Dirac–Majorana duality from the M-theory lift of the sBootstrap

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The sBootstrap identifies Standard Model leptons with mesino composites of a confining $\mathrm{SU}(3)_F$ family gauge theory at $N_f = N_c = 3$. In the Hanany–Witten brane construction, mesons lift to M2-branes coupling to the 3-form C_3, while baryons lift to M5-branes coupling to the 6-form C_6. We show that this M2/M5 correspondence maps onto the distinction between Dirac and Majorana mass types: the bilinear meson $M = Q\tilde{Q}$ produces Hermitian (Dirac) mass matrices, while the trilinear baryon $B = \varepsilon Q^3$ produces symmetric (Majorana) mass matrices, with the involution $M_i \leftrightarrow \mathrm{adj}(M)_i$ lifting to $C_3 \leftrightarrow C_6$ electromagnetic duality. The meson Kähler metric is approximately diagonal (protecting small CKM angles), whereas the baryon Kähler metric is generically democratic (allowing large PMNS angles). The holomorphic superpotential $W = BM\tilde{B}/\Lambda^5$ generates a type-I seesaw structure with $M_R^{ii} = m_i$ (exact from s-confinement); however, the physical neutrino mass scale requires non-perturbatively large Kähler suppression factors of order 10^5, which cannot be computed from holomorphy alone. Using multi-loop SM running of Yukawa couplings, the inverse Koide ratio $K^{-1}(d, s, b)$ crosses $2/3$ at $Q \approx 10\,\mathrm{TeV}$ in the Standard Model with no new thresholds, while in the MSSM it remains 0.23–0.65% above $2/3$ at M_{GUT} for all $\tan\beta$—disfavouring conventional low-energy supersymmetry.

I. INTRODUCTION

The sBootstrap [? ?] is a structural effective field theory that identifies Standard Model fermions with composites of a confining $\mathrm{SU}(3)_F$ family gauge theory at $N_f = N_c = 3$. The nine mesinos of the meson matrix $M^i{}_j = Q^i\tilde{Q}_j$ decompose under the electroweak branching $\mathbf{3}_L \to \mathbf{2}_{-1/2} \oplus \mathbf{1}_{+1}$ into three lepton doublets L_i and three charged-lepton singlets e_i^c. Quarks, which carry colour and therefore cannot be family-singlet composites, enter through partial compositeness [?]. The adjugate $\mathrm{adj}(M) = \det(M)\, M^{-1}$ provides the second MSSM Higgs doublet, and the charge-squared mass law $m_i = \mu\,\mathfrak{z}_i^2$ yields the Koide relation $K = 2/3$ as a trace identity of the canonical $\mathrm{U}(3)_F$ normalisation [?].

The Seiberg seesaw [?] produces a direct branch ($\langle M_i\rangle \propto 1/m_i$) and an inverse branch ($\mathrm{adj}(M)_i \propto m_i$). In the M-theory lift of the Hanany–Witten construction [?], mesons are M2-branes stretched between separated parts of the M5 worldvolume curve, coupling to the 3-form potential C_3; baryons are M5-branes wrapping compact 4-cycles, coupling to the 6-form C_6 [? ?]. The Hodge duality $*G_4 = G_7$ in 11-dimensional supergravity [?] is the field-strength avatar of M2/M5 electromagnetic duality.

In this paper we develop the observation that this M2/M5 correspondence maps onto the distinction between Dirac and Majorana mass types. Specifically: (i) the bilinear meson $M = Q\tilde{Q}$ produces Hermitian (Dirac) mass matrices, while the trilinear baryon $B = \varepsilon Q^3$ produces symmetric (Majorana) mass matrices; (ii) the meson Kähler metric is approximately diagonal (small CKM angles), while the baryon Kähler metric is generically democratic (large PMNS angles); (iii) the holomorphic superpotential $W = BM\tilde{B}/\Lambda^5$ generates a type-I seesaw structure.

We also present updated renormalisation group evidence for the inverse Koide relation in the down-quark sector, using the multi-loop running Yukawa couplings of Antusch, Hinze, and Saad (AHS) [?], and show that the SM desert uniquely produces the crossing $K^{-1}(d, s, b) = 2/3$ at $Q \approx 10\,\mathrm{TeV}$, while the MSSM fails to reach this value.

The paper is organised as follows. Section ?? establishes the brane dictionary at each stage of the breaking chain. Section ?? develops the $C_3 \leftrightarrow C_6$ Dirac–Majorana duality. Section ?? discusses the Kähler protection asymmetry. Section ?? derives the geometric seesaw and honestly assesses its limitations. Section ?? presents the RG evidence for the SM desert. Section ?? discusses implications, including the debunking of a previously noted coincidence with the Pendleton–Ross fixed point. Section ?? summarises and identifies experimental tests.

Throughout, we work in natural units ($\hbar = c = 1$) with metric signature $(-,+,+,+)$ and generators normalised as $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$. The 11d supergravity conventions follow Cremmer–Julia–Scherk [?]: C_3 is the 3-form potential, $G_4 = \mathrm{d}C_3$, $G_7 = *G_4 = \mathrm{d}C_6$.

II. THE BRANE DICTIONARY

We first establish the M-theory lift of each sBootstrap composite. The type IIA Hanany–Witten construction for SU(3) SQCD with $N_f = 3$ places $N_c = 3$ D4-branes suspended between two NS5-branes, with three D6-branes providing fundamental flavours [?]. Lifting to M-theory by opening the x^{10} circle [?]: the D4-branes become an M5-brane wrapping a holomorphic curve Σ in

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TABLE I. M-theory dictionary for SU(3) SQCD with $N_f = N_c = 3$. The column "C-field" indicates the dominant coupling. The mass-type column identifies the fermion bilinear structure in the low-energy effective theory.

Composite	IIA	M-theory	C-field	Mass type	p
$M = Q\tilde{Q}$	D6–D6	M2	C_3	Dirac	2
$B = \varepsilon Q^3$	wrapped D4	wrapped M5	C_6	Majorana	5
adj(M)	thr. D4	thr. M5	mixed	inv. branch	—
$M \leftrightarrow$ adj(M)	T-dual	EM dual	$C_3{\leftrightarrow}C_6$	—	—
Φ (aux. adj.)	D4–D4	M5 modulus	metric	—	—

the (x^4, x^5, x^6, x^{10}) space; the D6-branes become Taub-NUT centres; the NS5-branes remain as M5.

The composites have direct brane interpretations, collected in Table **??**.

a. M2-branes and mesons. The meson $M^i{}_j = Q^i\tilde{Q}_j$ lifts to an M2-brane ending on the M5 worldvolume. The M2 worldvolume action contains the Wess–Zumino term $\int C_3$, so the meson couples electrically to C_3. The M2-brane area encodes the meson mass: for a p-brane whose worldvolume dimensions scale with the family charge $\mathfrak{z}_i$, the mass scales as $m_i \propto \mathfrak{z}_i^p$ [**?**]. The generalised Koide ratio $K_p = \sum \mathfrak{z}_i^{2p}/(\sum \mathfrak{z}_i^p)^2$ is independent of the Cartan direction only for $p = 2$—the M2-brane. This consistency check does not constitute a derivation of the charge-squared law, since assuming both worldvolume directions scale with $\mathfrak{z}_i$ is equivalent to the bilinear structure $M = Q\tilde{Q}$.

b. M5-branes and baryons. The baryon $B = \varepsilon_{ijk}Q^iQ^jQ^k$ lifts to an M5-brane wrapping S^4 in the near-horizon geometry of the D6-branes, with three string endpoints [**? ?**]. The M5 couples electrically to C_6. For $p = 5$, the ratio K_5 depends on the Cartan angle; baryon masses are determined algebraically ($M_R^{ii} = m_i$, exact from s-confinement [**?**]) rather than by a power law in $\mathfrak{z}_i$. The baryon mass is a topological quantity (the volume of S^4 in the Taub-NUT geometry), while the meson mass is a modular quantity (the area of a 2-cycle on Σ).

c. The adjugate. The adjugate $\mathrm{adj}(M)_i = \varepsilon_{ijk}M_jM_k/\Lambda^3$ involves the ε tensor but is a product of two mesons with an ε contraction. In M-theory, this corresponds to three M2-branes joined at a baryon-like vertex—a mixed C_3/C_6 configuration, not a pure C_6 coupling. Only $B = \varepsilon Q^3$ is a pure C_6 object. The involution $M_i \leftrightarrow \mathrm{adj}(M)_i$ exchanges pure C_3 with mixed C_3/C_6, not pure C_3 with pure C_6.

d. The breaking chain. The full correspondence across the breaking chain M-theory $\to \mathcal{N}$=2 $\to \mathcal{N}$=1 $\to$ SM is:

- The M5-brane wrapping Σ gives $\mathcal{N}$=2 in 4d; the genus-2 curve encodes the Seiberg–Witten prepotential [**?**].
- The adjoint mass μ_Φ deforms Σ, breaking $\mathcal{N}$=2 $\to$ $\mathcal{N}$=1. The hypermultiplet $(M_i, \mathrm{adj}(M)_i)$ splits into two independent $\mathcal{N}$=1 chirals—the two periods of Σ.
- At confinement, the 9 mesinos identify with the SM lepton content: $3L_i + 3e_i^c$. Baryoninos provide sterile (Majorana) neutrinos.

e. Colour-3 impossibility. The 11d 3-form decomposes as $C_3 \in \Lambda^3(\mathbb{R}^9) = \mathbf{84}$, which under SU(3)$_c$ contains $\mathbf{1}_c$, $\mathbf{3}_c$, $\overline{\mathbf{3}}_c$ but never $\mathbf{6}_c$. The colour-sextet representation appears only in the symmetric (gravitational) sector $g_{MN} \in \mathrm{Sym}^2(\mathbb{R}^9) = \mathbf{45}$. This is consistent with the field-theory result that SU(3)$_F$ composites with $N_f = N_c = 3$ never carry the colour representation required for quarks to be fully composite [**?**].

III. DIRAC–MAJORANA DUALITY

We now develop the central observation: the C_3/C_6 distinction maps onto the Dirac/Majorana mass distinction.

A. Electric and magnetic mass types

In 11d supergravity, the 4-form field strength $G_4 = \mathrm{d}C_3$ and its Hodge dual $G_7 = *G_4 = \mathrm{d}C_6$ satisfy [**? ?**]:

$$\mathrm{d} * G_4 = *j_3 \quad \text{(M2 source, electric)}\,, \tag{1}$$

$$\mathrm{d}G_4 = j_6 \quad \text{(M5 source, magnetic)}\,. \tag{2}$$

The M2-brane is the electric source (appearing on the right-hand side of $\mathrm{d} * F = j$); the M5-brane is the magnetic source (modifying the Bianchi identity).

In the sBootstrap, the algebraic structure of each composite determines the symmetry of its mass matrix:

- **Meson** $M^i{}_j = Q^i\tilde{Q}_j$ (bilinear): the mass matrix is Hermitian, coupling left-handed to right-handed fermions—Dirac type.
- **Baryon** $B = \varepsilon_{ijk}Q^iQ^jQ^k$ (trilinear): the mass matrix is symmetric, coupling a fermion to itself and violating lepton number by two units—Majorana type.

This is an algebraic fact of s-confinement [**? ?**], independent of M-theory. The M-theory lift upgrades this field-theory observation to a geometric statement: the Dirac/Majorana distinction is the electric/magnetic distinction of the C-field (Table **??**).

B. The involution and A/B period duality

The Seiberg–Witten curve Σ for SU(3) with $N_f = 3$ is a genus-2 hyperelliptic surface [**? ?**]:

$$y^2 = \left[x^3 - u_2\,x - u_3\right]^2 - 4\Lambda^3 \prod_{i=1}^{3}(x + m_i)\,, \tag{3}$$

TABLE II. Structural correspondence between C-field sectors and fermion mass types. The identification is algebraic at the field-theory level and geometric in M-theory.

Property	Electric (C_3, M2)	Magnetic (C_6, M5)
Brane	M2	M5
Composite	Meson $Q\tilde{Q}$	Baryon εQ^3
Mass type	Dirac (bilinear)	Majorana (trilinear)
Matrix symmetry	Hermitian	Symmetric
Protected by	Chiral symmetry	Lepton number
SM sector	Charged fermions	Neutrinos

where $u_2 = \mathrm{Tr}(\Phi^2)$ and $u_3 = \det(\Phi)$ are Coulomb-branch moduli. In the confining vacuum, these are fixed by the fermion mass matrix $D = \mathrm{diag}(m_1, m_2, m_3)$ via the Cayley–Hamilton equation of the $\mathcal{N}$=2 adjoint Φ:

$$u_2 = \mathrm{Tr}(D^2) = \sum_i m_i^2\,, \quad u_3 = 2\det(D) = 2\prod_i m_i\,. \tag{4}$$

The four periods of Σ—two A-periods $a_i = \oint_{A_i} \lambda_{\mathrm{SW}}$ and two B-periods $a_{D,i} = \oint_{B_i} \lambda_{\mathrm{SW}}$—encode both mass hierarchies:

$$A\text{-periods:} \quad \langle M_i \rangle \propto 1/m_i \quad \text{(direct branch)}\,, \tag{5}$$
$$B\text{-periods:} \quad \mathrm{adj}(M)_i \propto m_i \quad \text{(inverse branch)}\,. \tag{6}$$

The involution $M_i \leftrightarrow \mathrm{adj}(M)_i$ exchanges A-periods with B-periods. In M-theory, this is precisely the electromagnetic duality $C_3 \leftrightarrow C_6$, exchanging M2 and M5 brane data.

C. Scope and caveats

Two important qualifications:

(i) The $C_3 \leftrightarrow C_6$ duality is a duality *within* the mesonic sector (direct $\leftrightarrow$ inverse branch), not a duality *between* the mesonic and baryonic sectors. The baryon $B = \varepsilon Q^3$ wraps S^4, not Σ; it is a topologically distinct object from $\mathrm{adj}(M)$. The connection between mesons and baryons is mediated by the quantum-modified constraint

$$\det M - B\tilde{B} = \Lambda^6\,, \tag{7}$$

not by a direct $C_3 \leftrightarrow C_6$ map.

(ii) The correspondence in Table **??** is structural, not quantitative. No formula relating the Dirac mass of a given fermion to its Majorana partner through the duality currently exists. The most promising path to such a relation is through the Riemann bilinear relations of the genus-2 period matrix, which constrain the product of A- and B-periods; this computation has not been carried out for the specific SU(3), $N_f = 3$ case at the Koide moduli point.

IV. KÄHLER PROTECTION ASYMMETRY

In the s-confined theory, the holomorphic masses of both mesinos and baryoninos are protected by the $\mathcal{N}$=2 non-renormalisation theorem (the Higgs-branch hyper-Kähler metric is one-loop exact in the $\mathcal{N}$=2 limit [**?**]). However, the physical masses $m_{\mathrm{phys}} = m_{\mathrm{hol}}/\sqrt{Z}$ depend on the Kähler metric Z, which is not protected by holomorphy after $\mathcal{N}$=2 $\to \mathcal{N}$=1 breaking. The key structural difference between the two sectors is in the flavour structure of Z.

a. Meson Kähler metric: approximately diagonal. The meson $M^i{}_j$ carries definite flavour quantum numbers (i, j). In the $\mathcal{N}$=2 limit, the Kähler metric on the Higgs branch is exactly diagonal in the flavour basis [**?**]. Breaking to $\mathcal{N}$=1 introduces corrections controlled by

$$\epsilon = \frac{m_e}{m_\tau} \approx 2.9 \times 10^{-4}\,, \tag{8}$$

giving $\delta Z_M/Z_M \sim \epsilon^2 \sim 10^{-7}$ to 10^{-9} [**?**]. This is consistent with the observed precision of $K(e, \mu, \tau) = 2/3$ to 0.001%. The near-diagonality of Z_M means that the meson-sector mass matrix remains approximately diagonal after Kähler rotation: *the CKM matrix is small because meson flavour numbers protect the Kähler metric.*

b. Baryon Kähler metric: generically democratic. The baryon $B = \varepsilon_{ijk} Q^i Q^j Q^k$ involves all three flavours symmetrically through the ε tensor. No flavour quantum number distinguishes B from any permutation of its constituent quarks. Consequently, the baryon Kähler metric Z_B^{ij} has no reason to be diagonal: it is generically of democratic form, with $\mathcal{O}(1)$ off-diagonal entries. The Kähler rotation needed to bring the physical mass matrix to canonical form then introduces large mixing angles. *The PMNS matrix is large because the ε tensor democratises the baryon Kähler metric.*

This is a structural explanation, not a numerical prediction. Computing Z_B requires the non-perturbative Kähler potential at strong coupling, which is beyond current techniques. Nevertheless, the qualitative asymmetry—diagonal mesonic vs. democratic baryonic—is a robust consequence of the composite structure and does not depend on the specific form of the Kähler potential. This asymmetry has been noted previously [**? ?**] in related compositeness frameworks; the present work identifies the M-theory origin as the C_3 vs. C_6 distinction.

V. THE GEOMETRIC SEESAW

A. Holomorphic superpotential

The full holomorphic superpotential on the mesonic branch of $\mathrm{SU}(3)_F$ with $N_f = N_c = 3$ is [**? ?**]:

$$W = X\big(\det M - B\tilde{B} - \Lambda^6\big) + \mathrm{Tr}(m\,M) + \frac{1}{\Lambda^5}\,B\,M\,\tilde{B}\,, \tag{9}$$

where X is a Lagrange multiplier enforcing the quantum-modified constraint, $m = \mathrm{diag}(m_1, m_2, m_3)$ are the UV mass deformations, and the last term is the baryon–meson coupling (dimension 8, suppressed by Λ^5).

B. Baryonino Majorana masses

On the mesonic branch ($B = \tilde{B} = 0$), the F-term equations give the standard result [**?** **?**]:

$$\langle M_i \rangle = \frac{\Lambda^2 P^{1/3}}{m_i}\,, \qquad P = m_1 m_2 m_3\,, \tag{10}$$

where $P^{1/3} = (m_e m_\mu m_\tau)^{1/3} = 45.78\,\mathrm{MeV}$ for the lepton sector. The baryonino masses follow from the second derivative of W with respect to the baryonic fields. The exact s-confinement result is [**?**]:

$$M_R^{ii} = m_i\,. \tag{11}$$

That is, the Majorana mass of the i-th baryonino equals the UV mass deformation. For the lepton sector: $M_R = \mathrm{diag}(m_e, m_\mu, m_\tau) = \mathrm{diag}(0.511, 105.66, 1776.86)\,\mathrm{MeV}$.

C. Dirac mass from the baryon–meson coupling

The third term in Eq. (**??**) generates a Dirac mass for the baryonino when the meson M acquires a VEV. After canonical normalisation of the confined fields (Kähler metric Z_M for mesons, Z_B and $Z_{\tilde{B}}$ for baryons), the physical Dirac mass is:

$$m_D^i = \frac{\sqrt{Z_M}}{\sqrt{Z_B Z_{\tilde{B}}}}\,\frac{\Lambda\, P^{1/3}}{m_i}\,. \tag{12}$$

Setting all Kähler factors to unity gives the holomorphic estimate $m_D^i = \Lambda P^{1/3}/m_i$.

D. Type-I seesaw structure

Integrating out the heavy baryonino at mass $M_R^i = m_i$ [Eq. (**??**)], the light neutrino mass matrix is:

$$m_\nu^{ii} = \frac{(m_D^i)^2}{M_R^i} = \frac{Z_M}{Z_B Z_{\tilde{B}}}\,\frac{\Lambda^2 P^{2/3}}{m_i^3}\,. \tag{13}$$

This has the structure of a type-I seesaw, $m_\nu = m_D^T M_R^{-1} m_D$, with the specific identifications: m_D^i from the meson VEV (electric/C_3 sector) and $M_R^i = m_i$ from the baryonino F-term (magnetic/C_6 sector).

On the M5 worldvolume, this coupling corresponds to the interaction $\int_{M5} C_3 \wedge H_3$, where H_3 is the self-dual 3-form on the M5 worldvolume. The seesaw suppression m_D^2/M_R arises because the M2 contributions (area-suppressed) combine with the M5 contribution (volume-enhanced). This identification is schematic; a rigorous derivation from the M5 worldvolume effective action for SU(3), $N_f = 3$ has not been performed.

TABLE III. Holomorphic seesaw masses at unit Kähler factors ($Z_M = Z_B = Z_{\tilde{B}} = 1$, $\Lambda = 100\,\mathrm{GeV}$), compared with observed neutrino mass estimates. The naive seesaw fails on both scale and hierarchy.

Generation	m_ν^{hol}	m_ν^{obs} (est.)	Ratio
e	$\sim 10^{11}$ GeV	~ 0.35 meV	4.5×10^{17}
μ	$\sim 10^{4}$ GeV	~ 8.6 meV	2.1×10^{12}
τ	$\sim 3.7\,\mathrm{GeV}$	~ 50 meV	7.5×10^{10}

E. The scale problem: an honest assessment

The holomorphic seesaw (**??**) at unit Kähler factors gives $m_\nu^{ii} \propto 1/m_i^3$, predicting an extreme inverted hierarchy ($m_{\nu_e} \gg m_{\nu_\tau}$) and masses in the GeV range—wrong by 10–17 orders of magnitude. We collect the numerical values in Table **??**.

To obtain $m_{\nu_3} \sim 50$ meV from the tau sector requires $\sqrt{Z_B Z_{\tilde{B}}} \sim 2.7 \times 10^5$—an enormous Kähler suppression that cannot be computed from holomorphy. Furthermore, the tree-level PMNS matrix on the mesonic branch is the identity: all neutrino masses are diagonal and there is no lepton mixing. Non-trivial PMNS angles require either off-diagonal Kähler rotations in the baryonic sector, radiative corrections, or SUSY-breaking baryon condensates [**?**].

a. Summary. The geometric seesaw provides the correct *structure*: Dirac masses from the meson VEV (C_3 sector), Majorana masses from baryonino F-terms (C_6 sector), and a type-I seesaw formula. It correctly identifies baryoninos as sterile neutrinos and the meson VEV as the Dirac mass source. It does *not* predict the neutrino mass scale or PMNS mixing without additional non-holomorphic input. The framework gives the right *players* but not the right *numbers*.

VI. SM DESERT VS. MSSM: KOIDE RG EVIDENCE

The charge-squared law $m_i = \mu\,\mathfrak{z}_i^2$ and the canonical $\mathrm{U}(3)_F$ normalisation give the Koide ratio $K = 2/3$ as a trace identity [**?**]. For the down-quark sector, which arises from the adjugate (inverse) branch, the relevant observable is the inverse Koide ratio [**?**]:

$$K^{-1}(d,s,b) \equiv \frac{1/m_d + 1/m_s + 1/m_b}{\left(1/\sqrt{m_d} + 1/\sqrt{m_s} + 1/\sqrt{m_b}\right)^2}\,. \tag{14}$$

At the compositeness scale (which we identify with the adjugate vacuum), $K^{-1} = 2/3$ exactly. SM renormalisation group running shifts K^{-1} away from $2/3$ at low energies.

Using the multi-loop Yukawa couplings of Antusch, Hinze, and Saad [**?**], we evaluate K^{-1} as a function of the RG scale Q in two scenarios (Table **??**).

TABLE IV. Inverse Koide ratio $K^{-1}(d, s, b)$ under RG running. "Best scale" is the scale at which K^{-1} is closest to 2/3. $\Delta = K^{-1} - 2/3$ in percent.

Scenario	Best scale	K^{-1}	Δ (%)
SM desert	$\sim$ **10** TeV	**0.66667**	**+0.0003**
MSSM, $M_S = 3$ TeV, $\tan\beta = 5$	$M_{\rm GUT}$	0.66689	+0.033
MSSM, $M_S = 3$ TeV, $\tan\beta = 10$	$M_{\rm GUT}$	0.66693	+0.040
MSSM, $M_S = 3$ TeV, $\tan\beta = 30$	$M_{\rm GUT}$	0.66749	+0.124
MSSM, $M_S = 3$ TeV, $\tan\beta = 50$	$M_{\rm GUT}$	0.66895	+0.342
MSSM, $M_S = 10$ TeV, $\tan\beta = 5$	$M_{\rm GUT}$	0.66668	+0.003
MSSM, $M_S = 10$ TeV, $\tan\beta = 10$	$M_{\rm GUT}$	0.66673	+0.009
MSSM, $M_S = 10$ TeV, $\tan\beta = 30$	$M_{\rm GUT}$	0.66724	+0.086
MSSM, $M_S = 10$ TeV, $\tan\beta = 50$	$M_{\rm GUT}$	0.66854	+0.281

a. SM desert. In the SM with no new thresholds between M_Z and the Planck scale, $K^{-1}(d, s, b)$ decreases monotonically with Q and crosses 2/3 at $Q \approx 10$ TeV to a precision of 0.0003% using AHS multi-loop data [? ?]. At $Q = M_Z$, the deviation is +0.11%: still within 0.2% of the trace identity, but measurably different from exact $K^{-1} = 2/3$.

b. MSSM. In the MSSM, K^{-1} at $M_{\rm GUT}$ lies 0.23–0.65% above 2/3 for all values of $\tan\beta$ and $M_{\rm SUSY}$ at two-loop accuracy [?]. The one-loop MSSM results in Table ?? are somewhat closer to 2/3 (missing higher-order effects), but the qualitative conclusion is robust: *the MSSM never crosses* 2/3. In no MSSM scenario does K^{-1} reach 2/3 between $M_{\rm SUSY}$ and $M_{\rm GUT}$.

c. Interpretation. If the trace identity $K^{-1}(d, s, b) = 2/3$ holds exactly at the compositeness scale, the RG evidence favours $\mathrm{SU}(3)_F$ compositeness emerging at $Q \sim$ 10–280 TeV in a standard model desert, rather than through a conventional SUSY threshold. This does *not* exclude SUSY as such—the sBootstrap itself relies on $\mathcal{N}$=1 holomorphy—but disfavours the MSSM spectrum as the next threshold above the electroweak scale.

VII. DISCUSSION

A. What the duality explains

The C_3/C_6 Dirac–Majorana correspondence provides three structural insights:

(i) *Origin of mass-type asymmetry.* The bilinear/trilinear distinction of s-confined composites, which is an algebraic fact, lifts to the electric/magnetic distinction of M-theory p-form gauge fields. This is not a coincidence but a consequence of the brane construction.

(ii) *Mixing angle hierarchy.* The Kähler protection asymmetry (Sec. ??) provides a qualitative explanation for the observed pattern: small CKM angles (meson sector, diagonal Z_M) vs. large PMNS angles (baryon sector, democratic Z_B).

(iii) *Seesaw architecture.* The superpotential $W = BM\tilde{B}/\Lambda^5$ provides the type-I seesaw skeleton with baryoninos as right-handed neutrinos at the charged-lepton mass scale, without introducing a GUT-scale seesaw.

B. What the duality does not explain

(i) *The neutrino mass scale* requires Kähler factors of order 10^5, which is a statement about the strongly-coupled Kähler potential and cannot be computed from holomorphy.

(ii) *The PMNS mixing angles* are trivial at tree level on the mesonic branch and require off-diagonal Kähler rotations or radiative corrections.

(iii) *A quantitative Dirac–Majorana mass relation* does not exist within the current framework. The duality is structural, not predictive for individual masses.

C. Pendleton–Ross fixed point: a debunked coincidence

The Pendleton–Ross [?] / Hill [?] infrared quasi-fixed point for the top Yukawa coupling gives, at one loop in the QCD-only approximation:

$$R^* = \frac{y_t^2}{g_3^2} = \frac{2}{9}\,. \tag{15}$$

This derives from the one-loop β-function $16\pi^2\,\mathrm{d}R/\mathrm{d}t = g_3^2\,R\,(9R-2)$, giving $R^* = 2/9$ at the zero. The coincidence of $R^* = 2/9$ with the Cartan angle $\delta = 2/9$ of the sBootstrap was noted as potentially significant.

We have verified computationally that this coincidence is *spurious*, for three independent reasons:

(i) Electroweak corrections shift the quasi-fixed point from $2/9 \approx 0.222$ to $R^* \approx 0.39$ at M_Z—a 76% increase. The shift is:

$$R^*(Q) = \frac{2}{9} + \frac{1}{9}\left[\frac{17}{10}\frac{g_1^2}{g_3^2} + \frac{9}{2}\frac{g_2^2}{g_3^2}\right]. \tag{16}$$

(ii) With running gauge coupling ratios, R^* is scale-dependent: there is no true fixed point in the full SM. A numerical scan with initial $R \in [0.1, 5.0]$ at $Q = 10^{16}$ GeV shows no convergence to a single value at M_Z [?].

(iii) The physical SM ratio is $R = y_t^2/g_3^2 = 0.59$ at M_Z, which is 2.66 times the QCD-only value. The QCD-only fixed point predicts $m_t \approx 100$ GeV, off by 73%.

The Pendleton–Ross ratio 2/9 *is exact only in the one-loop QCD-only limit, which is unphysical. The agreement with* $\delta = 2/9$ *is a numerical accident.*

D. Experimental tests

The strongest near-term test of the sBootstrap neutrino sector is the mass ordering. The geometric seesaw with normal-ordering Dirac mass structure predicts

$m_1 < m_2 < m_3$. Current data (DESI + CMB) favour normal ordering with $\sum m_\nu < 0.057$ eV [**?**]; the sBootstrap gives $\sum m_\nu \approx 0.060$ eV [**?**], marginally consistent. JUNO [**?**] will definitively determine the mass ordering by 2027–2028. *If JUNO establishes inverted ordering, the sBootstrap seesaw structure is disfavoured.*

The inverse Koide crossing at $Q \approx 10\,\mathrm{TeV}$ predicts no new coloured particles below this scale. This is testable at the HL-LHC and future colliders: observation of MSSM-like superpartners at $M_S < 10\,\mathrm{TeV}$ would conflict with the SM-desert scenario favoured by the RG analysis.

The qualitative prediction of small CKM vs. large PMNS mixing from the Kähler asymmetry could be sharpened by a computation of the baryon Kähler metric using holographic QCD techniques [**?**].

VIII. CONCLUSIONS

We have shown that the M-theory lift of the sBootstrap—a confining $\mathrm{SU}(3)_F$ family gauge theory at $N_f = N_c = 3$—contains a structural correspondence between the Dirac and Majorana mass types and the electromagnetic duality of p-form gauge fields in 11 dimensions. Mesons (M2-branes, C_3) produce Dirac masses; baryons (M5-branes, C_6) produce Majorana masses. The involution $M_i \leftrightarrow \mathrm{adj}(M)_i$ lifts to the exchange of A- and B-periods of the genus-2 Seiberg–Witten curve.

The composite structure explains why CKM angles are small (meson Kähler metric is diagonal) and PMNS angles are large (baryon Kähler metric is democratic). The superpotential $W = BM\tilde{B}/\Lambda^5$ provides a type-I seesaw skeleton with baryonino Majorana masses $M_R^{ii} = m_i$ (exact from s-confinement), but the physical neutrino mass scale requires non-perturbatively large Kähler suppression that cannot be computed from holomorphy alone.

Multi-loop SM running of down-quark Yukawa couplings shows that $K^{-1}(d, s, b) = 2/3$ at $Q \approx 10\,\mathrm{TeV}$ in the SM desert, while no MSSM scenario reaches this value—discriminating between $\mathrm{SU}(3)_F$ compositeness at the 10–280 TeV scale and conventional low-energy SUSY.

The previously noted coincidence between the Pendleton–Ross QCD-only fixed point $y_t^2/g_3^2 = 2/9$ and the Cartan angle $\delta = 2/9$ has been shown to be spurious: electroweak corrections shift the quasi-fixed point to $R^* \approx 0.39$, and the physical SM ratio is $R = 0.59$.

The framework makes three testable predictions: (i) neutrino normal ordering (testable by JUNO, 2027–2028); (ii) no new coloured particles below $\sim 10\,\mathrm{TeV}$ (testable at HL-LHC and future colliders); (iii) a qualitative asymmetry between CKM and PMNS mixing rooted in the composite structure of the family sector.

The principal open problems are the computation of the non-perturbative Kähler potential for the baryonic sector (which would predict PMNS angles and the neutrino mass scale), and the determination of the Cartan angle $\delta = 2/9$ from the instanton structure of the confining theory.

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[1] A. Rivero, arXiv:hep-ph/0510068 (2005).
[2] A. Rivero, Eur. Phys. J. C **84**, 1058 (2024), arXiv:2407.05397.
[3] D.B. Kaplan, Nucl. Phys. B **365**, 259 (1991).
[4] N. Seiberg, Nucl. Phys. B **435**, 129 (1995), arXiv:hep-ph/9411149.
[5] A. Hanany and E. Witten, Nucl. Phys. B **492**, 152 (1997), arXiv:hep-th/9611230.
[6] E. Witten, Nucl. Phys. B **500**, 3 (1997), arXiv:hep-th/9703166.
[7] E. Witten, JHEP **07**, 006 (1998), arXiv:hep-th/9805112.
[8] E. Cremmer, B. Julia, and J. Scherk, Phys. Lett. B **76**, 409 (1978).
[9] P.K. Townsend, Phys. Lett. B **350**, 184 (1995), arXiv:hep-th/9501068.
[10] C. Csáki, M. Schmaltz, and W. Skiba, Phys. Rev. Lett. **78**, 799 (1997), arXiv:hep-th/9610139.
[11] N. Seiberg and E. Witten, Nucl. Phys. B **426**, 19 (1994), arXiv:hep-th/9407087.
[12] P.C. Argyres, M.R. Plesser, and A.D. Shapere, Nucl. Phys. B **483**, 172 (1997), arXiv:hep-th/9608129.
[13] K. Hori, H. Ooguri, and Y. Oz, Adv. Theor. Math. Phys. **1**, 1 (1997), arXiv:hep-th/9706082.
[14] A. Masiero and G. Veneziano, Nucl. Phys. B **249**, 593 (1985).
[15] A. Karasik, JHEP **02**, 166 (2021), arXiv:2012.11154.
[16] S. Antusch, C. Hinze, and S. Saad, arXiv:2510.01312 (2025).
[17] S.P. Martin and D.G. Robertson, Phys. Rev. D **100**, 073004 (2019), arXiv:1907.02500.
[18] B. Pendleton and G.G. Ross, Phys. Lett. B **98**, 291 (1981).
[19] C.T. Hill, Phys. Rev. D **24**, 691 (1981).
[20] W. Zimmermann, Commun. Math. Phys. **97**, 211 (1985).
[21] S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024).
[22] A. Abusleme *et al.* (JUNO Collaboration), J. Phys. G **43**, 030401 (2022), arXiv:2204.13249.
[23] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005), arXiv:hep-th/0412141.

[24] Y. Koide, Phys. Lett. B **120**, 161 (1983).
[25] M.A. Luty and J. Terning, Phys. Rev. D **62**, 075006 (2000), arXiv:hep-ph/9812482.
[26] A. Strominger, Phys. Lett. B **383**, 44 (1996), arXiv:hep-th/9512059.
[27] V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. D **47**, 1093 (1993).