivations from the operator basis remain open.

8 The W and Z bosons

8.1 The Casimir eigenvalue equation

Consider the eigenvalue equation

$$x^2 + C_2\,x - C_2 = 0\,, \qquad C_2 = s(s{+}1)\,, \tag{25}$$

where C_2 is the quadratic Casimir of SU(2) at spin s. This equation is pinned by three structural constraints: (i) no spin-independent mass term ($f_0 = 0$); (ii) a massless eigenstate at $s = 0$ ($g_0 = 0$); (iii) coefficient antisymmetry ($g_1 = -f_1$).

Evaluating at $s = 1/2$ ($C_2 = 3/4$) and $s = 1$ ($C_2 = 2$):

$$x_+(1/2) = \tfrac{1}{8}(-3+\sqrt{57})\,, \quad x_+(1/2)\,m^2 = M_W^2\,, \tag{26}$$

$$x_+(1) = -1+\sqrt{3}\,, \quad x_+(1)\,m^2 = M_Z^2\,. \tag{27}$$

The electroweak ratio is

$$R \equiv 1 - \frac{x_+(1/2)}{x_+(1)} = \frac{(\sqrt{19}-3)(\sqrt{19}-\sqrt{3})}{16} \approx 0.22310\,, \tag{28}$$

to be compared with $\sin^2\theta_W^{\rm os} = 0.22320 \pm 0.00026$ (0.4σ). The predicted W mass is $M_W^{\rm pred} = 80.374 \pm 0.002\,\mathrm{GeV}$ $(0.39\sigma$ from PDG).

8.2 The fourth eigenvalue

At $s = 1/2$, the second root gives $|M(1/2,-)| = 122.4\,\mathrm{GeV}$, which is 2.3% below $M_H = 125.25\,\mathrm{GeV}$. This eigenstate carries spin 1/2 while the Higgs is spin 0; the near-degeneracy is noted as a numerical curiosity.

8.3 The scale parameter

Setting $x_+(1)\,m^2 = M_Z^2$ gives $m = M_Z/\sqrt{\sqrt{3}-1} = 106.6\,\mathrm{GeV} \approx m_\mu \times 10^3$.

8.4 Origin and scheme dependence

This observation was first made by H. de Vries (2004) and subsequently studied in [**?**]. The comparison uses the on-shell definition $\sin^2\theta_W = 1 - M_W^2/M_Z^2 = 0.22320$, **not** the $\overline{\rm MS}$ value 0.23122. Until a dynamical mechanism selecting the on-shell scheme is identified, the agreement should be regarded as suggestive but scheme-dependent.

9 Generation uniqueness and the SO(32) embedding

9.1 The Diophantine system

The sBootstrap postulates that every scalar of the Supersymmetric Standard Model is a composite of two quarks (diquark) or of a quark–antiquark pair (meson). Matching composites to fermions, charge sector by charge sector:

$$rs = 2N\,, \tag{29}$$

$$\frac{r(r+1)}{2} = 2N\,, \tag{30}$$

$$r^2 + s^2 - 1 = 4N\,. \tag{31}$$

Theorem 2 (Generation uniqueness)**.** *The system* (**??**)–(**??**) *has a unique positive-integer solution:* $(N, r, s) = (3, 3, 2)$.

With $r+s = 5$ light quarks forming the fundamental **5** of $\mathrm{SU}(5)_f$, the composites organise into $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$. The counting equation (**??**) selects the *symmetric* product **15**, not the antisymmetric $\overline{\mathbf{10}}$—the Pauli obstruction for point-like diquarks.

9.2 Discriminant matching

Two numbers arise naturally for N generations: the algebraic discriminant $D_{\mathrm{alg}} = (N+2)^2 - 4$ and the dynamical discriminant $D_{\mathrm{dyn}} = N(2N+1)$. The condition $D_{\mathrm{alg}} = D_{\mathrm{dyn}}$ reduces to $N^2 - 3N = 0$:

$$\boxed{N = 3\,.} \tag{32}$$

At $N = 3$: $D = 21 = \dim \mathrm{Sp}(6) = \dim \mathrm{adj}\, \mathrm{SO}(7)$.

9.3 The SO(32) adjoint

The SO(32) gauge group of Type I / heterotic string theory admits the chain $\mathrm{SO}(32) \supset \mathrm{SU}(16) \times \mathrm{U}(1) \supset \mathrm{SU}(5) \times \mathrm{SU}(3) \times \mathrm{U}(1)^2$. The adjoint $\mathbf{496} = \wedge^2(\mathbf{32})$ decomposes to contain, in the neutral sector ($Q_A = Q_B = 0$): $\mathbf{24} \oplus \mathbf{1}$, and in the matter sector ($|Q_A| = 2$): $\mathbf{15} \oplus \overline{\mathbf{15}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{5} \oplus \bar{\mathbf{5}}$.

The sBootstrap representations $\mathbf{24}$ and $\mathbf{15} + \overline{\mathbf{15}}$ appear naturally. The remaining 442 states must be projected out by an orbifold or GSO-type condition—an open problem.

Why SO(32) and not $E_8 \times E_8$. Every decomposition of E_8 under SU(5) produces the antisymmetric $\mathbf{10}$, never the symmetric $\mathbf{15}$. The obstruction is irreducible: E_8's simply-laced root lattice admits only antisymmetric tensor representations of SU(5). By contrast, SO(32)'s adjoint naturally contains the symmetric $\mathbf{15}$ through $\wedge^2(\mathbf{5} \otimes \mathbf{3}) \supset S^2(\mathbf{5}) \otimes \wedge^2(\mathbf{3}) = (\mathbf{15}, \bar{\mathbf{3}})$. The sBootstrap **constrains the string embedding to the Type I / SO(32) corner** of the landscape.

10 M-theory lift

10.1 Mesons as M2-branes, baryons as M5-branes

The Hanany–Witten brane construction [?] realises SU(3) SQCD in Type IIA using D4-branes between NS5-branes with D6-branes providing flavours. Lifted to M-theory, the configuration merges into a single M5-brane wrapping a holomorphic curve Σ.

Field theory	Type IIA	M-theory	Mass type
Meson $M^i{}_j = Q^i \tilde{Q}_j$	open string (D6–D6)	**M2-brane**	Dirac
Baryon $B = \epsilon\, Q^3$	wrapped D4 (vertex)	**M5-brane**	Majorana

The moduli-space involution $M_i \to \Lambda^4 / M_i$ exchanges the C_3 and C_6 descriptions—the *electromagnetic duality* of M-theory.

10.2 Why $p = 2$: the membrane signature

With U(3) family symmetry, D6-brane positions are $v_i = c\,\mathfrak{z}_i$. An M2-brane stretched to the i-th D6-brane spans two spatial directions, so its BPS mass is $m_i \propto \mathrm{Area}_i \propto \mathfrak{z}_i^2$.

For a general p-brane:

p	Object	$m_i \propto$	K
1	string	$\mathfrak{z}_i$	0.478
2	**M2-brane**	$\mathfrak{z}_i^2$	**2/3**
3	3-brane	$\mathfrak{z}_i^3$	0.805
5	M5-brane	$\mathfrak{z}_i^5$	0.945

Only $p = 2$ **yields** $K = 2/3$: the energy-balance condition is a signature of the membrane.

10.3 The number 84

In 11d SUGRA, the C_3 field has $\binom{9}{3} = 84$ polarisations. Under $\mathrm{SU}(9) \subset E_8$: $248 = 80 + 84 + \overline{84}$. In the SM with right-handed neutrinos, the 96 fermionic degrees of freedom split as $84 + 12$ under dual mass-protection symmetries. A fourth uniqueness criterion: $\binom{3N}{N} = (N{+}1) \cdot N(2N{+}1)$ holds only at $N = 3$, giving $84 = 4 \times 21$.

10.4 Strong CP and the involution as CP

Identifying the moduli-space involution with CP gives a Nelson–Barr structure: the superpotential is real in the CP-symmetric basis, so $\arg\det m_q = 0$ at tree level. The CKM phase arises from spontaneous CP violation. Radiative corrections: $\delta\bar{\theta} \sim (\alpha/4\pi)^2 J \sim 10^{-11}$, at or below the experimental bound $|\bar{\theta}| < 10^{-10}$.

11 The explicit Lagrangian

Below the family confinement scale Λ_F, the effective theory is specified by:

Kähler potential.

$$K = \underbrace{\mathrm{Tr}(M^\dagger M) + |X|^2 + |B|^2 + |\bar{B}|^2}_{K_{\mathrm{can}}} \underbrace{- \frac{|X|^4}{12\mu^2}}_{K_{\mathrm{stab}}} + \underbrace{\frac{\zeta^2}{\Lambda^2}\Big|\sum_k s_k \mathfrak{z}_k\Big|^2}_{K_{\mathrm{bion}}}, \tag{33}$$

where K_{stab} pins the pseudo-modulus at $\langle X \rangle = \sqrt{3}\,\mu$ (the Kähler pole), and K_{bion} enforces the v_0-doubling condition $\mathfrak{z}_b = 3\mathfrak{z}_s + \mathfrak{z}_c$.

Superpotential.

$$W = \underbrace{\mathrm{Tr}(\hat{m}\, M)}_{\text{chiral mass}} + \underbrace{X(\det M - B\bar{B} - \Lambda^6)}_{\text{Seiberg}} + \underbrace{c_3 \frac{(\det M)^3}{\Lambda^{18}}}_{3-\text{inst.}} + \underbrace{y_c H_u M^c{}_c + y_b H_d M^b{}_b + y_t H_u Q^t \bar{Q}_t}_{\text{Yukawa}} + \underbrace{\lambda_S S H_u \cdot H_d}_{\text{NMSSM}} \tag{34}$$

The three-instanton term $(\det M)^3/\Lambda^{18}$ generates a $\cos(9\delta)$ potential whose minima include the observed lepton Koide phase to 0.9σ in m_τ. The NMSSM singlet S is a separate field from the Lagrange multiplier X (dimensional analysis: $\lambda_{\mathrm{Higgs}}/\lambda_{\max} \sim 10^5$). At $\tan\beta = 1$, the Higgs mass is $m_h = \lambda_S v/\sqrt{2} = 125\,\mathrm{GeV}$ for $\lambda_S = 0.72$.

Mixed triples require $\tan\beta = 1$**.** The quark Koide triples obligatorily mix up-type and down-type quarks. The translation from masses to Yukawa couplings depends on $\tan\beta$: K on masses equals K on Yukawas only when $\sin\beta = \cos\beta$, i.e., $\tan\beta = 1$.

Soft breaking. The soft terms arise from a spurion $\langle F_S \rangle = f_\pi^2$: $V_{\text{soft}} = f_\pi^2 \, \text{Tr}(M^\dagger M)$. The supertrace $\text{STr}[M^2] = 18\, f_\pi^2$ receives contributions only from the nine soft meson masses.

12 SUSY breaking: F-terms are lepton masses

The SUSY-preserving vacuum is diagonal and does not break $\text{SU}(2)_L$. When electroweak symmetry breaks (all three Higgs doublet components acquire VEVs), the F-term system becomes overconstrained: 18 real equations for 12 real variables. SUSY *necessarily* breaks, with

$$|F_{2j}| = m_j\,, \qquad j = e, \mu, \tau\,. \tag{35}$$

The SUSY-breaking auxiliary fields *are* the lepton masses. The goldstino is predominantly the τ-mesino, and $m_{3/2} = |F|/(\sqrt{3}\, M_{\text{Pl}}) \approx 4 \times 10^{-16}$ GeV.

The three Higgs doublets $H_i = M^{1,2}{}_i$ acquire hierarchical VEVs $v_i \propto m_{\ell,i}$: the SM Higgs is predominantly the τ-generation doublet ($v_\tau/\sqrt{\sum v_i^2} = 0.998$).

13 Numerical results and status

13.1 Parameter counting

The framework has four continuous inputs: k (overall scale, from m_e), α (sector angle, from m_μ), m_d and m_s ($\overline{\text{MS}}$ at 2 GeV, from PDG). From these, it produces:

Observable	Method	Predicted	PDG 2024	Status
m_τ	$K = 2/3$	1776.97 MeV	1776.86 ± 0.12	0.9σ
$K(e, \mu, \tau)$	trace identity	2/3	0.666661	0.001%
$K^{-1}(d, s, b)$	Seiberg seesaw	$\approx 2/3$	0.6652	0.22%
$K(-\pi, D_s, B)$	meson triple	$\approx 2/3$	0.6674	0.10%
m_c	$(2+\sqrt{3})^2 m_s$	1301 MeV	1273	2.4%
$m_u + m_d$	zero-charge	6.7 MeV	6.8	1.4%
$\|V_{us}\|$	$\sqrt{m_d/m_s}$	0.2236	0.2243	0.9σ
$\|V_{cb}\|$	$1/(5R)$	0.0417	0.0422	0.6σ
$\|V_{ub}\|$	$2\sqrt{2}/(7R^3)$	0.00367	0.00368	0.1σ
δ_{CP}	$\arctan\sqrt{R}$	65.4°	65.7°	0.1σ
$\sin^2\theta_W$	Casimir eq.	0.22310	0.22320	0.4σ
M_W	from above	80.374 GeV	80.369	0.39σ
m_b (bion)	v_0-doubling	4177 MeV	4183	0.06σ
m_t (chain)	(t, b, c)	~ 171 GeV	172.8	1%

Table 1: Quantitative results. $R = (5 + \sqrt{21})/2$.

13.2 Epistemological classification

14 TeV-scale convergence

Three TeV-range scales emerge with no TeV-scale input:

Claim	Status	Comment
$K = 2/3$ from U(3) charges	Derived	trace identity
$n = 2$ uniqueness	Derived	binomial theorem
Seiberg transparency $(W_S)_{kk} = 0$	Derived	det multilinearity
Involution $M \leftrightarrow \Lambda^4/M$	Derived	moduli space
Interleaving theorem	Derived	opposite orderings
$N = 3$ uniqueness (Diophantine)	Derived	$(r-3)(r+1) = 0$
$N = 3$ uniqueness (discriminant)	Conditional	equal quartic couplings
O'Raifeartaigh seed at $gv/m = \sqrt{3}$	Derived	fermion eigenvalues
Pauli theorem (no symmetric J=0 diquark)	Derived	angular momentum
$\tan\beta = 1$ from mixed triples	Derived	$\sin\beta = \cos\beta$
SO(32) but not $E_8 \times E_8$	Derived	**15** vs. **10**
Lepton Koide $K = 2/3$ to 0.001%	Observed	2.8σ after LEE
Meson triple $(-\pi, D_s, B)$ to 0.10%	Observed	1.1σ after LEE
v_0-doubling: m_b to 0.07%	Observed	bion Kähler
Casimir $R = 0.22310$ vs. $\sin^2\theta_W$	Observed	scheme-dependent
Dual Koide $K^{-1}(d, s, b) = 0.665$	Observed	0.22% from 2/3
$\lvert V_{cb}\rvert = 1/(5R)$, integers 5, 7	Empirical	no derivation yet
$\alpha_{\rm lep}$ origin	Open	Kähler modulus

Table 2: Logical status of claims.

1. The emergent adjoint mass $\mu_{\rm crit} \approx 4.5\,\text{TeV}$ (below which the Koide vacuum ceases to exist).

2. The composite stop mass $M_S(X_t = 0) \approx 4.6\,\text{TeV}$ required for $m_h = 125\,\text{GeV}$ via the Coleman–Weinberg formula.

3. The inverse Koide crossing scale $\mu_K \approx 1\,\text{TeV}$ where $K^{-1}(d, s, b)$ passes through 2/3 under 5-loop SMDR running.

The full scale hierarchy is $\mu_{\mathcal{N}=2} \gtrsim 37\,\text{TeV} > \Lambda_{\rm CST} > v_{\rm EW} = 246\,\text{GeV} > \Lambda_F \approx 80\,\text{MeV}$.

15 Open problems

1. **The Kähler metric.** Computing the Kähler potential on the quantum-modified moduli space is the master obstruction: $v_{\rm EW}$, $\tan\beta$, the overall scale k, Z_c/Z_s, and the scalar spectrum all depend on it.

2. **The origin of $\alpha_{\rm lep}$.** The sector angle $\alpha_{\rm lep} = 0.2222\,\text{rad}$ determines the lepton mass ratios but is fitted, not derived. All SU(3)-invariant potentials have extrema at $\alpha = 0$ and $\pi/3$; fixing $\alpha_{\rm lep}$ requires non-perturbative Kähler effects or UV input. The three-instanton potential $-\cos(9\delta)$ provides one candidate mechanism.

3. **The CKM integers.** The integers 5 and 7 in $|V_{cb}| = 1/(5R)$ and $|V_{ub}| = 2\sqrt{2}/(7R^3)$ are pattern identifications. Whether they encode Clebsch–Gordan coefficients, dimensions, or M-theoretic data ($84 = \binom{9}{3}$, $\pi/84$) is unknown.

4. **PMNS mixing.** The baryonino Majorana matrix is diagonal on the mesonic branch. Large lepton mixing angles require non-diagonal Kähler corrections to the baryonic sector.

5. **The quartic ε.** Computing the inter-branch coupling requires the non-perturbative Kähler metric, entangled with Z_c/Z_s and the waterfall pattern.

6. **The** SO(32) **projection.** Of the 496 adjoint states, 442 must be projected out. No mechanism has been identified.

7. **The Casimir equation.** The dynamical origin of the eigenvalue equation $x^2 + C_2x - C_2 = 0$ and the preferred renormalisation scheme remain open.

16 Conclusions

We have presented a unified framework that begins with the identification of Standard Model fermions as composite fermions of a confining $\mathrm{SU}(3)_F$ family gauge theory—leptons as mesinos, quarks as quarkinos—and follows the consequences through five levels of structure:

1. The composite identification provides a single meson matrix M from which both electroweak Higgs doublets emerge ($H_d = M^{1,2}$, $H_u = \mathrm{adj}(M)^{1,2}$), with no independent Yukawa matrices. The 9 mesinos furnish the SM lepton sector; the diquark partners fill the symmetric **15** of $\mathrm{SU}(5)_f$.

2. The emergent $\mathcal{N}$=2 doublet $(M_i, \mathrm{adj}(M)_i)$ per family, broken to $\mathcal{N}$=1 by a quartic coupling, generates electroweak isospin: the direct and inverse branches produce up-type and down-type mass hierarchies from the same charge structure.

3. The charge-squared mass law $m = \mathfrak{z}^2$, uniquely selected by the requirement that $K = 2/3$ be direction-independent in the Cartan subalgebra, produces an energy-balance condition satisfied by the charged leptons (0.001%), the meson triple $(-\pi, D_s, B)$ (0.10%), the inverse down quarks (0.22%), and the quark chain to $\sim 1\%$. The O'Raifeartaigh model produces the seed spectrum at $gv/m = \sqrt{3}$.

4. The electroweak gauge bosons are accommodated through a Casimir eigenvalue equation predicting $\sin^2\theta_W = 0.22310$ (0.4σ).

5. The string/M-theory embedding: the Diophantine counting selects $N = 3$ generations; the composites fill $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$ of $\mathrm{SU}(5)_f$, contained in the SO(32) adjoint but not $E_8 \times E_8$. In M-theory, mesons are M2-branes and baryons are M5-branes, with the charge-squared mass law reflecting the membrane dimensionality $p = 2$.

The framework has four continuous inputs for 13 observables, with the CKM elements depending only on the self-dual ratio $R = (5 + \sqrt{21})/2$. The most pressing open problems are the Kähler metric, the origin of the sector angle α, the CKM integers, and the PMNS mixing pattern.

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