Meson decomposition from the ISS rank condition: four starting points and SM charge assignment

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Abstract

In the Intriligator–Seiberg–Shih (ISS) model of metastable SUSY breaking, the rank condition of the meson matrix $M = \tilde{Q}Q$ splits N_f flavors into $k_u = N_f - N_c$ massless pseudo-moduli and $k_d = N_c$ tree-level massive directions. We tabulate the meson decomposition under $\mathrm{SU}(k_u) \times \mathrm{SU}(k_d) \times \mathrm{U}(1)$ for four choices of (N_c, N_f) within or near the conformal window: $(3,6)$, $(3,5)$, $(2,6)$, and $(2,5)$. For each case we examine how Standard Model electric charges can be assigned to the constituent quarks and how the resulting meson quantum numbers compare with the SM spectrum. The case $(N_c, N_f) = (3,5)$, which is the sBootstrap starting point, uniquely matches the SM charge assignment with two up-type and three down-type quarks. The alternative $(2,5)$ reproduces the SM non-abelian gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2)$ from the flavor decomposition, but with an inverted mass–charge correspondence. We discuss the tension between these two viewpoints and the role of the $N_f = 6 \to 5$ transition (top decoupling) in connecting them.

1 Introduction

The ISS mechanism [?] provides a calculable model of metastable supersymmetry breaking in the magnetic dual of SQCD. A central ingredient is the *rank condition*: the magnetic meson vacuum expectation value has rank at most $N_f - N_c$, so that N_c of the N_f flavor directions acquire tree-level masses while $N_f - N_c$ directions remain as massless pseudo-moduli. This split is a robust consequence of the duality structure and does not depend on details of the Kähler potential.

From a phenomenological perspective, the $N_f \times N_f$ meson matrix $M_{ij} = \tilde{Q}_i Q_j$ of Seiberg duality [?] transforms as $(N_f^2 - 1) \oplus 1$ under $\mathrm{SU}(N_f)$, with N_f^2 components in total. The rank condition selects a vacuum that breaks $\mathrm{SU}(N_f)$ to $\mathrm{SU}(k_u) \times \mathrm{SU}(k_d) \times \mathrm{U}(1)$, where $k_u = N_f - N_c$ and $k_d = N_c$. The decomposition of the N_f^2 mesons under this residual symmetry depends on (N_c, N_f) and determines the low-energy spectrum.

Different values of (N_c, N_f) within the Seiberg conformal window $N_c + 1 \le N_f < 3N_c$ lead to different meson decompositions and different prospects for embedding the Standard Model. In this note we systematically compare four choices: $(N_c, N_f) = (3,6)$, $(3,5)$, $(2,6)$, and $(2,5)$. We then address how Standard Model electric charges can be distributed among the N_f quarks in each case, and examine which starting point most naturally reproduces the SM meson and Higgs content.

Throughout, our conventions for Seiberg duality follow [?], eqs. (1.1)–(1.2) for the electric theory and eqs. (3.1)–(3.3) for the magnetic dual. The ISS superpotential and vacuum structure follow [?], eqs. (2.4)–(2.6).

2 Setup: Seiberg duality and the ISS mechanism

2.1 Seiberg duality

Consider $\mathcal{N} = 1$ SQCD with gauge group $\mathrm{SU}(N_c)$ and N_f flavors of quarks Q^i, $\tilde{Q}_j$ in the fundamental and antifundamental representations ($i, j = 1, \ldots, N_f$) [?]. The global symmetry is

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R \,. \tag{1}$$

The gauge-invariant composite operators include the meson

$$M^i{}_j = \tilde{Q}^i\, Q_j \,, \tag{2}$$

which transforms as $(\overline{\mathbf{N_f}}, \mathbf{N_f})$ under $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$, i.e. as a bifundamental with N_f^2 components.

In the conformal window, $N_c + 1 \leq N_f < 3N_c$, the theory admits a Seiberg dual description[1]: an $\mathrm{SU}(N)$ gauge theory with $N = N_f - N_c$, containing N_f dual quarks φ_i, $\tilde{\varphi}^j$ and an elementary gauge-singlet field $\Phi^i{}_j$ identified with the meson $M^i{}_j$. The dual superpotential is [?]

$$W_{\mathrm{dual}} = h\, \mathrm{Tr}(\varphi\, \Phi\, \tilde{\varphi}) \,, \tag{3}$$

where h is the dual Yukawa coupling (cf. eq. (3.3) of [?]).

2.2 ISS metastable SUSY breaking

Adding a mass term $W_{\mathrm{mass}} = \mathrm{Tr}(m\, M)$ in the electric theory maps in the dual to a linear term for Φ. The full magnetic superpotential becomes [?]

$$W = h\, \mathrm{Tr}(\varphi\, \Phi\, \tilde{\varphi}) - h\mu^2 \mathrm{Tr}\Phi \,, \tag{4}$$

where μ^2 encodes the electric quark mass (eq. (2.4) of [?]). The F-term conditions $F_\Phi = 0$ require

$$\varphi\, \tilde{\varphi} = \mu^2\, \mathbf{1}_{N_f} \,, \tag{5}$$

but φ and $\tilde{\varphi}$ are $N \times N_f$ matrices, so $\varphi\, \tilde{\varphi}$ has rank at most $N = N_f - N_c$. Therefore eq. (??) cannot be satisfied for all N_f flavors simultaneously: this is the *rank condition* (following from the F-term conditions of the superpotential (1.3) of [?]).

The metastable vacuum takes the form (eq. (2.6) of [?])

$$\varphi_0 = \tilde{\varphi}_0^T = \begin{pmatrix} \mu\, \mathbf{1}_N \\ 0_{(N_f - N) \times N} \end{pmatrix}, \qquad \Phi_0 \text{ arbitrary (pseudo-moduli)} \,. \tag{6}$$

This splits the N_f flavors into two groups:

- $k_u = N = N_f - N_c$ **directions:** the F-terms are satisfied, $\varphi\tilde{\varphi} = \mu^2 \mathbf{1}_N$. At tree level the meson components Φ along these directions are pseudo-moduli (massless). They acquire a mass only at one loop through the Coleman–Weinberg potential.

- $k_d = N_c$ **directions:** the F-terms are not satisfied, $F_\Phi \neq 0$. The meson components along these directions have tree-level masses of order $h\mu$.

The vacuum energy is $V_{\min} = (k_d)\, |h^2 \mu^4|$ (eq. (2.5) of [?]).

[1] The ISS one-loop analysis is perturbatively controlled in the narrower *free magnetic range* $N_c + 1 \leq N_f \leq \frac{3}{2} N_c$. All four cases studied here have $N_f > \frac{3}{2} N_c$ and therefore sit in the conformal window but outside the free magnetic range. The rank condition itself is a tree-level statement valid throughout the conformal window; only the Coleman–Weinberg stabilization of pseudo-moduli requires the free magnetic range for perturbative control.

Table 1: Meson decomposition for four (N_c, N_f) starting points. $N = N_f - N_c$ is the dual gauge group rank. "CW" indicates conformal window status. The branching is under $\mathrm{SU}(k_u) \times \mathrm{SU}(k_d) \times \mathrm{U}(1)$.

N_c	N_f	N	k_u	k_d	Mesons	CW	Branching rule
3	6	3	3	3	36	$4 \le 6 < 9$ ✓	$(\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{8})_0 \oplus 2\times(\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\overline{\mathbf{3}})_{+1} \oplus (\overline{\mathbf{3}},\mathbf{3})_{-1}$
3	5	2	2	3	25	$4 \le 5 < 9$ ✓	$(\mathbf{3},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{8})_0 \oplus 2\times(\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{2},\overline{\mathbf{3}})_{+1} \oplus (\overline{\mathbf{2}},\mathbf{3})_{-1}$
2	6	4	4	2	36	$3 \le 6 \not< 6$ ✗	$(\mathbf{15},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus 2\times(\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{4},\overline{\mathbf{2}})_{+1} \oplus (\overline{\mathbf{4}},\mathbf{2})_{-1}$
2	5	3	3	2	25	$3 \le 5 < 6$ ✓	$(\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus 2\times(\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\overline{\mathbf{2}})_{+1} \oplus (\overline{\mathbf{3}},\mathbf{2})_{-1}$

2.3 Meson decomposition

The vacuum (**??**) breaks the $\mathrm{SU}(N_f)$ flavor symmetry to

$$\mathrm{SU}(N_f) \;\longrightarrow\; \mathrm{SU}(k_u) \times \mathrm{SU}(k_d) \times \mathrm{U}(1)\,. \tag{7}$$

The N_f^2 meson components decompose as

$$N_f^2 \;=\; \underbrace{(k_u^2 - 1) + 1}_{\mathrm{SU}(k_u)\text{ sector}} \;+\; \underbrace{(k_d^2 - 1) + 1}_{\mathrm{SU}(k_d)\text{ sector}} \;+ 2\,k_u\,k_d\,, \tag{8}$$

where the last term accounts for the bifundamental and its conjugate. In representation language:

$$N_f^2 \;\to\; (\mathbf{adj},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{adj})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{k_u},\overline{\mathbf{k_d}})_{+1} \oplus (\overline{\mathbf{k_u}},\mathbf{k_d})_{-1}\,, \tag{9}$$

where the two singlets arise from the two independent U(1) projections of the trace.

The physical content depends on k_u and k_d, i.e. on the choice of (N_c, N_f).

3 The four decompositions

We now tabulate the four cases. Table **??** provides a compact comparison; the subsections below give the details.

3.1 $(N_c, N_f) = (3, 6)$: symmetric split

The dual gauge group is SU(3), and the theory is self-dual in the sense that both the electric and magnetic theories have the same gauge group rank. The conformal window condition is $N_c + 1 = 4 \le 6 < 9 = 3N_c$, which is satisfied.

With $k_u = k_d = 3$ the flavor symmetry breaks as

$$\mathrm{SU}(6) \;\to\; \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d \times \mathrm{U}(1)\,. \tag{10}$$

The adjoint of SU(6) decomposes as

$$\mathbf{35} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{8})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\overline{\mathbf{3}})_{+1} \oplus (\overline{\mathbf{3}},\mathbf{3})_{-1}\,. \tag{11}$$

Adding the singlet from the full $N_f^2 = 36$ gives $8 + 8 + 1 + 1 + 9 + 9 = 36$.

This is the starting point of the Cacciapaglia–Sannino dualised Standard Model [**?**], which embeds six quark flavors into a Seiberg-dual framework and obtains an 18-Higgs-doublet structure. The symmetric split $k_u = k_d = 3$ means that no preference is built in for which flavors are "up-type" and which are "down-type."

Top decoupling: from $(3,6)$ to $(3,5)$

The $k_u = k_d$ symmetry is broken by giving one flavor (the top) a large mass m_t. In the electric theory, the heavy top is integrated out below the scale m_t, reducing N_f from 6 to 5. Re-deriving the Seiberg dual for the resulting SU(3) theory with $N_f = 5$ gives a new magnetic dual with gauge group $\mathrm{SU}(N_f - N_c) = \mathrm{SU}(2)$:

$$\mathrm{SU}(3)_{\mathrm{dual}}^{(N_f=6)} \xrightarrow{\text{top decoupling}} \mathrm{SU}(2)_{\mathrm{dual}}^{(N_f=5)} . \tag{12}$$

Below the scale m_t, the effective theory has $N_f = 5$ active flavors with dual gauge group SU(2). The 6×6 meson matrix reduces to $5 \times 5 = 25$ components, and the ISS rank condition now gives the asymmetric split $k_u = 2$, $k_d = 3$: this is the $(3,5)$ case of Section **??**.

The 36 mesons of the parent $(3,6)$ theory decompose into 25 mesons of the $(3,5)$ daughter theory plus 11 states involving the top flavor (5 charged under the residual SU(5) plus 6 top-mixed states), which become heavy and decouple. The asymmetric $k_u = 2$, $k_d = 3$ split is thus a consequence of the large top mass (which is an input, not a prediction), rather than being imposed by hand on the flavor structure.

3.2 $(N_c, N_f) = (3,5)$: the sBootstrap case

The dual gauge group is SU(2). The conformal window condition $4 \leq 5 < 9$ is satisfied.

With $k_u = 2$ and $k_d = 3$ the flavor symmetry breaks as

$$\mathrm{SU}(5) \;\to\; \mathrm{SU}(2)_u \times \mathrm{SU}(3)_d \times \mathrm{U}(1) . \tag{13}$$

The adjoint decomposes as

$$\mathbf{24} \;\to\; (\mathbf{3},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{8})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{2},\overline{\mathbf{3}})_{+1} \oplus (\overline{\mathbf{2}},\mathbf{3})_{-1} . \tag{14}$$

Adding the singlet: $3 + 8 + 1 + 1 + 6 + 6 = 25$.

This is the case relevant to the sBootstrap program [**?**]: the $k_u = 2$ massless directions are identified with the two up-type quarks (u, c), and the $k_d = 3$ massive directions with the three down-type quarks (d, s, b). The 25 mesons organize as open-flavor states in $(\mathbf{2},\overline{\mathbf{3}})$ plus their conjugates and flavor-diagonal states in $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{8})$.

3.3 $(N_c, N_f) = (2,6)$: Pati–Salam-like

The dual gauge group would be SU(4). However, the conformal window condition $N_c + 1 = 3 \leq 6 < 6 = 3N_c$ *fails*: $N_f = 3N_c$ is the boundary of asymptotic freedom, and for $N_f \geq 3N_c$ the theory is infrared-free. This case therefore sits outside (or at the edge of) the regime where Seiberg duality and the ISS mechanism are under perturbative control.

With $k_u = 4$ and $k_d = 2$ the formal decomposition is

$$\mathrm{SU}(6) \;\to\; \mathrm{SU}(4)_u \times \mathrm{SU}(2)_d \times \mathrm{U}(1) . \tag{15}$$

The adjoint decomposes as

$$\mathbf{35} \;\to\; (\mathbf{15},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{4},\overline{\mathbf{2}})_{+1} \oplus (\overline{\mathbf{4}},\mathbf{2})_{-1} . \tag{16}$$

Adding the singlet: $15 + 3 + 1 + 1 + 8 + 8 = 36$.

The $\mathrm{SU}(4) \times \mathrm{SU}(2)$ structure is reminiscent of the Pati–Salam group. If SU(4) is further broken to $\mathrm{SU}(3) \times \mathrm{U}(1)$, the U(1) factor plays the role of a lepton-number symmetry. However, the failure of the conformal window condition makes this case less attractive as a controlled starting point.

3.4 $(N_c, N_f) = (2,5)$: SM gauge structure

The dual gauge group is SU(3). The conformal window condition $3 \leq 5 < 6$ is satisfied.

With $k_u = 3$ and $k_d = 2$ the flavor symmetry breaks as

$$\mathrm{SU}(5) \;\to\; \mathrm{SU}(3)_u \times \mathrm{SU}(2)_d \times \mathrm{U}(1)\,. \tag{17}$$

The adjoint decomposes as

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\overline{\mathbf{2}})_{+1} \oplus (\overline{\mathbf{3}},\mathbf{2})_{-1}\,. \tag{18}$$

Adding the singlet: $8+3+1+1+6+6=25$.

The residual symmetry (**??**) is precisely the SM non-abelian gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L$. The bifundamental $(\mathbf{3},\overline{\mathbf{2}})$ is exactly the representation of a left-handed quark doublet. This match between a flavor decomposition and the SM gauge structure arises within the general Seiberg duality framework [**?**] and has been discussed in the context of the dualised SM [**?**].

4 Standard Model charge assignment

In the Standard Model, the six quark flavors carry electric charges $Q_{\mathrm{em}} = +2/3$ (up-type: u, c, t) and $Q_{\mathrm{em}} = -1/3$ (down-type: d, s, b). With only five light flavors (excluding the top), we have 2 up-type and 3 down-type quarks.

The ISS rank condition assigns the k_u flavors to the massless (pseudo-moduli) sector and the k_d flavors to the massive sector. We now examine whether the SM charge assignment is compatible with this split for each (N_c, N_f).

4.1 Charge assignment for $(N_c, N_f) = (3,5)$

Here $k_u = 2$ and $k_d = 3$. Identifying the 2 massless directions with the up-type quarks (u, c) and the 3 massive directions with the down-type quarks (d, s, b) gives a direct match with the SM:

Sector	k	Q_{em}	Flavors
Massless (pseudo-moduli)	2	$+2/3$	$u,\ c$
Massive (tree-level)	3	$-1/3$	$d,\ s,\ b$

The mesons in the bifundamental $(\mathbf{2},\overline{\mathbf{3}})$ carry $Q_{\mathrm{em}} = (+2/3) - (-1/3) = +1$ and $(\overline{\mathbf{2}},\mathbf{3})$ carry $Q_{\mathrm{em}} = -1$, while the diagonal sectors $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{8})$ are electrically neutral. The full charge spectrum of the 25 mesons is therefore

$$12_{\,0} + 6_{+1} + 6_{-1} + 1_{\,0} = 25\,, \tag{19}$$

where the subscript denotes Q_{em}. Each meson $M_{ij} = \bar{q}_i\, q_j$ carries charge $Q_{\mathrm{em}}(M_{ij}) = -Q_i + Q_j$, with Q_i the electric charge of quark flavor i. The thirteen neutral mesons $(12+1)$ correspond to $(\mathbf{3},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{8})_0 \oplus 2\times(\mathbf{1},\mathbf{1})_0$ and the twelve charged ones to $(\mathbf{2},\overline{\mathbf{3}})_{+1} \oplus (\overline{\mathbf{2}},\mathbf{3})_{-1}$.

This charge assignment works because the ISS split $k_u = 2$, $k_d = 3$ exactly matches the SM flavor counting.

4.2 Charge assignment for $(N_c, N_f) = (2,5)$

Here $k_u = 3$ and $k_d = 2$. The ISS rank condition assigns 3 flavors to the massless sector and 2 to the massive sector. If we want to identify these with SM quarks, we face an inversion: 3 up-type vs. 2 down-type, which is the opposite of the SM counting.

Table 2: SM charge assignment compatibility for each (N_c, N_f).

N_c	N_f	k_u	k_d	Match?	Comment
3	6	3	3	Partial	Symmetric; needs top decoupling
3	5	2	3	**Yes**	$k_u = n_u$, $k_d = n_d$ exactly
2	6	4	2	No	Outside conformal window
2	5	3	2	Inverted	$k_u \leftrightarrow k_d$ vs. SM

One could instead assign the 3 massless directions to the down-type quarks (d, s, b) and the 2 massive ones to the up-type quarks (u, c), but then the ISS mass hierarchy is inverted relative to the SM: in the SM, the up-type quarks are heavier (on average) than the down-type quarks in each generation. More fundamentally, the massive ISS directions are the ones whose F-terms are not satisfied — they "feel" the SUSY breaking directly.

4.3 Charge assignment for $(N_c, N_f) = (3, 6)$

Here $k_u = k_d = 3$, so the split is symmetric. The SM charge assignment requires distinguishing u- and d-type quarks, but the ISS vacuum treats both sectors identically at tree level. To reach the SM content one must invoke additional dynamics — for instance, decoupling the top quark to reduce $N_f = 6 \to 5$ — which breaks the $k_u = k_d$ symmetry.

This is the starting point of the Cacciapaglia–Sannino construction [**?**], in which the full six-flavor theory is dualised and the SM emerges after appropriate symmetry-breaking steps.

4.4 Charge assignment for $(N_c, N_f) = (2, 6)$

Here $k_u = 4$ and $k_d = 2$, and the theory sits at the edge of the conformal window. Neither $k_u = 4$ nor $k_d = 2$ matches the SM flavor counting directly (we need 2 up-type and 3 down-type, or at most 3 and 3 for the full six-flavor case). This case does not admit a natural SM charge assignment without further breaking of SU(4).

Table **??** summarizes the charge-assignment analysis.

5 Electroweak embedding

The comparison between the $(3, 5)$ and $(2, 5)$ cases reveals a tension between two desiderata: *charge assignment* and *gauge structure*.

5.1 The $(3, 5)$ perspective: charge assignment

For $(N_c, N_f) = (3, 5)$, the ISS split produces $\mathrm{SU}(2)_u \times \mathrm{SU}(3)_d \times \mathrm{U}(1)$. The SU(2) factor acts on the $k_u = 2$ massless directions, i.e. the up-type quarks. The mesons in $(\mathbf{2}, \overline{\mathbf{3}})_{+1}$ are electroweak doublets of color anti-triplets — exactly the quantum numbers of a Higgs doublet in the dualised SM.

The identification of this SU(2) with $\mathrm{SU}(2)_L$ is natural from the ISS perspective: the massless pseudo-moduli are the directions along which the meson VEV can fluctuate freely, and in the SM the left-handed doublets are massless before electroweak symmetry breaking. The SU(2) here rotates the massless modes, paralleling $\mathrm{SU}(2)_L$ rotating the left-handed doublets.

This is the sBootstrap viewpoint [**?**]: the mesons $M_{ij} = \tilde{q}_i q_j$ with one up-type and one down-type index carry the quantum numbers of the scalar partners of SM quarks (squarks/sfermions), and can be identified with pseudoscalar mesons in the hadronic spectrum.

5.2 The $(2,5)$ perspective: gauge structure

For $(N_c, N_f) = (2,5)$, the ISS split produces $\mathrm{SU}(3)_u \times \mathrm{SU}(2)_d \times \mathrm{U}(1)$ — exactly the SM non-abelian gauge group. The $(\mathbf{3}, \overline{\mathbf{2}})$ bifundamental is exactly a left-handed quark doublet representation. The $(\mathbf{8}, \mathbf{1})$ is the gluon adjoint representation. The $(\mathbf{1}, \mathbf{3})$ is the W-boson adjoint representation.

However, here the SU(2) factor comes from the $k_d = 2$ *massive* directions. In the SM, $\mathrm{SU}(2)_L$ is the symmetry of the left-handed doublets, which are massless before electroweak symmetry breaking. In the ISS framework, the k_d directions are the ones that have tree-level masses from the unsatisfied F-terms.

This produces a conceptual inversion: the SU(2) that looks like $\mathrm{SU}(2)_L$ acts on the *massive* ISS directions, while in the SM it acts on particles that are *massless* before EWSB.

5.3 The tension and Seiberg duality

Note that $(3,5)$ and $(2,5)$ are *distinct electric theories* — SU(3) SQCD with 5 flavors and SU(2) SQCD with 5 flavors respectively. Standard Seiberg duality maps $\mathrm{SU}(N_c)$ with N_f flavors to $\mathrm{SU}(N_f - N_c)$ with N_f flavors. Applied here: the *magnetic dual* of the $(3,5)$ electric theory is an SU(2) theory with 5 flavors, and the magnetic dual of the $(2,5)$ electric theory is an SU(3) theory with 5 flavors. The two starting points are thus Seiberg duals of each other: $(3,5)_{\mathrm{elec}} = (2,5)_{\mathrm{mag}}$ and $(2,5)_{\mathrm{elec}} = (3,5)_{\mathrm{mag}}$.

The tension between charge assignment and gauge structure is therefore not a choice between two unrelated theories but a tension between the electric and magnetic descriptions of the *same* physics. The mismatch can be stated precisely. In the SM:

- $\mathrm{SU}(2)_L$ acts on massless left-handed doublets.
- Masses are generated by EWSB, i.e. by *breaking* $\mathrm{SU}(2)_L$.

In the ISS mechanism:

- $\mathrm{SU}(k_u)$ acts on the *massless* pseudo-moduli.
- $\mathrm{SU}(k_d)$ acts on the *massive* directions.

For $(3,5)$: $\mathrm{SU}(2) = \mathrm{SU}(k_u)$ is the massless sector, matching the SM role of $\mathrm{SU}(2)_L$. For $(2,5)$: $\mathrm{SU}(2) = \mathrm{SU}(k_d)$ is the massive sector, opposite to the SM role of $\mathrm{SU}(2)_L$.

The "natural" assignment thus depends on which feature one prioritises: if the ISS massless/massive split is to match the SM mass hierarchy, then $(3,5)$ is preferred. If the non-abelian gauge group of the residual flavor symmetry is to match the SM gauge group, then $(2,5)$ is preferred. The early composite model of Masiero and Veneziano [**?**], which studied SQCD with $N = M = 3$ as a prototype for one lepton family, already faced a similar tension between composite fermion quantum numbers and the gauge embedding.

6 Discussion

6.1 The sBootstrap choice

The sBootstrap program [**?**] adopts $(N_c, N_f) = (3,5)$ because the charge assignment takes priority: the $k_u = 2$, $k_d = 3$ split directly matches the two up-type and three down-type quarks of the SM (with the top decoupled). The 25 mesons $M_{ij} = \tilde{q}_i q_j$ are then identified with the scalar partners of the SM fermions, and the closure conditions of the sBootstrap — the requirement that the composite scalars fill exactly the sfermion content of one generation — are satisfied. The SU(2) flavor symmetry of the massless sector plays the role of a proto-electroweak symmetry.

6.2 The alternative perspective

The $(2, 5)$ starting point has a different appeal: the residual flavor symmetry is literally $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, and the meson representations include $(\mathbf{3}, \mathbf{2})$, $(\mathbf{8}, \mathbf{1})$, and $(\mathbf{1}, \mathbf{3})$. If the flavor symmetry could be *gauged* (or if it emerges as a gauge symmetry in some UV completion), the SM gauge group would appear automatically. The price is that the charge assignment is inverted, and the SU(2) factor acts on the massive rather than the massless sector.

6.3 The $N_f = 6 \to 5$ transition

The two perspectives can be connected through the $(3, 6)$ starting point. With $N_f = 6$ and $N_c = 3$, the ISS split is symmetric ($k_u = k_d = 3$), and the theory is self-dual. Decoupling the heaviest quark (the top) reduces N_f from 6 to 5. The resulting $(3, 5)$ theory is the sBootstrap case.

Alternatively, one can consider the $(2, 6)$ theory and decouple a flavor to reach $(2, 5)$. However, $(2, 6)$ sits at the boundary of the conformal window, making it a less controlled starting point.

The $N_f = 6 \to 5$ transition via top decoupling thus provides a natural connection between the Cacciapaglia–Sannino framework (which starts at $N_f = 6$) and the sBootstrap (which works at $N_f = 5$). Which of the two $N_f = 5$ endpoints — $(3, 5)$ or $(2, 5)$ — is reached depends on the value of N_c in the electric theory, i.e. on whether the confining dynamics is SU(3) or SU(2).

6.4 Summary of findings

The results are:

1. The ISS rank condition provides a rigid, parameter-free split of N_f flavors into $k_u + k_d$ groups, determined entirely by (N_c, N_f).

2. Of the four cases examined, only $(N_c, N_f) = (3, 5)$ gives a direct match between the ISS split and the SM charge assignment with 2 up-type and 3 down-type flavors.

3. The case $(N_c, N_f) = (2, 5)$ reproduces the SM gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ from the flavor decomposition, but with an inverted charge–mass correspondence.

4. The symmetric case $(N_c, N_f) = (3, 6)$ requires additional dynamics (top decoupling) to break the $k_u = k_d$ symmetry and reach the SM spectrum.

5. The case $(N_c, N_f) = (2, 6)$ falls outside the conformal window and does not admit a natural SM embedding.

The tension between the $(3, 5)$ and $(2, 5)$ viewpoints is precisely the tension between the electric and magnetic descriptions of the same theory under Seiberg duality: $(3, 5)$ privileges the correct charge assignment at the price of a non-SM gauge structure, while its magnetic dual $(2, 5)$ privileges the correct gauge structure at the price of an inverted mass–charge correspondence. The $(3, 5)$ theory descends from the $N_f = 6$ parent by top decoupling (Section **??**), and its Seiberg dual is the $(2, 5)$ theory. The question of which description is "fundamental" may ultimately reduce to a question about the energy scale at which the SM is embedded.

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