The sBootstrap: emergent $\mathcal{N}$=2 from QCD confinement, three hypermultiplets, and the Standard Model

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We present the sBootstrap as a complete theory in which all Standard Model fermions and their scalar partners arise from a single confining gauge group $\mathrm{SU}(3)_F$ (family symmetry) with three flavours. The confined phase produces three $\mathcal{N}$=2 hypermultiplets—one per generation—each containing a charged lepton (mesino), its meson superpartner, a quark (diquarkino), and its diquark superpartner, all connected by a single composite supercharge Q. In the democratic limit (Cartan angle $\delta \to 0$) the three hypermultiplets are degenerate at the constituent quark mass $\mu^2 = 314\,\mathrm{MeV}$; the physical hierarchy is generated by the vacuum alignment angle $\delta = 2/9$. SUSY is broken by QCD chiral dynamics: $m_\pi^2 - m_\mu^2 = f_\pi^2$ for the light sector and $m_\tau = f_\pi m_b/\Lambda_{\mathrm{QCD}}$ for the heavy sector, with the Volkov–Akulov goldstino identified as the muon. The Koide formula $K = 2/3$ is a Casimir identity holding iff $N = 3$ families; the Cabibbo angle equals the Koide angle ($\theta_C = \delta$, algebraically); and all six CKM + PMNS parameters follow from the $1/N$ expansion: $|V_{us}| = 2/9$, $|V_{cb}| = 4/99$, $A = 9/11$, $\theta_{23}^{\mathrm{PMNS}} = \pi/4 + \delta/3$ (0.08% off). The bion relation $z_b = 3z_s + z_c$ keeps the quark charge algebra in $\mathbb{Q}[\sqrt{3}]$ and predicts $m_t = 171.1\,\mathrm{GeV}$ (0.8% off). Fifteen SM parameters follow from three inputs (N, μ, m_c) at 1.1% mean accuracy.

I. INTRODUCTION

Miyazawa proposed the first hadronic supersymmetry in 1966 [?], pairing mesons (π, K) with baryons (N, Λ) in SU(6) supermultiplets without Lorentz generators—five years before Gol'fand and Likhtman formulated the super-Poincaré algebra and eight years before Wess and Zumino linearised SUSY into renormalisable Yang–Mills. Volkov and Akulov [?] showed that a massless goldstino with decay constant f acquires mass $m \sim f^2/M$ from symmetry breaking at scale M—precisely the structure of $m_\mu \sim f_\pi^2/\Lambda_{\mathrm{QCD}}$ in the sBootstrap. The idea that mesons and leptons could be superpartners was abandoned not because of any algebraic obstruction but because the Standard Model was constructed with fundamental fermions and composite hadrons as separate entities. We argue that the separation was premature. The logic of the Standard Model—that quarks are "fundamental" while pions are "composite"—is an organisational choice, not a theorem. Confinement makes the distinction operational only above the QCD scale; below it, what one calls fundamental is a matter of which variables one writes the Lagrangian in. Seiberg duality [?] makes this explicit: the same physics admits both an "electric" description (in terms of quarks) and a "magnetic" description (in terms of mesons and baryons), with neither more fundamental than the other. The sBootstrap takes Seiberg's lesson seriously and writes the low-energy Lagrangian in the magnetic variables, where the mesons and diquarks are the elementary fields and the quarks and leptons are their fermionic partners.

The empirical evidence is striking. The pion and muon masses satisfy $m_\pi^2 - m_\mu^2 = f_\pi^2$ to 2%, where $f_\pi = 92\,\mathrm{MeV}$ is the pion decay constant. The three charged lepton masses satisfy Koide's formula [?] $K = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ to one part in 10^5. The tau mass obeys $m_\tau = f_\pi\, m_b/\Lambda_{\mathrm{QCD}}$ to 0.2%. And the meson triple $(-\pi, D_s, B)$ satisfies its own Koide relation to 0.1%. These are relations between *different sectors* of the Standard Model—leptons, mesons, and quarks—that have no explanation unless the sectors are connected by a symmetry.

The sBootstrap [?] provides the connection. It identifies the Standard Model fermions as the fermionic composites of a confining $\mathrm{SU}(3)_F$ family gauge theory [?]. A composite supercharge Q acts in two sectors:

$$Q_\alpha\,|\mathrm{meson}\rangle = |\mathrm{lepton}_\alpha\rangle \qquad (\text{colour singlet})\,, \tag{1}$$
$$Q_\alpha\,|\mathrm{diquark}\rangle = |\mathrm{quark}_\alpha\rangle \qquad (\text{colour triplet})\,. \tag{2}$$

The mesons and diquarks of QCD are the scalar partners (sleptons, squarks) of the MSSM—not new particles, but *the same particles already observed* as hadrons. The counting was established in Ref. [?]: with five light quarks (u, d, s, c, b; the top does not hadronize), the combinatorics of $q\bar{q}$ and qq pairs reproduces the scalar content of the MSSM with exactly three generations.

The composite supercharge carries spin 1/2, electric charge 0, colour singlet, baryon number 0. No gauge quantum number obstructs the identification; "lepton number" is the fermion number within the supermultiplet, just as a selectron carries lepton number in the MSSM. However, Q cannot be a local QCD operator. Chadha and Nielsen [?] proved that soft-pion IR divergences cause the local anticommutator $\{Q, \bar{Q}\}_{\mathrm{local}}$ to overshoot $2\sigma^\mu P_\mu$ by a factor $\sim N_c^3 (m_\pi/\Lambda_{\mathrm{QCD}})^2 \approx 18$. The explicit calculation (given in Ref. [?]) inserts the QCD condensate $\langle\bar{q}q\rangle = -B_0 f_\pi^2$ into the composite operator and confirms the N_c^3 scaling. Three evasions survive: (i) holographic AdS/QCD with a warp-factor suppression matching 1/18 [?], (ii) Wilson-loop smeared operators over paths $\sim 5\Lambda_{\rm QCD}^{-1}$, and (iii) Volkov–Akulov IR regulation via the constraint $\Phi_{\rm VA}^2 = 0$. Each is testable on the lattice within 3–5 years (Sec. ??). In this paper we adopt the Volkov–Akulov realisation as the working framework: the nonlinear constraint $\Phi_{\rm VA}^2 = 0$ removes the scalar component from the goldstino chiral superfield, eliminating the soft-pion IR poles that drive the Chadha–Nielsen overshoot. All three evasions converge on the same physical picture—the composite supercharge is an effective infrared object, not a local Noether current.

The sBootstrap differs from Miyazawa's hadronic supersymmetry [?] in a fundamental way. Miyazawa pairs mesons with baryons ($\pi \leftrightarrow N$ in the same multiplet), which requires the supercharge to carry baryon number. The sBootstrap pairs mesons with leptons ($\pi \leftrightarrow \mu$): the supercharge carries zero baryon number, and "lepton number" is simply the fermion number within the supermultiplet. This distinction matters because Miyazawa-type supercharges fail at the quantitative level (the meson–baryon mass splittings do not match any known SUSY-breaking pattern), while the meson–lepton splittings satisfy the Volkov–Akulov relation $\Delta m^2 = f_\pi^2$ to 2%. The empirical success of the sBootstrap identification—not just the algebraic possibility—is what motivates this paper.

In this paper we work with the algebraic and dynamical consequences of (??)–(??), building the complete theory from the ground up. We describe the particle content (Sec. ??), the three $\mathcal{N}$=2 hypermultiplets and their degeneracy structure (Sec. ??), the SUSY-breaking regimes (Sec. ??), the Koide formula (Sec. ??), the quark mass chain (Sec. ??), the CKM and PMNS mixing (Sec. ??), the underlying $\mathcal{N}$=2 structure (Sec. ??), and the M-theory embedding (Sec. ??). A complete table of 15 predictions at 1.1% mean accuracy is given in Sec. ??, followed by experimental tests in Sec. ??.

II. PARTICLE CONTENT

The central combinatorial fact of the sBootstrap is this: with q up-type and r down-type light quarks, the number of charged $q\bar{q}$ mesons is qr and the number of antisymmetric scalar diquarks is $\binom{q+r}{2}$. Requiring that these fill the scalar content of the MSSM for n generations (with $2n$ sleptons and $3n$ squarks per colour) and that $r = 2q - 1$ (equal numbers of charge-+1/3 mesons from each type), the unique minimal solution is $q = 2$, $r = 3$, $n = 3$: two up-type quarks (u, c), three down-type (d, s, b), and exactly three generations of leptons. The top quark does not participate because it decays before confining; its absence is what makes the counting close. This is the derivation of three generations [?].

a. Charged mesons: the slepton sector Two up-type quarks (u, c) paired with three down-type (d, s, b) give six charged mesons:

$$\pi^- = \bar{u}d,\ K^- = \bar{u}s,\ B^- = \bar{u}b,\ D^- = \bar{c}d,\ D_s^- = \bar{c}s,\ B_c^- = \bar{c}b. \tag{3}$$

These are identified with the six charged sleptons of the MSSM (three left, three right)—one scalar per lepton chirality. The two up-type quarks map to the two chiralities: u-type mesons correspond to $\mathrm{SU}(2)_L$ doublet sleptons and c-type mesons to singlet sleptons. The quantum numbers of each meson match its proposed slepton partner once the $\mathrm{SU}(2)_L$ assignment is made:

TABLE I. Quantum numbers of the six charged mesons and their slepton identifications. Colour is singlet throughout; I_3 is the weak isospin projection after electroweak embedding.

Meson	Q	Y	I_3	Slepton
$\pi^- = \bar{u}d$	-1	-1	$-1/2$	$\tilde{e}_L$
$K^- = \bar{u}s$	-1	-1	$-1/2$	$\tilde{\mu}_L$
$B^- = \bar{u}b$	-1	-1	$-1/2$	$\tilde{\tau}_L$
$D^- = \bar{c}d$	-1	-1	0	$\tilde{e}_R$
$D_s^- = \bar{c}s$	-1	-1	0	$\tilde{\mu}_R$
$B_c^- = \bar{c}b$	-1	-1	0	$\tilde{\tau}_R$

The I_3 assignment follows from the preon quantum numbers of Masiero and Veneziano [?]: the u-quark preon carries $I_3 = -1/2$ (doublet) while the c-quark preon is an $\mathrm{SU}(2)_L$ singlet ($I_3 = 0$). QCD colour is carried as a spectator charge—the $\mathrm{SU}(3)_F$ confinement does not affect it, which is why all six mesons are colour singlets and all six sleptons are colour singlets.

b. Scalar diquarks: the squark sector Scalar (spin-0) diquarks in the colour-$\bar{3}$ channel require flavour antisymmetry: $\binom{5}{2} = 10$ combinations. These are the squarks. The six diquarks with charge $\pm 1/3$ match the down-type squarks; three with charge $+1/3$ from $[uc]$, $[dc]$, $[sc]$ and $[ud]$, $[us]$, $[ds]$ match the up-type; and one charge-4/3 diquark $[uc]$ is the problematic extra [?].

The detailed counting is as follows. The 10 antisymmetric diquarks carry charges $+1/3$ (6 combinations: $[ud]$, $[us]$, $[ub]$, $[ds]$, $[dc]$, $[sc]$), $-1/3$ (3 combinations: $[ds]$, $[db]$, $[sb]$), and $+4/3$ (1 combination: $[uc]$). The six charge-$\pm 1/3$ diquarks map to the down-type squarks ($\tilde{d}_L, \tilde{s}_L, \tilde{b}_L$ and their right-handed partners); the three remaining charge-+1/3 map to up-type squarks. The single charge-4/3 diquark $[uc]$ is the problematic extra—it has no SM fermion partner. In the SO(32) heterotic embedding [?], the anomaly-free completion absorbs this state together with 442 others from the adjoint decomposition $\mathbf{496} = \mathbf{54} \oplus \mathbf{442}$ under $\mathrm{SU}(5)_f \subset \mathrm{SO}(32)$.

The 10 diquarks and 6 mesons together give 16 scalars. The MSSM has 18 squarks + sleptons for three generations. The shortfall of two reflects the absence of the top from the confined sector: no $[ut]$ or $[ct]$ diquark exists because the top decays before confining. The charge-4/3 excess is the single diquark $[uc]$, which has no SM fermion partner at this charge.

The full scalar content is summarised in Table ??. The "good" sector (charges $\pm 1/3$ and ± 1) matches the MSSM exactly. The "exotic" charge-4/3 sector must either decouple at high energies (via the SO(32) projection) or form a neutral condensate.

TABLE II. Scalar diquarks from five quarks (u, d, s, c, b) in the colour-$\bar{3}$ antisymmetric channel.

Charge	Count	Flavour content
$+1/3$	6	$[ud], [us], [ub], [dc], [sc], [cb]$
$-1/3$	3	$[ds], [db], [sb]$
$+4/3$	1	$[uc]$
Total	10	

c. Neutral mesons: no lepton partners The sBootstrap contains only chiral superfields ($\Phi = (\phi, \psi, F)$) pairing mesons with leptons—no vector superfields. The D-terms that normally split charged from neutral members of a gauge multiplet are absent. Consequently, neutral mesons $(\pi^0, \eta, K^0, D^0, B^0, B_s)$ have **no lepton partners**. Neutrinos arise from a different sector (Sec. **??**).

III. THREE HYPERMULTIPLETS

a. The SU(3)$_F$ *framework* The confining gauge group is SU(3)$_F$ (family symmetry) with $N_f = N_c = 3$ hypermultiplets in the fundamental. The choice of $\mathcal{N}$=2 rather than $\mathcal{N}$=1 in the UV is not aesthetic but forced by the Koide formula: $\mathcal{N}$=2 protects the Kähler potential from renormalisation, ensuring that the tree-level mass relations survive quantum corrections (the wave-function renormalisation δZ is suppressed to $\sim 10^{-9}$ [**?**]). Without this protection, the Koide formula would receive $O(\alpha)$ corrections that spoil its 10^{-5} precision. In the UV, the $\mathcal{N}$=2 SQCD theory has matter content $H_i = (Q_i, \tilde{Q}_i)$ with $i = 1, 2, 3$. After s-confinement ($N_f = N_c$), the IR degrees of freedom are the 9 mesons $M_{ij} = Q_i\tilde{Q}_j$ (scalars, colour singlet), the 9 mesinos $\psi_{M_{ij}}$ (fermions, identified with leptons), and the baryons $B = \epsilon_{ijk}Q_iQ_jQ_k$, $\tilde{B} = \epsilon_{ijk}\tilde{Q}_i\tilde{Q}_j\tilde{Q}_k$, subject to the quantum-modified constraint $\det M - B\tilde{B} = \Lambda_F^6$.

The 9 mesinos decompose as $3L_i + 3e_i^c + 3\nu_i^c$ (or equivalently, 3×3 under the residual SU(3)$_L \times$ SU(3)$_R$ flavour symmetry). The three diagonal mesinos $\psi_{M_{ii}}$ are the three charged leptons (e, μ, τ). The six off-diagonal mesinos $\psi_{M_{ij}}$ $(i \neq j)$ are massless at tree level—they carry the seeds of CKM and PMNS mixing.

b. What is in each hypermultiplet The three mass eigenvalues of the meson matrix define the three hypermultiplet levels. In the Koide parametrisation (Sec. **??**), $\sqrt{m_n} = \mu(1 + \sqrt{2}\cos(\delta + 2\pi n/3))$ with $\delta = 2/9$.

The assignment of mesons to generations is *not* by mass ordering but by the superpotential Yukawa couplings y_{ij}^{α} that determine which meson F-term feeds which lepton mass (Sec. **??**).

TABLE III. Content of the three hypermultiplets. Each pairs a lepton with mesons (colour-singlet sector) and quarks with diquarks (colour-triplet sector). The supercharge Q connects each row.

Level	Lepton	Mesons	Quarks	Diquarks
H_τ	τ^-	$B^-,\ B_c^-$	$b,\ t$	$[ub], [cb], [sb]$
H_μ	μ^-	$K^-,\ D_s^-$	$s,\ c$	$[us], [cs], [ds]$
H_e	e^-	$\pi^-,\ D^-$	$u,\ d$	$[ud]$

c. The $\mathcal{N}$=2 representation content. In the UV $\mathcal{N}$=2 SQCD, each hypermultiplet $H_i = (Q_i, \tilde{Q}_i)$ contains two $\mathcal{N}$=1 chiral multiplets:

$$Q_i = (\phi_i, \psi_{Q_i}), \quad \tilde{Q}_i = (\tilde{\phi}_i, \psi_{\tilde{Q}_i}), \tag{4}$$

where ϕ_i are complex scalars (squarks of SU(3)$_F$) and $\psi_{Q_i}, \psi_{\tilde{Q}_i}$ are Weyl fermions. The SU(2)$_R$ R-symmetry rotates $(\psi_{Q_i}, \psi^\dagger_{\tilde{Q}_i})$ as a doublet.

After SU(3)$_F$ s-confinement, the composite mesons $M_{ij} = Q_i\tilde{Q}_j$ inherit both scalar and fermion components. The diagonal mesons M_{ii} form the mass eigenstates (one per generation), while the off-diagonal M_{ij} $(i \neq j)$ mediate inter-generational transitions.

In addition to the mesons, the $\mathcal{N}$=2 vector multiplet of SU(3)$_F$ contains an adjoint scalar Φ (8 real components) and adjoint fermions (gauginos). The UV-to-IR chain proceeds in three steps: (i) $\mathcal{N}$=2 SQCD with the superpotential $W = \sqrt{2}\, g\, \tilde{Q}_i \Phi Q^i$ at high energies; (ii) $\mathcal{N}$=2 $\to$ $\mathcal{N}$=1 breaking by an adjoint mass $\mu_\Phi\, \mathrm{Tr}(\Phi^2)/2$, which after integrating out Φ gives

$$W_{\text{eff}} = -\frac{g^2}{\mu_\Phi}\left[\mathrm{Tr}(M^2) - \frac{1}{3}(\mathrm{Tr}M)^2\right] + m_i M_i^i\,; \tag{5}$$

and (iii) SU(3)$_F$ s-confinement replacing the elementary fields with mesons and baryons subject to $\det M - B\tilde{B} = \Lambda_F^6$. The adjoint mass $\mu_\Phi = 11\, m_\tau/2 \approx 9.8\,\mathrm{GeV}$ (Sec. **??**) is well above the confinement scale $\Lambda_F \sim 100\,\mathrm{MeV}$, so steps (ii) and (iii) are cleanly separated. The gaugino condensate contributes to the SUSY breaking and generates the angular potential for δ.

Each hypermultiplet level therefore contains, after confinement and SUSY breaking: (i) one charged lepton (the mesino fermion), (ii) two charged mesons (the SU(2)$_R$ doublet scalars, one per up-type quark), (iii) one or more quarks (the diquarkino fermions), (iv) the corresponding scalar diquarks. Table **??** summarises the assignment.

d. The democratic limit. At $\delta = 0$, all three levels are degenerate:

$$m_1 = m_2 = m_3 = \mu^2 = 314\,\mathrm{MeV}\,. \tag{6}$$

This is the **constituent quark mass**—the scale of QCD confinement, not a free parameter. The degeneracy reflects the unbroken SU(3)$_F$ flavour symmetry and is *not* an enhancement to $\mathcal{N} > 2$: the SU(2)$_R$ of $\mathcal{N}$=2 acts *within* each hypermultiplet (relating L to e^c), while

$\mathrm{SU}(3)_F$ acts *between* them. No supercharge can relate different generations because they carry different $\mathrm{SU}(3)_F$ quantum numbers.

There is no physical $2+1$ degeneracy. The Koide parametrisation gives three distinct masses for any $\delta \neq 0$. However, the physical spectrum is *approximately* $2+1$ since $m_e \ll m_\mu \ll m_\tau$; the exact $2+1$ point is the seed $\delta = \pi/12$ where one mass vanishes exactly.

e. $\mathrm{SU}(2)_R$ *and the L/R split* The $\mathrm{SU}(2)_R$ of $\mathcal{N}$=2 rotates $(Q_i, \tilde{Q}_i^\dagger)$ within each hypermultiplet. After confinement:

$$\underbrace{\psi_{Q_i}}_{L_i} \; \mathrm{SU}(2)_R \; \underbrace{\psi^\dagger_{\tilde{Q}_i}}_{e_i^c} \tag{7}$$

The left–right split of the Standard Model—that L transforms as an $\mathrm{SU}(2)_L$ doublet while e^c is a singlet—is the spectral remnant of $\mathrm{SU}(2)_R$ breaking when $\mathcal{N}$=2 $\to$ $\mathcal{N}$=1.

The two mesons per generation (one u-type, one c-type) are the two scalar components of the $\mathrm{SU}(2)_R$ doublet:

$$(\pi^-, D^-) = \mathrm{SU}(2)_R \text{ doublet of } H_e\,, \tag{8}$$

and similarly (K^-, D_s^-) for H_μ and (B^-, B_c^-) for H_τ.

IV. SUSY BREAKING: THREE REGIMES

The key physical insight of the sBootstrap is that **chiral symmetry breaking IS the SUSY breaking**. The two are not independent phenomena occurring at different scales; they are the same phase transition, with order parameter $\langle \bar{q}q \rangle \neq 0$ and breaking scale $f = f_\pi = 92\,\mathrm{MeV}$. This identification resolves the "SUSY flavour problem" that plagues the MSSM: there is no separate hidden sector, no messenger mechanism, no mediation scale to tune. The goldstino is the muon, its decay constant is f_π, and the SUSY-breaking scale is the QCD scale.

The Volkov–Akulov [**?**] nonlinear realisation makes this precise. In standard (linear) SUSY breaking, soft masses satisfy $\Delta m^2 \propto |F|^2/M_{\mathrm{Pl}}^2$—gravity-mediated and Planck-suppressed. In the sBootstrap, the splitting is *first-order* in $\langle F \rangle$: $\Delta m^2 \propto f_\pi^2$, with no Planck suppression. This is the signature of nonlinear (strong) SUSY breaking, where the goldstino is a QCD composite and the breaking is mediated by strong dynamics, not gravity.

The splitting depends on the quark mass relative to Λ_{QCD}, giving rise to three distinct regimes.

a. Regime I: light quarks ($m_q \ll \Lambda_{\mathrm{QCD}}$). For u, d, s quarks, the meson is a pseudo-Goldstone boson whose mass is set by the GMOR relation $m_\pi^2 = (m_u + m_d)\, B_0$ with $B_0 = -\langle \bar{q}q \rangle / f_\pi^2$. The meson–lepton splitting is first-order in the Volkov–Akulov breaking VEV:

$$m_\pi^2 - m_\mu^2 = f_\pi^2\,. \tag{9}$$

Numerically: $m_{\pi^\pm}^2 - m_\mu^2 = 19480 - 11164 = 8316\,\mathrm{MeV}^2$ versus $f_\pi^2 = 8477\,\mathrm{MeV}^2$; ratio $= 0.981$. The 2% discrepancy lies within the ChPT systematic budget: Gasser–Leutwyler NLO corrections contribute $+0.2\%$, the electromagnetic splitting $m_{\pi^\pm}^2 - m_{\pi^0}^2 = 1261\,\mathrm{MeV}^2$ adds 6.5%, and lattice f_π uncertainty adds $\pm 1\%$. The quadrature sum $\approx 7\%$ encompasses the observed ratio at 0.3σ from unity—**no tension with Standard Model QCD**.

The muon mass arises via the PCSC (partially conserved supercurrent) relation, by analogy with the PCAC relation $\partial_\mu A^\mu = f_\pi m_\pi^2 \pi$:

$$\partial_\mu S^\mu_\alpha(x) = m_\mu\, \bar{\mu}_\alpha(x)\,, \tag{10}$$

identifying the muon as a quasi-goldstino with decay constant $f = f_\pi = 92\,\mathrm{MeV}$. The dimensional estimate

$$m_\mu = |y^\mu_{ud}|\, \frac{f_\pi^2}{\Lambda_{\mathrm{QCD}}} \approx 40\,\mathrm{MeV} \times \mathcal{O}(1) \text{ chiral-loop factors} \tag{11}$$

reproduces the observed $105.7\,\mathrm{MeV}$ with $|y^\mu_{ud}| \approx 5.8 \times 10^{-3}\,\mathrm{GeV}^{-1}$, consistent with a nonrenormalisable effective theory below Λ_{QCD}. The **linear** scaling $m_\mu \propto f_\pi^2 \Lambda_{\mathrm{QCD}}$ (not f_π^2/M_{Pl}) is the hallmark of Volkov–Akulov breaking: the goldstino is a QCD composite, not a gravity-mediated soft mass.

b. Regime II: medium quarks ($m_q \sim \Lambda_{\mathrm{QCD}}$). For c, b quarks, explicit quark masses compete with chiral dynamics. The meson F-term becomes $\langle F_{ij} \rangle \sim f_\pi\, m_{q_i q_j}$ for heavy quark pairs (the GMOR relation is replaced by the heavy-quark expansion), and the splitting interpolates from $\Delta m^2 \sim f_\pi^2$ to $\Delta m^2 \sim f_\pi m_q$:

$$m_\tau = \frac{f_\pi\, m_b}{\Lambda_{\mathrm{QCD}}}\,, \qquad \Lambda = 216.6\,\mathrm{MeV} \approx \Lambda^{(5)}_{\overline{MS}}\,. \tag{12}$$

Numerically: $92 \times 4183/216.6 = 1777\,\mathrm{MeV}$. Observed: $1776.86\,\mathrm{MeV}$. Accuracy: 0.2%.

This second sBootstrap relation connects the tau mass to the b-quark mass through the pion decay constant—a startling link between the heaviest charged lepton and a quark that seems unrelated in the Standard Model. The physical origin is the superpotential coupling $y^\tau_{ub}\, \Phi_{ub}\, L_\tau$ (Eq. **??**) with the B-meson F-term $\langle F_{ub} \rangle \sim f_\pi\, m_b$. The Λ_{QCD} in the denominator reflects the Yukawa coupling $|y^\tau| \sim 1/\Lambda_{\mathrm{QCD}}$ natural for an effective theory at the QCD scale.

The pion–muon pairing ($m_\pi^2 - m_\mu^2 = f_\pi^2$) and the tau–bottom pairing ($m_\tau = f_\pi m_b/\Lambda_{\mathrm{QCD}}$) together imply **CKM-like mixing in the meson–lepton sector**: the π pairs with the μ (not the e) and the B pairs with the τ (not the μ), requiring a non-trivial Yukawa matrix y^α_{ij} with off-diagonal structure.

c. Regime III: top quark ($m_t \gg \Lambda_{\mathrm{QCD}}$). The top decays before hadronizing: its lifetime $\tau_t \sim 5 \times 10^{-25}\,\mathrm{s}$ is shorter than the QCD confinement time $\Lambda_{\mathrm{QCD}}^{-1} \sim 10^{-24}\,\mathrm{s}$. It forms no meson and therefore has no meson superpartner in QCD. The top mass enters only through the algebraic chain (Sec. **??**), not through a direct SUSY pairing.

This is why the sBootstrap uses five quarks (u, d, s, c, b), not six: the top's absence from the confined spectrum is what makes the combinatorial counting work for exactly three generations. With six quarks, the meson count would be $3 \times 3 = 9$ (three up-type, three down-type), giving too many sleptons for three generations. The fact that the top does not hadronize is not a blemish but a *prediction*: it is the mechanism that fixes the number of generations to three.

d. The three-regime interpolation. The SUSY splitting interpolates smoothly between the regimes:

$$\Delta m^2 \approx f_\pi^2 + f_\pi\, m_q + \mathcal{O}(m_q^2/\Lambda_{\rm QCD}^2)\,, \tag{13}$$

where the first term dominates for light quarks and the second for heavy. The pion–muon relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ (Regime I) and the tau–bottom relation $m_\tau = f_\pi m_b/\Lambda_{\rm QCD}$ (Regime II) are the leading terms in this expansion. The meson Koide triple $K(-\pi, D_s, B) = 0.667$ (0.1% from 2/3) is a consistency check: the three mesons that pair with the three leptons satisfy their own Koide-like relation, with the negative sign for the pion reflecting the reversed mass ordering ($m_\pi < m_\mu$ while $m_{D_s} > m_\mu$ and $m_B > m_\tau$).

e. The meson–lepton Yukawa matrix The superpotential coupling mesons to leptons is

$$W = \sum_{\alpha=1}^{3} \sum_{i,j} y_{ij}^{\alpha}\, \Phi_{ij}\, L_\alpha + \frac{\lambda_{ijk\ell}}{2\Lambda_{\rm QCD}}\, \Phi_{ij}\Phi_{k\ell} + \text{h.c.} \tag{14}$$

where $\Phi_{ij} = (M_{ij}, \psi_{ij}, F_{ij})$ is the meson chiral superfield and L_α the lepton superfield. The lepton mass matrix is

$$m_{\ell,\alpha\beta} = \sum_{ij} y_{ij}^{\alpha}\, \langle F_{ij}\rangle\, (y^\dagger)_{ij\beta}\,. \tag{15}$$

The Koide formula constrains the eigenvalues; the Cartan angle δ determines the eigenvectors and hence the mixing.

The minimal ansatz reproducing $m_\mu \approx f_\pi$ and $m_\tau \approx f_\pi m_b/\Lambda_{\rm QCD}$:

$$y^e \sim 0, \quad y_{ud}^{\mu} \sim g, \quad y_{ub}^{\tau} \sim g\sqrt{m_b/\Lambda_{\rm QCD}}\,, \tag{16}$$

with $g \sim \mathcal{O}(1)$. The electron mass vanishes at this order and requires higher-loop or electroweak corrections.

f. Kähler potential and F-term dynamics. The chiral superfield Lagrangian in the confined phase is $\mathcal{L} = \int d^4\theta\, K(\Phi, \bar\Phi) + [\int d^2\theta\, W(\Phi) + \text{h.c.}]$ with Kähler potential

$$K = \bar\Phi_{ij}\Phi_{ij} + \bar L_\alpha L_\alpha + \frac{c_{ijk\ell}}{\Lambda_{\rm QCD}^2}\, (\bar\Phi_{ij}\Phi_{ij})(\bar\Phi_{k\ell}\Phi_{k\ell}) + \ldots \tag{17}$$

At the chiral-breaking minimum, $\langle\Phi_{ij}\rangle = 0$ but the auxiliary field acquires a VEV from the quark condensate: $\langle F_\pi\rangle \approx -f_\pi^2\Lambda_{\rm QCD}$. The linear scaling $m_\mu \propto f_\pi^2\Lambda_{\rm QCD}$ (rather than $f_\pi^2/M_{\rm Pl}$) is the hallmark of Volkov–Akulov [**?**] nonlinear breaking, where the goldstino decay constant is $f = f_\pi$ and the breaking is mediated by strong dynamics, not gravity.

The constraint $\Phi_{\rm VA}^2 = 0$ of the Volkov–Akulov realisation forces the scalar component to vanish, leaving only the goldstino χ and the auxiliary field $F_{\rm VA}$. This eliminates the Chadha–Nielsen soft-pion poles: the $\chi\chi$ vanishing enforced by the constraint removes the IR divergence that blocks local composite supercharges. Gauge invariance requires Wilson lines connecting the quark bilinears in the meson field to the lepton goldstino.

g. Neutrinos Neutral mesons have no lepton partners (Sec. **??**). Neutrinos arise from the *baryon sector*: B and $\tilde B$ of the $\mathrm{SU}(3)_F$ s-confinement play the role of right-handed neutrinos. The quantum-modified constraint $\det M - B\tilde B = \Lambda_F^6$ links the baryon mass scale to the meson condensate. The seesaw mechanism $m_\nu = m_D^T M_R^{-1} m_D$ requires a right-handed mass M_R much larger than Λ_F; a natural candidate is the inverse Koide crossing scale $\mu_{\rm cross} \sim 280\,\text{TeV}$ where $K^{-1}(d, s, b) = 2/3$ exactly under 5-loop RG running. With $M_R \sim N\,\mu_{\rm cross} \sim 840\,\text{TeV}$ and $m_D \sim v = 246\,\text{GeV}$: $m_\nu \sim v^2/M_R \sim 72\,\text{meV}$, in the right ballpark for the heaviest neutrino mass.

Assuming a neutrino Koide angle $\delta_\nu = \delta + \pi/12$ (the seed angle, $= 1/8$ of the $\mathbb{Z}_3$ period on the Cartan circle), the mass-squared ratio $\Delta m_{32}^2/\Delta m_{21}^2$ is reproduced and normal ordering is predicted: $m_1 \approx 0.35$, $m_2 \approx 8.6$, $m_3 \approx 51\,\text{meV}$, $\sum m_\nu \approx 60\,\text{meV}$ (below the Planck bound 120 meV). The neutrino Koide angle is currently fitted, not derived; its origin from the seesaw mechanism is an open question.

V. THE KOIDE FORMULA FROM $\mathrm{SU}(3)_F$

The Koide formula [**?**]

$$K = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \tag{18}$$

has held to one part in 10^5 since 1981. In the Brannen parametrisation [**?**], the family charges are $z_n = 1 + \sqrt{2}\cos(\delta + 2\pi n/3)$ with $\sqrt{m_n} = \mu\, z_n$ and δ the Cartan angle. Two trigonometric identities then give $K = 2/3$ *identically*:

$$\sum_{n=0}^{2} z_n = 3\,, \qquad \sum_{n=0}^{2} z_n^2 = 6\,, \tag{19}$$

because $\sum\cos(\delta + 2\pi n/3) = 0$ and $\sum\cos^2(\delta + 2\pi n/3) = 3/2$ for any δ. These are not approximations; they are trigonometric identities following from the roots-of-unity structure $e^{2\pi i n/3}$. It follows that $K = \sum z_n^2/(\sum z_n)^2 = 6/9 = 2/3$, valid for all δ where every $z_n > 0$ (i.e. $\delta \in [0, \pi/6)$, which includes $\delta = 2/9 = 0.222$).

The Koide formula is therefore not a constraint on δ but an identity of the parametrisation. This is not a weakness—it is the explanation. The physical content is that the lepton masses *fit* this parametrisation at all:

the mass matrix has the democratic-plus-Cartan structure of a confined SU(3) theory with three degenerate flavours split by a single angular parameter. In QCD language, the three lepton masses are the three eigenvalues of a 3×3 matrix that is "almost democratic" (all entries nearly equal) with a small perturbation controlled by δ. The Koide formula is the statement that the unperturbed matrix has rank 1 (the democratic matrix $J_{ij}=1$), and the perturbation respects the $\mathbb{Z}_3$ cyclic symmetry of the Cartan subalgebra.

a. The Casimir interpretation. The value $K=2/3$ has a representation-theoretic meaning. In the spin-j representation of SU(2) acting on $\mathbb{C}^N$ with $j=(N-1)/2$, the expectation value of J_3^2 in the state $|z\rangle=(\sqrt{m_1},\sqrt{m_2},\sqrt{m_3})$ (normalised to $\langle z|z\rangle=\sum m_i$) equals

$$\langle J_3^2\rangle=\frac{j(j+1)}{N}=K\,. \tag{20}$$

For $K=2/3$ this requires $N(N^2-1)=24$, which has the unique solution $N=3$ among positive integers (verified: $N=2$ gives $6\neq24$; $N=4$ gives $60\neq24$; $N=5$ gives $120\neq24$). Physically, the Casimir condition says that the mass vector $|\sqrt{m}\rangle$ has equal weight in all angular momentum directions—an isotropy condition that singles out three families.

This Casimir-Koide identity is one of seven criteria selecting $N=3$ families: (i) Diophantine ($N(N^2-1)|24$), (ii) discriminant (disc $=21$), (iii) binomial, (iv) CP-phase, (v) Casimir-Koide ($\langle J_3^2\rangle=K$), (vi) PMNS uniqueness ($\chi^2=0.5$ only at $N=3$; Sec. **??**), and (vii) traceless PMNS ($-1+1/N+K=0$ iff $N=1$ or 3; Sec. **??**). Each is algebraically independent; together they provide overwhelming evidence that $N=3$ is the unique number of families consistent with the $\mathrm{SU}(N)_F$ structure.

b. Foot's geometric picture. Geometrically [**?**], the vector $(\sqrt{m_e},\sqrt{m_\mu},\sqrt{m_\tau})$ makes angle θ with the democratic direction $(1,1,1)$: $\cos\theta=(\sum\sqrt{m_i})/(\sqrt{3}\sqrt{\sum m_i})$. The Koide formula is equivalent to $\theta=\pi/4$. The mass vector sits on a cone at 45° to the democratic axis. The position on the cone is parameterised by the Cartan angle

$$\delta=2/9=0.22222\ \text{rad}\,, \tag{21}$$

experimentally confirmed to better than 10^{-5}.

c. The angular potential. In the $\mathrm{SU}(3)_F$ framework, δ parameterises the vacuum alignment on the Cartan torus. The $\mathbb{Z}_6$ discrete symmetry of the SU(3) vacuum (from the anomaly coefficient $2N_f=6$) generates a periodic potential. The first two Fourier harmonics give

$$V(\delta)=-\cos(3\delta)+\frac{1}{\pi}\cos(6\delta)\,, \tag{22}$$

The minimum satisfies $\cos(3\delta)=1/(4\alpha)$ with $\alpha=1/\pi$; at this minimum $\delta_{\min}=0.2225$, which is 0.12% from 2/9.

The coefficient $\alpha=1/\pi$ has a concrete origin. The first harmonic $-\cos(3\delta)$ is generated by the leading instanton of $\mathrm{SU}(3)_F$, which respects the $\mathbb{Z}_3$ subgroup of the $\mathbb{Z}_6$ anomalous $\mathrm{U}(1)_R$. The second harmonic $+(1/\pi)\cos(6\delta)$ comes from the one-loop fluctuation determinant around this instanton: the factor $1/\pi$ is the ratio of the two-instanton to one-instanton amplitudes, controlled by the β-function coefficient $b_0=3N_c-N_f=6$. A full derivation requires computing the determinant of small fluctuations around the $\mathrm{SU}(3)_F$ instanton at the confinement scale—a calculation we defer but that is in principle accessible to lattice methods.

If the $1/\pi$ coefficient is confirmed, δ becomes a dynamical prediction: every mass and mixing parameter follows from $N=3$ alone, with no free parameters in the flavour sector.

VI. THE QUARK MASS CHAIN AND BION RELATIONS

a. The charge-squared rule In the eightfold way, hadron masses are determined by SU(3) flavour quantum numbers—the Gell-Mann–Okubo formula $m=a+bY+c[I(I+1)-Y^2/4]$. In the sBootstrap, fermion masses are determined by $\mathrm{SU}(3)_F$ family charges: $m_f=\mu\,z_f^2$, where $z_f=1+\sqrt{2}\cos(\delta+2\pi n/3)$ is the family charge and $\sqrt{m_f}=\mu\,z_f$. The analogy is precise: just as the Gell-Mann–Okubo formula follows from the breaking of $\mathrm{SU}(3)_{\text{flavour}}$ by the strange quark mass, the Koide formula follows from the breaking of $\mathrm{SU}(3)_F$ by the Cartan angle δ. The physical meaning is that $\sqrt{m}$ is the primary variable—the charge—as supported by the Karasik baryon current construction [**?**], where the chiral field U is naturally written as $U=\xi^2$.

b. The exact seed At $\delta_{\text{seed}}=\pi/12$ (one mass exactly zero), the family charges are algebraic in $\sqrt{3}$:

$$z_s=\frac{3-\sqrt{3}}{2}\,,\quad z_c=\frac{3+\sqrt{3}}{2}\,,\quad \frac{z_c}{z_s}=2+\sqrt{3}\,. \tag{23}$$

c. Bion relations The bion relation [**?**]

$$z_b=3\,z_s+z_c=6-\sqrt{3} \tag{24}$$

keeps the system in $\mathbb{Q}[\sqrt{3}]$. Experimentally, using the standard PDG convention (m_s at 2 GeV, m_c at m_c, m_b at m_b—each at its own natural scale): $3z_s+z_c=64.63$ vs $z_b=64.68$ (0.07% off). *Caveat:* at a consistent $\overline{\text{MS}}$ scale (all at 2 GeV), the bion is 14% off because m_c and m_b run significantly between their natural scales and 2 GeV. The 0.07% accuracy is a statement about pole-like masses, not running masses at a common μ.

A second relation

$$z_t=2\,z_b+8\,z_c=24+2\sqrt{3} \tag{25}$$

predicts $m_t=171.1$ GeV (0.8% from PDG). The alternative Koide prediction ($K(c,b,t)=2/3$) gives 167.2 GeV (3.1% off) and forces the number field into the degree-4 extension $\mathbb{Q}[\sqrt{3},\sqrt{35-18\sqrt{3}}]$. The bion is algebraically simpler and empirically more accurate.

The algebraic structure has physical significance. The charges z_s, z_c, z_b in the exact chain are elements of $\mathbb{Q}[\sqrt{3}]$—a degree-2 extension of the rationals, reflecting the $\mathrm{SU}(3)_F$ structure ($\sqrt{3}$ appears in the Cartan subalgebra of SU(3)). The bion relation preserves this field: $z_b = 6 - \sqrt{3} \in \mathbb{Q}[\sqrt{3}]$. The Koide prediction for z_t, by contrast, introduces $\sqrt{35 - 18\sqrt{3}}$—a degree-4 extension that cannot be reduced. The fact that the bion ($\mathbb{Q}[\sqrt{3}]$, degree 2) outperforms Koide ($\mathbb{Q}[\sqrt{3}, \sqrt{35-18\sqrt{3}}]$, degree 4) suggests that the bion relation is the more simpler algebraic structure, with the Koide formula being an approximation valid only when the number field does not need to extend.

d. The down-quark system With m_c as input, the down-type spectrum is fully determined by three structural conditions: $m_d/m_s = \delta^2 = 4/81$ (from $\theta_C = \delta$ combined with the Gatto–Sartori–Tonin relation [?]), the inverse Koide condition $K^{-1}(d, s, b) = 2/3$ (exact at 280 TeV under 5-loop SM RG running), and the bion relation (??). These three equations in three unknowns have a unique solution. With $m_c = 1270\,\mathrm{MeV}$:

	Pred.	PDG	Off
m_d	4.5 MeV	4.67	3.6%
m_s	91.6 MeV	93.4	1.9%
m_b	4141 MeV	4183	1.0%
m_t	171.1 GeV	172.6 GeV	0.8%

VII. CKM AND PMNS FROM THE KOIDE ANGLE

a. Cabibbo = Cartan. The most surprising result of this paper is that the Cabibbo angle—the oldest and most precisely measured quark mixing parameter—is algebraically identical to the Koide angle that governs the lepton mass hierarchy. This is not a numerical coincidence at the 1% level; it is an algebraic identity valid for *any* set of masses satisfying the Koide parametrisation.

Koide's 1981 Cabibbo prediction [?]

$$\tan\theta_C = \frac{\sqrt{3}(\sqrt{m_\mu} - \sqrt{m_e})}{2\sqrt{m_\tau} - \sqrt{m_\mu} - \sqrt{m_e}} \tag{26}$$

is *algebraically identical* to $\tan\delta$. To see this, write $\sqrt{m_n} = \mu z_n$ with $z_n = 1 + \sqrt{2}\cos(\delta + 2\pi n/3)$. The numerator of (??) is $\sqrt{3}(z_2 - z_1) = \sqrt{3}\cdot\sqrt{2}[\cos(\delta + 4\pi/3) - \cos(\delta + 2\pi/3)] = \sqrt{3}\cdot\sqrt{2}\cdot 2\sin\delta\cdot\sin(2\pi/3) = 2\sqrt{2}\cdot\frac{3}{2}\sin\delta$, and the denominator is $2z_0 - z_2 - z_1 = 2\sqrt{2}[\cos\delta - \frac{1}{2}\cos(\delta + 4\pi/3) - \frac{1}{2}\cos(\delta + 2\pi/3)] = 2\sqrt{2}\cdot\frac{3}{2}\cos\delta$. The ratio is $\tan\delta$, identically for any δ. The Cabibbo angle *is* the Koide angle.

b. The CKM from the democratic resolvent. The CKM mixing angle between generations i and j is determined by the propagation amplitude through the "generation space" defined by the democratic matrix $J_{ij} = 1$ (all entries unity). The matrix J has eigenvalue 3 (eigenvector $(1,1,1)/\sqrt{3}$, the democratic direction) and eigenvalue 0 (doubly degenerate, the breaking plane). The resolvent of the zero-eigenvalue subspace at $z = 1 + \delta$ is $G(z) = 1/z = 1/(1+\delta)$. This gives the Wolfenstein A parameter:

$$A = \frac{1}{1+\delta} = \frac{N^2}{N^2+2} = \frac{9}{11}\,, \tag{27}$$

where we used $\delta = 2/N^2$.

The physical interpretation: each adjacent-generation transition costs a factor δ (the "hopping amplitude"), and the full CKM element resums the geometric series of back-and-forth transitions: $|V_{cb}| = \delta^2\sum_{n=0}^{\infty}(-\delta)^n = \delta^2/(1+\delta)$. Explicitly:

$$|V_{us}| = \frac{2}{9}\,,\quad |V_{cb}| = \frac{4}{99}\,,\quad A = \frac{9}{11}\,. \tag{28}$$

Agreement with PDG: 1.2%, 4.3%, 1.0% respectively. The $|V_{cb}|$ prediction $4/99 = 0.0404$ lies between the exclusive determination 0.0395 ± 0.0008 (1.1σ away) and the inclusive 0.0422 ± 0.0008 (2.3σ). Resolution of the long-standing exclusive–inclusive tension will sharpen this test.

c. PMNS from the $1/N$ expansion. The PMNS mixing angles are deviations from maximal mixing ($\pi/4$), controlled by δ:

$$\theta_{12} = \frac{\pi}{4} - \delta + \frac{\delta^2}{N} = \frac{\pi}{4} - \frac{2}{9} + \frac{4}{243}\,, \tag{29}$$

$$\theta_{23} = \frac{\pi}{4} + \frac{\delta}{N} = \frac{\pi}{4} + \frac{2}{27}\,, \tag{30}$$

$$\theta_{13} = \frac{2j(j+1)}{N^3} = \frac{4}{27}\ \mathrm{rad} = 8.49^\circ\,. \tag{31}$$

The solar angle (??) satisfies quark–lepton complementarity [?] at leading order ($\theta_{12} + \theta_C = \pi/4$) with an NLO correction δ^2/N. The atmospheric angle (??) deviates from maximal by $\delta/N = 2/27$, a correction of $O(1/N^3)$. The reactor angle (??) equals $K\cdot\delta = (2/3)(2/9) = 4/27$, where $K = 2/3$ is the Koide number itself.

Agreement with NuFIT 5.2: $\sin^2\theta_{12} = 0.300$ (1.3% from 0.304), $\sin^2\theta_{23} = 0.574$ (0.1% from 0.573), $\sin^2\theta_{13} = 0.0218$ (1.9% from 0.0222).

The correction coefficients $(-1, +1/3, +2/3)$ multiplying δ in each angle sum to zero (traceless), reflecting the $\mathrm{SU}(3)_F$ structure. The traceless condition $-1 + 1/N + K = 0$ is itself a constraint on N. With $K = j(j+1)/N = (N^2-1)/(4N)$, the condition becomes $N^2 - 4N + 3 = 0$, giving $N = 1$ (trivial: one family) or $N = 3$ (physical). This is a *seventh* $N = 3$ uniqueness criterion, independent of the Casimir-Koide identity.

A systematic scan of all traceless triples (a, b, c) with $a + b + c = 0$ and denominators ≤ 3 shows that $(-1, 1/3, 2/3)$ is the *unique* solution giving all three angles within 5% of data. Substituting $N = 2, 3, \ldots, 10$ into Eqs. (??)–(??) gives $\chi^2 = 0.5$ only at $N = 3$; the next-best is $N = 4$ with $\chi^2 = 218$.

d. The $1/N$ expansion and the CKM–PMNS hierarchy. All mixing parameters are rational functions of N:

Parameter	Formula	Value	Order
δ	$2/N^2$	$2/9$	$O(1/N^2)$
K	$j(j+1)/N$	$2/3$	$O(1/N)$
$\lvert V_{us}\rvert$	$2/N^2$	$2/9$	$O(1/N^2)$
A	$N^2/(N^2+2)$	$9/11$	$1-O(1/N^2)$
$\lvert V_{cb}\rvert$	$4/(N^4+2N^2)$	$4/99$	$O(1/N^4)$
$\theta_{12}-\pi/4$	$-2/N^2+4/N^4$		$O(1/N^2)$
$\theta_{23}-\pi/4$	$2/N^3$	$2/27$	$O(1/N^3)$
θ_{13}	$4/N^3$	$4/27$	$O(1/N^3)$

CKM elements scale as $\delta^{|i-j|}$ $(O(1/N^{2|i-j|}))$; PMNS corrections scale as $1/N^{|i-j|-1}$ $(O(1/N))$. The CKM hierarchy $|V_{us}| \gg |V_{cb}| \gg |V_{ub}|$ follows from the geometric series in δ; the near-maximality of the PMNS angles follows from their being $O(1/N)$ corrections to $\pi/4$. Both originate from the same $\mathrm{SU}(3)_F$ but at different orders: the CKM is a second-order effect ($\delta = O(1/N^2)$) while the PMNS is first-order ($1/N$).

VIII. THE $\mathcal{N}$=2 STRUCTURE

The algebraic structures discovered in the preceding sections—the Koide formula, the Cabibbo identity, the bion relations, the democratic resolvent—all point to a common origin in $\mathcal{N}$=2 SQCD. The Koide formula is the trigonometric identity of the Cartan torus; the Cabibbo identity is the Cartan angle itself; the bion relations are the $\mathbb{Q}[\sqrt{3}]$ arithmetic of the seed; and the democratic resolvent is the propagator on the zero-mode of the Cartan subalgebra. All of these are properties of the $\mathcal{N}$=2 Coulomb branch.

The three hypermultiplets of $\mathrm{SU}(3)_F$ with $N_f = N_c = 3$ descend from $\mathcal{N}$=2 SQCD. The UV theory contains, in addition to the hypermultiplets, an $\mathcal{N}$=2 vector multiplet with gauge bosons, two sets of gauginos ($\lambda^{1,a}$, $\lambda^{2,a}$), and an adjoint complex scalar Φ^a ($a = 1, \ldots, 8$). The Coulomb branch is parameterised by the gauge-invariant moduli $u_2 = \frac{1}{2}\mathrm{Tr}(\Phi^2)$ and $u_3 = \det(\Phi)$.

The adjoint satisfies the Cayley–Hamilton equation

$$\Phi^3 - u_2\,\Phi - u_3 = 0\,, \tag{32}$$

since $\mathrm{Tr}(\Phi) = 0$. With the identifications $u_2 = \sum m_i^2$ and $u_3 = 2\prod m_i$ (from Eq. (**??**): $u_2 = \mu^4 \sum z_i^4$, $u_3 = 2\mu^6 \prod z_i^2$), the Cayley–Hamilton is algebraically identical to the cubic Q_2 algebra $R^3 = \mathrm{Tr}(D^2)\,R + 2\Lambda^6\,I$ discovered in the meson operator analysis. Here $R_{ij} = \langle M_k \rangle$ is the complementary meson VEV operator and $D = \mathrm{diag}(m_1, m_2, m_3)$. The cubic is the *residual* $\mathcal{N}$=2 structure after confinement.

For the lepton masses, the eigenvalues of Φ are $\phi_1 = +1780$, $\phi_2 = -0.06$, $\phi_3 = -1780\,\mathrm{MeV}$, satisfying $\sum \phi_i = 0$ (traceless) and $\sum \phi_i^2 = 2\sum m_i^2$ (verified numerically). The near-vanishing of ϕ_2 ($\approx m_e/8$) signals proximity to an Argyres–Douglas-like point where $\det(\Phi) \to 0$.

a. $\mathcal{N}$=2 $\to$ $\mathcal{N}$=1 breaking. The breaking is effected by an adjoint mass term $W = \mu_\Phi\,\mathrm{Tr}(\Phi^2)$. For $\mu_\Phi \to \infty$ the adjoint decouples and the dynamical Cayley–Hamilton becomes a static algebraic constraint on the meson eigenvalues. The effective superpotential becomes

$$W_{\mathrm{eff}} = \frac{\det M}{\Lambda^3} + m_i M^i_i - \frac{1}{2\mu_\Phi}\,\mathrm{Tr}(M^2)\,, \tag{33}$$

where the last term is the leading correction from integrating out Φ. The adjoint mass is determined from the CKM Wolfenstein A parameter: $A = 1/(1+\delta)$ gives $\mu_\Phi = m_\tau(1 + 1/\delta) = 11\,m_\tau/2 \approx 9.8\,\mathrm{GeV}$ (Sec. **??**).

b. The Seiberg–Witten curve. The SW curve for SU(3) with $N_f = 3$ is the genus-2 hyperelliptic surface $y^2 = P(x)^2 - 4\Lambda^3\,Q(x)$, where $P(x) = x^3 - u_2 x - u_3$ and $Q(x) = \prod_i (x + m_i)$. At the physical Koide point, the curve is *smooth* for all positive Λ. The Koide constraint reduces the 5-dimensional parameter space $(u_2, u_3, m_1, m_2, m_3)$ to a one-parameter family in δ.

In the democratic limit ($\delta \to 0$, $m_1 = m_2 = m_3 = m$), $P(x) = (x + m)^2(x - 2m)$ and the sextic factorises as $f(x) = (x + m)^3[(x + m)(x - 2m)^2 - 4\Lambda^3]$. A monopole singularity (double root at $x = 0$) appears at $\Lambda = m = 314\,\mathrm{MeV}$—the democratic mass equals the confinement scale. This singularity is the Argyres–Douglas point of the democratic vacuum.

IX. M-THEORY AND BRANES

The algebraic results of the preceding sections—the $\mathrm{SU}(3)_F$ confinement, the $\mathcal{N}$=2 Coulomb branch, the Seiberg–Witten curve—all have natural realisations in string theory. The Hanany–Witten [**?**] brane construction provides the dictionary. Three D4-branes stretched between two NS5-branes give the $\mathrm{SU}(3)_F$ gauge group; three D6-branes provide $N_f = 3$ flavours. The relative separation of the NS5-branes sets $1/g_F^2$; the D4-D6 open strings are the preons $Q_i, \tilde{Q}_i$. After s-confinement ($N_f = N_c$): no magnetic D4-branes survive; the IR theory is described entirely by D6–D6 strings (mesons) and wrapped D4-branes (baryons).

Quarks are open strings between the colour-D4$_c$ stack and the D6 flavour branes. Leptons are D6–D6 strings. In M-theory both lift to M5-branes, and the distinction between "elementary" and "composite" becomes geometric: which sheet of the M5 carries the M2-brane boundary.

The Seiberg–Witten curve (**??**) *is* the M5 shape. The Cartan angle δ is the M5 position on the M-theory circle. The Cabibbo angle equals δ because both are set by the same M5 geometry.

The seed angle $\pi/12$ is the angular quantum connecting the charged-lepton and neutrino sectors: $\delta_\nu = \delta + \pi/12 = 1/8$ of the $\mathbb{Z}_3$ period on the Cartan circle.

In this picture, the distinction between "elementary" and "composite" dissolves: a quark is an open string between two brane stacks, while a lepton is an open string between two sheets of the *same* brane stack (the flavour D6s). Both are equally fundamental—or equally composite—depending on which brane coordinates one chooses as "position" versus "field." The sBootstrap is the statement that this M-theory democracy extends to the effective field theory: the supercharge Q that connects mesons to leptons and diquarks to quarks is the low-energy remnant of the open-string vertex operator that changes the endpoint boundary conditions from D6–D6 (meson) to D4$_c$–D6 (quark).

The diquark sector maps to D4$_c$–D4$_c$ strings in the colour-antisymmetric channel. The charge-4/3 diquark $[uc]$ corresponds to a string stretching between D4$_c$ branes with both endpoints carrying up-type quantum numbers; its problematic charge reflects the fact that the SO(32) embedding requires the full type I string spectrum, of which the low-energy QCD composites are only a subset.

The NS5-brane rotation that effects $\mathcal{N}$=2 $\to$ $\mathcal{N}$=1 (the adjoint mass μ_Φ of Sec. ??) has a simple geometric interpretation: the relative angle between the two NS5-branes is $\mu_\Phi L$, where L is the separation in the compact direction. At $\mu_\Phi = 11\, m_\tau/2 \approx 9.8\,\mathrm{GeV}$, the rotation angle is small ($\sim 10^{-1}$ rad in appropriate units), consistent with the N=2 structure being "almost" preserved in the confined phase—which is precisely what the Koide formula requires.

The Hanany–Witten construction also provides a brane-theoretic interpretation of the Chadha–Nielsen obstruction. A local composite supercharge corresponds to a vertex operator localised at a point on the D6-brane worldvolume. The factor-of-18 overshoot arises because the vertex operator probe is sensitive to the full area of the D6-brane, which scales as N_c^3 in the large-N_c limit. The holographic evasion (Sec. ??) localises the supercharge wavefunction near the IR wall z_{IR}, where the warp factor suppresses the anticommutator by exactly the factor needed to close the algebra.

X. COMPLETE PREDICTIONS

Not predicted: CKM and PMNS CP phases (two parameters), Majorana phases (two more), m_u (isospin splitting, outside the family-symmetry framework), gauge couplings g_1, g_2, g_3 (part of the elementary $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ sector), Higgs self-coupling λ, θ_{QCD} (strong CP), and the neutrino Koide angle δ_ν (fitted to $\delta + \pi/12$ but not derived).

The statistical assessment, treating $|V_{us}|$ as a single input (determining δ), gives $\chi^2/\mathrm{dof} = 6.2/5 = 1.25$ ($p = 0.28$) for the remaining five mixing predictions. The pulls are $|V_{cb}|$ (-2.3σ), A (-0.6σ), $\sin^2\theta_{12}$ (-0.3σ), $\sin^2\theta_{23}$ ($+0.04\sigma$), $\sin^2\theta_{13}$ (-0.6σ). The fit is acceptable.

With $\delta = 2/9$ used without adjustment, $|V_{us}|$ contributes -4.1σ, reflecting the 1.2% discrepancy between $2/9 = 0.2222$ and the PDG value 0.2250. This tension may be resolved if the angular potential (??) shifts the minimum to $\delta = 0.2225$.

TABLE IV. Predictions from $N = 3$, $\mu = 17.72\,\sqrt{\mathrm{MeV}}$, $m_c = 1270\,\mathrm{MeV}$. Mean off: 1.1%.

Parameter	Formula	Off
$m_e,\ m_\mu,\ m_\tau$	Koide + μ + δ	$< 0.01\%$
$\lvert V_{us}\rvert$	2/9	1.2%
$\lvert V_{cb}\rvert$	4/99	4.3%
A	9/11	1.0%
$\sin^2\theta_{12}$	$\sin^2(\pi/4 - \delta + \delta^2/N)$	1.3%
$\sin^2\theta_{23}$	$\sin^2(\pi/4 + \delta/N)$	0.1%
$\sin^2\theta_{13}$	$\sin^2(4/27)$	1.9%
m_d/m_s	4/81	1.2%
m_s	91.6 MeV	1.9%
m_b	4141 MeV	1.0%
m_t	171.1 GeV	0.8%
$m_\pi^2 - m_\mu^2$	f_π^2	2.2%
m_τ	$f_\pi m_b/\Lambda_{\mathrm{QCD}}$	0.2%
$K(-\pi, D_s, B)$	2/3	0.1%

a. Parameter reduction. The Standard Model has 19 free parameters with massless neutrinos (or 28 with Dirac neutrino masses and PMNS phases). The sBootstrap predicts 15 of these from 3 inputs ($N = 3$, μ, m_c), reducing the free count to 13. The $15/28 = 54\%$ reduction is the main quantitative achievement of the framework.

XI. DISCUSSION

a. The parameter reduction. The sBootstrap predicts 15 of the 28 free parameters of the Standard Model from 3 inputs (N, μ, m_c), a 54% reduction. No GUT achieves this: SU(5) predicts ~ 2 mass relations ($m_b = m_\tau$ at M_{GUT}, plus gauge unification); SO(10) adds ~ 2 more. String compactifications on Calabi–Yau threefolds have $O(100)$ moduli and predict zero mass values. The sBootstrap achieves quantitative predictions because it operates at the QCD scale, where the dynamics is controlled by a single confinement parameter Λ_F and the mass hierarchy by a single angle δ.

b. Relation to the MSSM. The sBootstrap is not the MSSM. In the MSSM, squarks and sleptons are new particles at the TeV scale; in the sBootstrap, they are the mesons and diquarks already observed in QCD. The MSSM has ~ 120 free parameters (soft-breaking masses, A-terms, μ-parameter); the sBootstrap has three (N, μ, m_c). The two share the same algebraic structure—chiral superfields with the same gauge quantum numbers—but differ fundamentally in the breaking mechanism. MSSM breaking is soft (gravity- or gauge-mediated, at TeV); sBootstrap breaking is strong (Volkov–Akulov, at f_π).

The absence of TeV-scale superpartners at the LHC is therefore not a problem for the sBootstrap: the superpartners are at the QCD scale, where they have been measured for decades as hadrons.

c. Relation to Masiero–Veneziano. Masiero and Veneziano [?] constructed the first prototype model for one leptonic family from SU(3) SQCD with $N_f = N_c = 3$. Their model has the same gauge group and matter content as ours but differs in the treatment of SUSY breaking (they use generic soft terms, we use Volkov–Akulov), the electroweak gauging (they gauge $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as a subgroup of the flavour group), and crucially the absence of the Koide formula (they have no mass predictions). We extend their framework by adding the charge-squared rule, the Cartan angle, and the bion relations.

d. The electron mass. The electron mass vanishes at leading order in the Yukawa ansatz (**??**): $y^e \sim 0$. This is a feature, not a bug: the tiny electron mass ($m_e/m_\mu = 1/207$) is naturally explained as a higher-order effect. The determinant $\det(m_\ell) \propto m_e\, m_\mu\, m_\tau$ is protected by an approximate $\mathrm{U}(1)_e$ symmetry (the electron number); its smallness does not require fine-tuning. The electron mass could arise from electroweak loops mixing the (e, μ, τ) sector, or from sub-leading Yukawa couplings $y^e_{cd} \sim g^2/16\pi^2$ at one loop.

e. The charge-4/3 problem. The single diquark $[uc]$ with charge 4/3 has no Standard Model fermion partner. In the SO(32) embedding [?], it is part of the **442** states in the adjoint decomposition $\mathbf{496} = \mathbf{54} \oplus \mathbf{442}$. These extra states must be projected out by the UV completion. In the M-theory picture (Sec. **??**), the charge-4/3 diquark is a $\mathrm{D4}_c$–$\mathrm{D4}_c$ string that can form a neutral condensate [?], removing it from the low-energy spectrum.

f. Why $\sqrt{m}$ is primary. The charge-squared rule $m = \mu\, z^2$ makes $\sqrt{m}$ the primary variable. This is not merely a parametric convenience. Karasik [?] showed that the baryon current in QCD is most naturally written in terms of the "square root" ξ of the chiral field ($\xi^2 = U$), not in terms of U itself. The standard skyrmion current B^μ (built from U) and Karasik's current H^μ (built from ξ) agree for smooth configurations but differ for singular ones (the $N_f = 1$ baryon). In our framework, the same ξ-structure appears in the Koide parametrisation: $\sqrt{m_n} = \mu\, z_n$ is the family charge in the "square root space" of the meson matrix.

XII. EXPERIMENTAL TESTS

a. JUNO: the critical test Our $\sin^2\theta_{12} = 0.300$ is 1.3% below the NuFIT central value 0.304. JUNO will measure this to ± 0.003, testing our prediction at $\sim 1.3\sigma$.

b. Lattice: the binary verdict. The Chadha–Nielsen calculation [? ?] gives an explicit numerical target. Using the composite supercharge ansatz $Q_\alpha = \int d^3z\, K(z)\, \epsilon^{abc} \bar{q}_a \sigma^\mu_{\alpha\dot{\beta}} q^{\dot{\beta}}_b \partial_\mu \phi_{\pi,c}$ with form factor $K(z) \sim \Lambda_{\mathrm{QCD}}^{-3/2}$ and the QCD condensate $\langle \bar{q}q \rangle = -B_0 f_\pi^2$:

$$\frac{\{Q, \bar{Q}\}_{\mathrm{local}}}{2\sigma^0 m_\pi} \sim N_c \left(\frac{f_\pi^2}{\Lambda_{\mathrm{QCD}}^2} \right)^2 \sim N_c^3 \approx 27\,. \tag{34}$$

With the $O(1)$ factor $m_\pi/\Lambda_{\mathrm{QCD}} \approx 0.64$, the numerical result is ~ 18. A nonlocal supercharge must suppress this by exactly 1/18 to close the algebra.

Five lattice observables, each requiring $\sim 10^7$ core-hours, provide a binary verdict. The PCSC residue should satisfy $Z_S = 1.00 \pm 0.05$; the auxiliary-field VEV should give $\langle F_\pi \rangle / (f_\pi^2 \Lambda_{\mathrm{QCD}}) = -1.0 \pm 0.2$ from the four-quark condensate; the restored-phase muon mass should vanish, $m_\mu(T > T_c) < 20\,\mathrm{MeV}$; the APE-smeared anticommutator should close, $\Delta(t)/(2m_\pi) = 1.0 \pm 0.2$; and a single holographic IR cutoff z_{IR} should simultaneously reproduce $m_\rho = 775\,\mathrm{MeV}$ and $m_{a_1} = 1230\,\mathrm{MeV}$. Agreement within 5% on any of these establishes nonlocal hadronic SUSY; a 3σ deviation on the anticommutator falsifies all nonlocal evasions. A realistic timeline is 36–48 months.

c. $|V_{cb}|$ exclusive/inclusive Our $4/99 = 0.0404$ lies between the exclusive (0.0395) and inclusive (0.0422) PDG determinations, 1.1σ from the exclusive. Resolution of this long-standing tension will sharpen the test.

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