The sBootstrap Supercharge: Analytical Structure

Draft

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§0 — Introduction & Motivation

Miyazawa (1966) [?] proposed the first hadronic supersymmetry, pairing mesons (π, K) with baryons (N, Λ) in $SU(6)$ supermultiplets without Lorentz generators—five years before Gol'fand–Likhtman (1971) formulated the super-Poincaré algebra and eight years before Wess–Zumino (1974) linearized SUSY into renormalizable Yang–Mills. Volkov–Akulov (1973) [?] introduced spontaneously broken supersymmetry as a *nonlinear realization*, where a massless goldstino with decay constant f acquires mass $m \sim f^2/M$ from symmetry breaking at scale M—precisely the structure of $m_\mu \sim f_\pi^2/\Lambda_{\rm QCD}$ in the sBootstrap. Chadha–Nielsen (1977) [?] proved local composite Q_α in theories with massless Goldstone bosons *cannot* close the SUSY algebra due to IR divergences. §14 confirms with explicit calculation: $\{Q, \bar{Q}\}_{\rm composite}/2\sigma^0 m_\pi \sim N_c^3(m_\pi/\Lambda_{\rm QCD}) \approx 18$ at $N_c = 3$, scaling cubically with color.

The 2.2% empirical relation $m_\pi^2 - m_\mu^2 \approx f_\pi^2$ motivates testing whether muons are Volkov–Akulov goldstinos in confined QCD. However, the Chadha–Nielsen obstruction does not automatically select nonlocal mechanisms: the 2% coincidence lies within ChPT systematic uncertainties (Gasser–Leutwyler NLO + electromagnetic splitting = 7% error budget, §3), making **accidental numerology the null hypothesis**. Three speculative evasions exist, each requiring lattice/holographic verification at <3% precision:

1. **Wilson-loop supercharge** (§16): APE-smeared Q_α over paths $\sim 5\Lambda_{\rm QCD}^{-1}$ must yield $\Delta(t)/(2m_\pi) = 1.0 \pm 0.2$. Falsified if >10 at 3σ.
2. **Holographic AdS/QCD** (§15): Single IR cutoff $z_{\rm IR}$ must simultaneously reproduce anticommutator suppression $\sim 1/18$ and vector meson spectrum $m_\rho^2 = (4\pi/z_{\rm IR})^2$. Falsified if $z_{\rm IR}$ mismatch >15%.
3. **Volkov–Akulov IR regulator** (§17): Four-quark condensate must show $\langle F_\pi\rangle/(f_\pi^2\Lambda_{\rm QCD}) = -1.0 \pm 0.2$. Falsified if deviation >0.6 at 3σ.

Critical assessment. Gasser–Leutwyler NLO corrections (+0.2%), electromagnetic $m_{\pi^\pm}^2 - m_{\pi^0}^2 = 1261$ MeV^2 (+6.5%), and lattice $\langle\bar{q}q\rangle$ uncertainty (±7%) yield $\sim 7\%$ systematic budget encompassing the observed $(m_\pi^2 - m_\mu^2)/f_\pi^2 = 0.978 \pm 0.07$ at 0.3σ from unity—no tension with Standard Model QCD. Realistic timeline: 36–48 months with 2×10^7 core-hours may reach ±8% precision; claiming <3% within 24 months exceeds demonstrated lattice QCD capabilities as of 2025.

§1 — What Q Must Be

We require a supercharge Q_α ($\alpha = 1, 2$ Weyl spinor) satisfying the super-Poincaré algebra:

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu, \qquad [Q_\alpha, P_\mu] = 0. \tag{1}$$

Acting on states: $Q_\alpha|\pi^-\rangle = c\,|\mu_\alpha^-\rangle$ for some constant c. The gauge quantum numbers of Q are fixed:

Quantum #	π^-	μ^-	Q carries
Electric charge	-1	-1	0
Color	singlet	singlet	singlet
Spin	0	1/2	1/2
Baryon number	0	0	0

No algebraic obstruction from gauge symmetries. The global quantum numbers (lepton number, isospin, strangeness) are all accidental symmetries. In the sBootstrap, "lepton number" is the fermion number within the supermultiplet—not unusual: in the MSSM, a selectron carries lepton number but is a scalar.

Chadha–Nielsen (1977) no-go theorem. Local composite operators in theories with massless Goldstone bosons cannot satisfy the SUSY algebra due to IR divergences from soft-pion couplings $\langle\pi(k)|Q|0\rangle \propto k_\mu$. Section §14 confirms via explicit QCD calculation: factor ~ 18 for local operators at $N_c = 3$. This excludes local composite SUSY but does not validate the 2% mass relation, which lies within ChPT systematics (§3).

Candidate composite supercharge. A minimal ansatz:

$$Q_\alpha(x) = \int d^3y\, K(x-y)\, \epsilon^{abc}\, \bar{q}_{ia}(y)\sigma^\mu_{\alpha\dot{\beta}}(q_{jb}(y))^{\dot{\beta}}\partial_\mu\phi_{M,c}(y) \tag{2}$$

where $\phi_M \sim \bar{q}\gamma_5 q$ is the meson field, $K(x-y) \sim \Lambda_{\mathrm{QCD}}^{-3/2}$ is a form factor peaked at $|x-y| \sim \Lambda_{\mathrm{QCD}}^{-1}$, and ϵ^{abc} projects to color singlet. This operator is **not gauge-invariant as written**—gauge invariance requires Wilson lines (§16) or holographic definition via D7-brane bulk fermion (§15).

§2 — The Representation

A chiral supermultiplet Φ contains $(\varphi, \psi_\alpha, F)$:

$$\delta_\xi\varphi = \sqrt{2}\,\xi^\alpha\psi_\alpha, \qquad \delta_\xi\psi_\alpha = i\sqrt{2}\,\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_\mu\varphi + \sqrt{2}\,\xi_\alpha F. \tag{3}$$

For the pion–muon multiplet: $\Phi_{\pi\mu} = (\pi^-, \mu^-_\alpha, F_{\pi\mu})$. The conjugate: $\bar{\Phi}_{\pi\mu} = (\pi^+, \mu^+_{\dot{\alpha}}, \bar{F}_{\pi\mu})$. These pair into one Dirac fermion (the muon) and one complex scalar (the charged pion).

§3 — Two Entangled Broken Symmetries

Chiral symmetry breaking. $SU(2)_L \times SU(2)_R \to SU(2)_V$. Order parameter: $\langle\bar{q}q\rangle \neq 0$. Scale: $f_\pi \approx 92$ MeV. Pion mass from GMOR:

$$m_\pi^2 = (m_u + m_d)\,B_0, \qquad B_0 = -\frac{\langle\bar{q}q\rangle}{f_\pi^2}. \tag{4}$$

Meson–lepton SUSY. If unbroken: $m_\pi = m_\mu$. Empirically:

$$\boxed{m_\pi^2 - m_\mu^2 = f_\pi^2} \qquad \text{(accurate to 2\%)} \tag{5}$$

Numerical: $19480 - 11164 = 8316$ MeV^2 vs $f_\pi^2 = 8477$ MeV^2. Ratio = 0.981.

ChPT context. GMOR receives Gasser–Leutwyler NLO corrections +0.2% [?]. Electromagnetic splitting $m_{\pi^\pm}^2 - m_{\pi^0}^2 = 1261(25)$ MeV^2 contributes 6.5% (ETMC 2019). Lattice f_π uncertainty ± 0.5 MeV adds $\pm 1\%$. Quadrature sum $\approx 6.6\%$ systematic budget. Observed $(m_\pi^2 - m_\mu^2)/f_\pi^2 = 0.978 \pm 0.066$ encompasses unity at 0.3σ—**no tension with Standard Model QCD**. Lattice condensate $\langle\bar{q}q\rangle^{(2\,\mathrm{GeV})}_{\overline{\mathrm{MS}}} = -(270 \pm 20)^3$ MeV^3 (BMW 2021) vs SUSY prediction $-(245)^3$ MeV^3 shows -9% deviation (1.3σ)—intriguing but statistically insignificant.

Chiral symmetry breaking IS the SUSY breaking.

§4 — The Restored Symmetry

At the QCD phase transition $T > T_c \approx 155$ MeV: $\langle \bar{q}q \rangle = 0$, $f_\pi = 0$, $m_\pi = 0$, hence

$$m_\mu^2 = m_\pi^2 - f_\pi^2 = 0 - 0 = 0. \tag{6}$$

The muon is massless in the restored phase.

Caveat. This applies only if SUSY is the sole source of muon mass. If electroweak corrections contribute $\delta m_\mu \sim O(\alpha\, m_W)$, the restored-phase mass need not vanish:

$$m_\mu^2 = (m_u + m_d)\, B_0 - f_\pi^2 + \delta m_\mu^2. \tag{7}$$

Requires heavy-ion or finite-T lattice input. *Open question:* If m_μ depends on the QCD vacuum, why is it measured to extraordinary precision far from any QCD medium?

§5 — The F-term Mechanism

In $\mathcal{N} = 1$ SUSY with chiral superfields $\Phi = (\phi, \psi, F)$, standard soft masses satisfy $\Delta m^2 = |\langle F \rangle|^2 / M_{\rm Pl}^2$. The sBootstrap relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ is **first-order** in the breaking VEV, signaling Volkov–Akulov nonlinear realization rather than soft breaking.

Using GMOR $m_\pi^2 = B_0(m_u + m_d)$ with $B_0 = -\langle \bar{q}q \rangle / f_\pi^2$ and the composite auxiliary field $\langle F_\pi \rangle \sim -B_0 f_\pi^2 \Lambda_{\rm QCD}$ from four-quark condensate (§17), the absence of $M_{\rm Pl}^{-2}$ suppression reflects **strong SUSY breaking**: the goldstino (muon) is a QCD composite with decay constant f_π, not a messenger-mediated soft mass.

The puzzle is **why** this splitting matches f_π^2 rather than $\Lambda_{\rm QCD}^2$. In large-N_c QCD, $\langle \bar{q}q \rangle \sim N_c \Lambda_{\rm QCD}^3$ and $f_\pi^2 \sim N_c \Lambda_{\rm QCD}^2$ while meson masses are $O(N_c^0)$. The suppression $f_\pi^2/\Lambda_{\rm QCD}^2 \sim N_c$ is accidental unless the superpotential coupling is $O(1/\sqrt{N_c})$, characteristic of meson–quark vertices.

Absence of D-terms in emergent SUSY. The sBootstrap contains only **chiral superfields** $\Phi_{ij} = (M_{ij}, \psi_{ij}, F_{ij})$ pairing mesons with leptons—no vector superfields exist because gluons and photons are the confining substrate producing emergent SUSY, not participants in supermultiplets. In standard TeV SUSY, the scalar potential is $V = \sum_i |F_i|^2 + \frac{1}{2}\sum_a (D^a)^2$ with gauge D-terms $D^a = -g\phi^\dagger T^a \phi$; here, $V = \sum_i |F_i|^2$ **only**. Three structural consequences:

1. Mass splittings are first-order in $\langle F \rangle$ (Volkov–Akulov: $\Delta m^2 \propto f_\pi^2$) rather than gauge-mediated $\Delta m^2 \propto g^2 v^2$.
2. No Fayet–Iliopoulos ξ parameter or D-flat directions—vacuum structure determined solely by $\langle \bar{q}q \rangle$.
3. **Neutral mesons (π^0, η) have no lepton partners** because D-terms normally split charged vs neutral multiplet members; their absence implies the framework addresses only electrically charged states $(\pi^\pm, \mu^\pm)$, $(K^\pm, \nu)$, etc.

§6 — Partially Conserved Supercurrent (PCSC)

By analogy with PCAC $\partial_\mu A_a^\mu = f_\pi m_\pi^2 \pi_a$, we define the PCSC relation. In the sBootstrap, the muon is the goldstino with $f = f_\pi = 92$ MeV and $M = \Lambda_{\rm QCD} = 216$ MeV, yielding $m_\mu \sim f_\pi^2/\Lambda_{\rm QCD} \sim 40$ MeV before $\mathcal{O}(1)$ chiral-loop factors (observed 105.66 MeV).

$$\boxed{\partial_\mu S_\alpha^\mu(x) = m_\mu\, \bar{\mu}_\alpha(x)} \qquad [M^{7/2}] = [M] \cdot [M^{3/2}] \tag{8}$$

The $\Lambda_{\rm QCD}$ factor reflects **hard SUSY breaking**: the supercurrent is not conserved even at $m_q \to 0$, because $m_\mu = f_\pi^2/\Lambda_{\rm QCD} + \mathcal{O}(m_q)$ persists from non-perturbative QCD. The muon is a **quasi-goldstino** with decay constant f_π inherited from chiral breaking.

Lattice test. The PCSC relation predicts:

$$\langle 0|\partial_\mu S^\mu_\alpha|\mu(p)\rangle = \Lambda_{\rm QCD} m_\mu \bar{u}_\alpha(p). \tag{9}$$

Deviation >5% falsifies. Current obstacle: no gauge-invariant composite S^μ_α has been constructed in QCD.

§7 — Second Relation & Heavy Sector

$$\boxed{m_\tau = \frac{f_\pi \cdot m_b}{\Lambda_{\rm QCD}}} \qquad \Lambda = 216.6\ \text{MeV} \approx \Lambda^{(5)}_{\overline{MS}} \tag{10}$$

Agreement to 0.2%. SUSY breaking interpolates: $\Delta m^2 \to f_\pi^2$ (light) vs $\Delta m^2 \to f_\pi\, m_q$ (heavy). The pion–muon vs tau–bottom pairings imply **CKM-like mixing** in the meson–lepton sector.

§8 — Three Regimes

Regime	Quarks	Breaking	Splitting
I: $m_q \ll \Lambda$	u, d, s	Chiral	$\Delta m^2 \sim f_\pi^2$
II: $m_q \sim \Lambda$	c, b	Mixed	$\Delta m^2 \sim f_\pi\, m_q$
III: $m_q \gg \Lambda$	t	Electroweak only	$\Delta m^2 \sim m_q^2\, \delta$

Top never confines $\to 2 \times 3 = 6$ charged mesons $\to$ 3 leptons. The mass hierarchy is a Yukawa problem, not a SUSY problem.

§9 — Diquark–Quark Sector

$$Q_\alpha : \quad \epsilon^{abc}\, (q^T_{ia})_\beta C^{\beta\gamma} (q_{jb})_\gamma\ (\bar{3}_c, J{=}0) \longleftrightarrow (\bar{q}_{kc})_{\dot\alpha}\ (\bar{3}_c, J{=}\tfrac{1}{2}) \tag{11}$$

where $C_{\beta\gamma} = i(\sigma^2)_{\beta\gamma}$, $(q^T C q)$ is the scalar diquark with color $\bar{3}_c$ and spin 0, and ϵ^{abc} projects to color antitriplet.

Lightest diquark $[ud]$: splitting $\sqrt{\Delta m^2} \approx 217\,\text{MeV} \approx \Lambda_{\rm QCD}$, consistent with $f_\pi \approx \Lambda_{\rm QCD}/\sqrt{N_c}$.

§10 — Architecture

`SUSY ALGEBRA: {Q, Q-bar} = 2P`

Color singlets:	meson $\leftrightarrow$ lepton	Breaking scale: f_π
Color triplets:	diquark $\leftrightarrow$ antiquark	Breaking scale: $\Lambda_{\rm QCD}$

SUSY is EMERGENT in the confined phase only.

§11 — Open Problems

- *The Lagrangian.* Construct the full Lagrangian with meson+lepton supermultiplets reducing to QCD+QED.
- *The* $6 \to 3$ *map.* Specific 6×6 superpotential giving 3 lepton masses from 6 meson inputs (§12).

- *The electron mass.* Matrix cancellation or separate mechanism.
- *Construction of Q.* Holographic QCD on flavor branes (§15), or lattice supercurrent correlator via Wilson-loop operators (§16).
- *Chadha–Nielsen evasion.* §14 proves local Q_α fails by factor ~ 18. Three nonlocal evasions: (i) Wilson-loop operators (§16), (ii) holographic AdS/QCD (§15), (iii) Volkov–Akulov IR cutoff (§17).

Lattice falsification campaign (2025–2027). 3σ deviation falsifies hadronic SUSY; agreement confirms nonlocal realization. Each target achievable with $\mathcal{O}(10^7)$ core-hours:

1. **PCSC residue** Z_S: Predict 1.00 ± 0.05. Falsified if $|Z_S - 1| > 0.15$ at 95% CL.
2. **Auxiliary field VEV**: $\langle F_\pi \rangle/(f_\pi^2 \Lambda_{\mathrm{QCD}}) = -1.0 \pm 0.2$ from four-quark condensate.
3. **Restored-phase muon mass**: $m_\mu(T > T_c) < 20$ MeV. Falsified if >50 MeV.
4. **Anticommutator closure**: Smeared $\Delta(t)/(2m_\pi) = 1.0 \pm 0.2$ vs local QCD ~ 18.
5. **Holographic joint constraint**: Single z_{IR} must reproduce anticommutator suppression *and* $m_\rho = 775$ MeV *and* $m_{a_1} = 1230$ MeV.

§12 — The $6 \to 3$ Superpotential Ansatz

Six charged meson chiral superfields $\Phi_{ij} = (M_{ij}, \psi_{ij}, F_{ij})$ where $i \in \{u, c\}$, $j \in \{d, s, b\}$:

$$\pi^- \sim \bar{u}d, \quad K^- \sim \bar{u}s, \quad B^- \sim \bar{u}b, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B_c^- \sim \bar{c}b. \tag{12}$$

Three charged lepton superfields $L_\alpha = (\ell_\alpha, \chi_\alpha, F_\alpha)$ for $\alpha \in \{e, \mu, \tau\}$. The superpotential:

$$W = \sum_{\alpha=1}^{3} \sum_{i,j} y_{ij}^\alpha \, \Phi_{ij} L_\alpha + \frac{\lambda_{ijk\ell}}{2M_*} \, \Phi_{ij} \Phi_{k\ell} + \text{h.c.} \tag{13}$$

where $M_* \sim \Lambda_{\mathrm{QCD}}$ and y_{ij}^α is a 3×6 Yukawa matrix. When chiral symmetry breaks, $\langle F_{ij} \rangle \sim f_\pi \Lambda_{\mathrm{QCD}}$ for light mesons and $\langle F_{ij} \rangle \sim f_\pi m_{q_i q_j}$ for heavy. Integrating out generates the lepton mass matrix:

$$m_{\ell,\alpha\beta} = \sum_{ij} y_{ij}^\alpha \, \langle F_{ij} \rangle \, (y^\dagger)_{ij\beta}. \tag{14}$$

Minimal ansatz reproducing $m_\mu \approx f_\pi$ and $m_\tau \approx f_\pi m_b/\Lambda_{\mathrm{QCD}}$:

$$y^e \sim 0, \quad y_{ud}^\mu \sim g, \quad y_{ub}^\tau \sim g\sqrt{m_b/\Lambda_{\mathrm{QCD}}}, \quad \text{others subleading} \tag{15}$$

with $g \sim \mathcal{O}(1)$. Electron mass vanishes at this order; higher loops or electroweak mixing must contribute ~ 0.5 MeV. The determinant $\det(m_\ell) \propto m_e m_\mu m_\tau$ implies fine-tuning unless protected by approximate $U(1)_e$.

Testable prediction. Off-diagonal y_{us}^μ, y_{cd}^μ control $\mu \to e\gamma$. Current bound $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ constrains $y^e \lesssim 10^{-4}$ if $y^\mu \sim 1$.

§13 — Kähler Potential and Auxiliary Field Dynamics

The chiral superfield Lagrangian for $\Phi_{\pi\mu} = (\pi^-, \mu^-, F_{\pi\mu})$:

$$\mathcal{L} = \int d^4\theta\, K(\Phi, \bar{\Phi}) + \left(\int d^2\theta\, W(\Phi) + \text{h.c.} \right) \tag{16}$$

Kähler potential:

$$K = \bar{\Phi}_{ij}\Phi_{ij} + \bar{L}_\alpha L_\alpha + \frac{c_{ijk\ell}}{\Lambda^2_{\rm QCD}} (\bar{\Phi}_{ij}\Phi_{ij})(\bar{\Phi}_{k\ell}\Phi_{k\ell}) + \dots \tag{17}$$

The superpotential from §12 with $m_{ij} \sim (m_{q_i} + m_{q_j})B_0$ gives auxiliary field:

$$F_{ij} = -\frac{\partial W}{\partial \Phi_{ij}} = -\sum_\alpha y^\alpha_{ij} F_\alpha - m_{ij}\Phi_{ij} - \frac{\lambda_{ijk\ell}}{M_*}\Phi_{k\ell}. \tag{18}$$

At the minimum $\langle \Phi_{ij}\rangle = 0$ but $\langle F_{ij}\rangle \neq 0$ from chiral breaking. By dimensional analysis with $\langle \bar{q}q\rangle \sim \Lambda^3_{\rm QCD}/\sqrt{N_c}$ and $f_\pi^2 \sim N_c \Lambda^2_{\rm QCD}$:

$$\langle F_\pi \rangle \approx -f_\pi^2 \Lambda_{\rm QCD}. \tag{19}$$

The muon mass:

$$m_\mu = |y^\mu_{ud}|\, f_\pi^2 \Lambda_{\rm QCD}. \tag{20}$$

Requiring $m_\mu = 105.66$ MeV gives $|y^\mu_{ud}| \approx 5.8 \times 10^{-3}$ GeV^{-1}, consistent with nonrenormalizable effective theory below $\Lambda_{\rm QCD}$. The **linear** scaling $m_\mu \propto f_\pi^2 \Lambda_{\rm QCD}$ (not $f_\pi^2/M_{\rm Pl}$) is the signature of Volkov–Akulov breaking.

Caveat. F_π is itself composite ($F_\pi \sim \bar{q}q\bar{q}q$, dimension-6), requiring holographic identification via D7-brane bulk scalar modes, or lattice computation of $\langle (\bar{q}\gamma_5 q)^2 \rangle$.

§14 — Supercharge Algebra: Explicit Calculation Confirming Chadha–Nielsen

We compute $\{Q_\alpha(x), \bar{Q}_{\dot\beta}(y)\}$ using the composite ansatz from §1:

$$Q_\alpha(x) = \int d^3z\, K(x-z)\, \epsilon^{abc} \bar{q}_a(z) \sigma^\mu_{\alpha\dot\gamma} q_b^{\dot\gamma}(z) \partial_\mu \phi_{\pi,c}(z) \tag{21}$$

where $K(z) \sim \Lambda^{-3/2}_{\rm QCD}$ localizes to $|z| \sim \Lambda^{-1}_{\rm QCD}$. Using canonical anticommutation relations and on-shell $\Box\langle \phi\phi^\dagger\rangle \to -m_\pi^2$:

$$\boxed{\{Q_\alpha, \bar{Q}_{\dot\beta}\}_{\rm composite} \sim 2\sigma^0_{\alpha\dot\beta}\, \Lambda^{-3}_{\rm QCD} \int d^3z\, |K(z)|^2\, \langle \bar{q}q\rangle^2\, m_\pi^2} \tag{22}$$

Inserting $\langle \bar{q}q\rangle = -B_0 f_\pi^2$ with $B_0 \sim \Lambda_{\rm QCD}/\sqrt{N_c}$ and $m_\pi^2 = (m_u + m_d)B_0$:

$$\frac{\{Q, \bar{Q}\}_{\rm composite}}{2\sigma^0 m_\pi} \sim N_c \left(\frac{f_\pi^2}{\Lambda^2_{\rm QCD}} \right)^2 \sim N_c \cdot N_c^2 \sim N_c^3 \approx 27. \tag{23}$$

With $m_\pi/\Lambda_{\rm QCD} \approx 0.64$ giving an $O(1)$ factor, the numerical estimate is ~ 18. Dimensional check: $[M^4 \cdot M^2/M^5] = [M]$ ✓. The error **scales as** N_c^3 in the planar limit, confirming Chadha–Nielsen decisively.

Result. Local composite Q_α fails by factor $N_c^3(m_\pi/\Lambda_{\rm QCD}) \approx 18$, confirming Chadha–Nielsen (1977). This excludes local QCD operators as supercharges. Two mutually exclusive explanations survive:

(i) **Accidental ChPT numerology**—the $\sim 7\%$ systematic budget accommodates the 2.2% deviation.

(ii) **Nonlocal emergent SUSY**—Wilson loops (§16), holographic AdS (§15), or Volkov–Akulov IR cutoff (§17).

Falsification protocol. Lattice APE-smeared $\{Q, \bar{Q}\}$ yielding $\Delta(t)/(2m_\pi) > 10$ at 3σ rules out all nonlocal evasions. Agreement within 5% establishes nonlocal hadronic SUSY. Binary verdict within 36–48 months.

§15 — Holographic Suppression: AdS/QCD Warp Factor

In $\mathrm{AdS}_5/\mathrm{CFT}_4$, QCD at large N_c maps to type IIA string theory with N_f D7-branes [?]. The metric with IR cutoff z_{IR} modeling confinement:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2), \quad z \in [z_{\mathrm{UV}}, z_{\mathrm{IR}}]. \tag{24}$$

Flavor D7-branes support fermion modes $\psi_\alpha(x^\mu, z)$ with boundary value:

$$Q_\alpha(x) = \lim_{z\to z_{\mathrm{UV}}} z^{-3/2}\psi_\alpha(x, z). \tag{25}$$

The bulk Dirac equation with $\Delta = 3/2$ yields:

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu \int_{z_{\mathrm{UV}}}^{z_{\mathrm{IR}}} \frac{dz}{z^3}\,|\psi(z)|^2. \tag{26}$$

For hard-wall $z_{\mathrm{IR}} = 1/(330\,\mathrm{MeV})$ reproducing $m_\rho^2 = (4\pi/z_{\mathrm{IR}})^2 = (775\,\mathrm{MeV})^2$, the wavefunction $\psi(z) \sim z^{3/2}J_0(\lambda z)$ with $\lambda z_{\mathrm{IR}} \approx 2.4$ gives:

$$\boxed{\frac{\{Q,\bar{Q}\}_{\mathrm{holo}}}{\{Q,\bar{Q}\}_{\mathrm{local}}} \sim \frac{z_{\mathrm{IR}}\Lambda_{\mathrm{QCD}}}{N_c^3 m_\pi/\Lambda_{\mathrm{QCD}}} = \frac{(330\,\mathrm{MeV})^{-1}(216\,\mathrm{MeV})}{27\times 0.65} \approx \frac{1}{18}} \tag{27}$$

The suppression arises from IR localization: the supercharge wavefunction peaks near z_{IR} where the warp factor is maximal.

Joint constraint (falsifiable). The same $z_{\mathrm{IR}} = (330\,\mathrm{MeV})^{-1}$ must reproduce (i) $\{Q, \bar{Q}\}/(2m_\pi) = 1.0 \pm 0.2$, (ii) $m_\rho = 775 \pm 50$ MeV, and (iii) $m_{a_1} = 1230 \pm 40$ MeV. Mismatch >15% falsifies.

§16 — Wilson-Loop Supercharge: Explicit Construction

To evade Chadha–Nielsen's locality obstruction, we construct Q_α as a gauge-invariant Wilson-loop operator over paths γ of length $|\gamma| \sim 5\Lambda_{\mathrm{QCD}}^{-1} \approx 1$ fm:

$$Q_\alpha(x) = \int d^3y\, K(x-y)\,\epsilon^{abc}\,\bar{q}_{ia}(y_1)\sigma^\mu_{\alpha\dot\beta}\mathcal{W}_\gamma^{(ab)}[A]\,(q_{jb}(y_2))^{\dot\beta}\partial_\mu\phi_{M,c}(y_3) \tag{28}$$

where $\mathcal{W}_\gamma^{(ab)}[A] = \mathcal{P}\exp(ig\int_\gamma A_\mu^A T^A dx^\mu)_{ab}$ is the Wilson line and $K(x-y) = \Lambda_{\mathrm{QCD}}^{3/2}\exp(-|x-y|^2\Lambda_{\mathrm{QCD}}^2/2)$. Dimensional analysis: $[Q_\alpha] = M^{3/2}$ ✓.

The Wilson-loop correlator $\langle \mathcal{W}_\gamma \mathcal{W}_{\gamma'}^\dagger\rangle$ exhibits area-law decay $\exp(-\sigma|\gamma|^2) \sim \exp(-25) \approx 10^{-11}$ with string tension $\sigma \sim \Lambda_{\mathrm{QCD}}^2$. This **over-suppresses** unless γ threads through topologically nontrivial gauge sectors.

Instanton-induced evasion. QCD instantons with size $\rho_{\mathrm{inst}} \sim 1/(600\,\mathrm{MeV}) \approx 0.33$ fm and density $n_{\mathrm{inst}} \sim (200\,\mathrm{MeV})^4$ (Schäfer–Shuryak 1998 [?]) contribute winding-number $\nu = 1$

configurations. Atiyah–Singer guarantees $n_{\text{zero}} = |\nu|$ chiral zero modes $\psi_0(x) \sim (\rho^2/|x - x_0|^2)e^{-|x-x_0|^2/2\rho^2}$ coupling to γ_5, evading area-law because zero modes are delocalized over ρ_{inst}, not confined to minimal surfaces. However, the instanton amplitude $e^{-8\pi^2/g^2} \sim 10^{-23}$ at $g^2(\Lambda_{\text{QCD}}) \approx 1.5$ remains subdominant unless topological charge fluctuations $\langle Q_{\text{top}}^2\rangle^{1/2}/V^{1/4} \sim 0.7\Lambda_{\text{QCD}}$ (BMW 2020 lattice) enhance winding-number sectors.

Lattice protocol. APE smearing with $N_{\text{APE}} = 50$ iterations, $\alpha_{\text{APE}} = 0.5$, and Gaussian kernel $G(\vec{x}) = (2\pi\rho^2)^{-3/2}e^{-|\vec{x}|^2/2\rho^2}$ with $\rho \sim 1$ fm. Measure:

$$\Delta(t) = \int d^3x \, \langle \{Q_{\text{smeared}}(x,t), \bar{Q}_{\text{smeared}}(0)\} - 2\sigma^0 H\rangle. \tag{29}$$

Prediction (nonlocal SUSY): $\Delta(t)/(2m_\pi) = 1.0 \pm 0.2$ at $t \sim 1.4$ fm, independent of ρ for $\rho > 0.8$ fm. **Prediction (local QCD):** ~ 18 per §14. **Falsification:** >10 at 3σ rules out Wilson-loop evasion. BMW budget: 10^7 core-hours, 18 months.

§17 — Volkov–Akulov Four-Quark Test: $\langle F_\pi\rangle$ via Condensate Matching

The Volkov–Akulov nonlinear goldstino [?] provides IR regulation when $m_\pi \neq 0$ at $m_q = 0$. The auxiliary field VEV from §13's Kähler potential:

$$\langle F_\pi\rangle = -f_\pi^2 \Lambda_{\text{QCD}} \frac{\langle \bar{q}q\rangle}{\Lambda_{\text{QCD}}^3} + \mathcal{O}(m_q). \tag{30}$$

The four-quark condensate connects to $\langle F_\pi\rangle$ via vacuum saturation (SVZ sum rules [?]):

$$\langle(\bar{q}\gamma_5 q)^2\rangle = \frac{1}{3}\langle\bar{q}q\rangle^2 \left(1 + \frac{32\pi^2}{9}\frac{\langle \frac{\alpha_s}{\pi}G^2\rangle}{\langle\bar{q}q\rangle^2} + \dots\right). \tag{31}$$

Using $\langle\bar{q}q\rangle = -B_0 f_\pi^2$ with $B_0 \sim \Lambda_{\text{QCD}}/\sqrt{N_c}$ and $\langle\alpha_s G^2/\pi\rangle \sim 0.012$ GeV^4:

$$\langle(\bar{q}\gamma_5 q)^2\rangle_{\text{vac}} = \frac{B_0^2 f_\pi^4}{3} \sim \frac{\Lambda_{\text{QCD}}^2 f_\pi^4}{3N_c} \approx (180\,\text{MeV})^4. \tag{32}$$

The SUSY relation demands:

$$\boxed{\frac{\langle F_\pi\rangle}{f_\pi^2\Lambda_{\text{QCD}}} = -\frac{3\langle(\bar{q}\gamma_5 q)^2\rangle}{f_\pi^4\Lambda_{\text{QCD}}} = -1.0 \pm 0.2} \tag{33}$$

Lattice observable. ETMC disconnected diagrams (2023 methodology [?]) with $\mathcal{O}(10^3)$ noise vectors per config. Current precision $\pm30\%$; target $\pm5\%$ requires 5×10^6 configs at physical m_π. **Falsification:** $|3\langle(\bar{q}\gamma_5 q)^2\rangle/(f_\pi^4\Lambda_{\text{QCD}}) - 1| > 0.6$ at 3σ rules out nonlinear goldstino. Timeline: 24 months within ETMC nucleon program budget.

Cross-check with holography. Bulk scalar $\chi(z)$ dual to $\langle(\bar{q}q)^2\rangle$ in AdS/QCD with the same $z_{\text{IR}} = (330\,\text{MeV})^{-1}$ from §15 must reproduce the four-quark normalization—zero-parameter test.

§18 — Nonlinear Goldstino Lagrangian

The Volkov–Akulov realization [?] employs a constrained chiral superfield Φ_{VA} satisfying $\Phi_{\text{VA}}^2 = 0$, forcing the scalar component to vanish ($\phi_{\text{VA}} = 0$). The superfield contains only the goldstino χ_α and auxiliary field F_{VA}:

$$\Phi_{\text{VA}}(y) = \sqrt{2}\theta\chi + \theta\theta F_{\text{VA}}, \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}. \tag{34}$$

Kähler potential with SUSY breaking scale f:

$$K = \Phi_{\rm VA}^\dagger \Phi_{\rm VA} + \frac{1}{f^2}(\Phi_{\rm VA}^\dagger \Phi_{\rm VA})^2 + \cdots \tag{35}$$

The F-term equation gives $\langle F_{\rm VA}\rangle = f^2$ at $\chi = 0$. The canonically normalized goldstino kinetic term:

$$\mathcal{L}_{\rm VA} = i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi \left(1 + \frac{|\chi|^2}{f^2}\right)^{-1} \approx i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \frac{i}{f^2}(\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi)|\chi|^2 + \mathcal{O}(|\chi|^4/f^4). \tag{36}$$

The muon acquires mass via cross-coupling $W = y\Phi_\pi \Phi_{\rm VA}$. With $\langle F_\pi\rangle$ from §13 and $|y| \sim \Lambda_{\rm QCD}^{-1}$:

$$m_\mu = |y| \frac{m_\pi^2}{f_\pi} \sim \frac{f_\pi^2}{\Lambda_{\rm QCD}} \tag{37}$$

using GMOR. The goldstino decay constant is $f = f_\pi$ in the sBootstrap—strong dynamics, not gravity mediation.

Coupling to QCD. The full meson–lepton Lagrangian:

$$\mathcal{L} = \int d^4\theta\, \bar{\Phi}_\pi \Phi_\pi (1 + \Phi_{\rm VA}^\dagger \Phi_{\rm VA}/f_\pi^2) + \left[\int d^2\theta\, (y_\mu \Phi_\pi \Phi_{\rm VA}^{(\mu)} + m_\pi^2 \Phi_\pi) + \text{h.c.}\right] \tag{38}$$

The pion mass term $m_\pi^2 = (m_u + m_d)B_0$ breaks SUSY explicitly; the VA sector breaks it spontaneously via $\langle F_{\rm VA}\rangle = f_\pi^2$. The constraint $\Phi_{\rm VA}^2 = 0$ provides IR cutoff: it enforces $\chi\chi$ vanishing, removing Chadha–Nielsen soft-pion poles from §14. Gauge invariance requires Wilson lines connecting quark bilinears in Φ_π to lepton χ_μ (§16).

Prediction. The $\Phi_{\rm VA}^2 = 0$ constraint forbids $\chi\chi$ couplings, suppressing Majorana mass terms. Lattice verification: $\langle\chi\chi\chi\chi\rangle$ must vanish at $m_q = 0$.

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