Sector selectivity of Koide-type mass relations from the sBootstrap

Alejandro Rivero[1, *]

[1] *BIFI, Universidad de Zaragoza, E-50018 Zaragoza, Spain*

(Dated: March 22, 2026)

The Koide mass relation $Q = (\sum \sqrt{m_i})^2 / \sum m_i = 3/2$ holds for charged leptons to 10^{-5} precision, and near-Koide tuples appear among pseudoscalar mesons and certain quark triples, but not among vector mesons or baryons. This selectivity pattern has lacked a structural explanation. I show that the sBootstrap framework, which identifies sfermions with known mesons and diquarks via SU(5) flavor symmetry, provides such an explanation. The sBootstrap mass formula from the SO(32) embedding (conditions $z_1 + z_2 + z_3 = 0$ and $z_0^2 = \sum z_k^2/3$) is proved algebraically equivalent to the Koide condition $Q = 3/2$, with the two angular parametrizations differing by exactly $\pi/6$. The sfermion identification selects spin-0 two-body composites—precisely the pseudoscalar mesons—while excluding vectors (wrong spin) and baryons (wrong composite structure). Through a five-link algebraic bridge—lepton Koide condition, slepton = meson identification, shared charge structure, spin-selection rule, and chiral/heavy-light compatibility—the framework connects the lepton Koide relation to the hadronic sector without invoking purely hadronic mechanisms. It does not, however, derive the Koide angle from first principles: the angle δ remains a free parameter per sector. Nor does it derive the inverse-mass relation $Q(1/m_d, 1/m_s, 1/m_b) \approx 3/2$: the turtles classify (d, s, b) as a distinguished triple, but the Z_3 parametrization does not close under inversion. The observed ratio $\delta_{scb} \approx 3\,\delta_{e\mu\tau}$ and the discriminant coincidence $\Delta_{\rm II} = 9 = k_d^2$ remain open problems. Three testable predictions are identified: spin-selection (only spin-0 mesons can show Koide structure), $N = 3$ uniqueness (the Koide chain terminates at the top quark), and the existence of charge-$\pm 4/3$ diquarks forming a Koide triple.

I. INTRODUCTION

In 1983, Koide observed that the charged lepton pole masses satisfy

$$Q \equiv \frac{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2}{m_e + m_\mu + m_\tau} = \frac{3}{2} \qquad (1)$$

to a precision of 10^{-5} [? ?]. Foot recast this as a geometric statement: the vector $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ makes an angle of exactly 45° with the democratic direction $(1, 1, 1)$ [?]. This model-independent reformulation separated the empirical fact from its composite-model origin and prompted a sustained search for extensions and mechanisms [?].

Rodejohann and Zhang [?] noticed that the mixed-charge quark triple (c, b, t) also satisfies Eq. (??) approximately, and Cao gave the first published account [?]. I subsequently identified the full chain $(0, s, c) \to (-s, c, b) \to (c, b, t)$, where each link is a Koide triple with a sign-flipped lightest charge, and showed that the chain angle satisfies $\delta_{scb} \approx 3\,\delta_{e\mu\tau}$ [?]. Zenczykowski independently confirmed this relation using the Z_3 parametrization [?]. These extensions opened the question of whether the Koide relation reflects a principle operating across particle sectors or a collection of numerical coincidences.

A systematic scan of particle mass triples reveals a striking selectivity pattern: pseudoscalar meson triples with a sign-flipped pion, such as $(-\pi^0, D_s, \eta_c)$ with $|Q - 3/2| = 6 \times 10^{-4}$, satisfy the relation well, but no vector meson or baryon triple comes close. The proximity of the pion and muon masses—one composite, one elementary—was noted early [?] and is a foundational clue for the supersymmetric interpretation pursued here. The inverse-mass tuple $K(1/m_d, 1/m_s, 1/m_b)$ gives $Q = 1.503$, while the inverse lepton tuple gives $Q = 1.17$. Why should pseudoscalar mesons be selected over vectors? Why baryons never? Why should inverse masses of down-type quarks—and only down-type—approach Koide?

In this paper I show that the sBootstrap [? ?] provides a structural explanation for this pattern. The sBootstrap identifies sfermions of a supersymmetric standard model with known mesons and diquarks via the SU(5) flavor group of five light ("turtle") quarks, deriving uniquely $N = 3$ generations from composite self-consistency [?], building on the early observation by Miyazawa [?] that hadronic supersymmetry connects meson and baryon spectra, and the demonstration by Masiero and Veneziano [?] that one generation of leptons arises from SQCD with $N_c = N_f = 3$. I prove that the sBootstrap mass formula from the SO(32) embedding [?] is algebraically equivalent to the Koide condition $Q = 3/2$, with the angle correspondence $\alpha_{\rm Cartan} = \delta_{Z_3} + \pi/6$. The slepton = meson identification then explains sector selectivity without additional assumptions: spin-0 mesons are sfermions and inherit the Koide constraint; spin-1 mesons and three-body baryons do not.

I am explicit about what the framework does not achieve. The inverse down-type relation $Q(1/m_d, 1/m_s, 1/m_b) \approx 3/2$ is *not* derived: the Z_3 parametrization does not close under mass inversion,

* arivero@unizar.es

and the coincidence $\Delta_{\rm II} = 9 = k_d^2$ connecting the Koide discriminant to the turtle count has no known mechanism. The angle ratio $\delta_{scb}/\delta_{e\mu\tau} \approx 3$ is compatible with the SU(5) structure but not derived from it. The framework provides a structural map, not a dynamical theory.

The paper is organized as follows. Section **??** reviews the sBootstrap framework and the SO(32) mass formula. Section **??** presents the algebraic proof that this mass formula implies $Q = 3/2$ and derives the angle correspondence. Section **??** explains the sector selectivity pattern, including an honest assessment of the inverse down-type relation. Section **??** presents the five-link non-hadronic bridge from leptons to mesons. Section **??** assesses what the framework explains and what it does not.

II. THE SBOOTSTRAP FRAMEWORK

A. Sfermion = hadron identification

The starting observation [**?** **?**] is that the global flavor symmetry of $N_f = 5$ light quarks is SU(5), decomposed as $\mathrm{SU}(3)_D \times \mathrm{SU}(2)_U$ under the separation into three down-type quarks (d, s, b) and two up-type quarks (u, c). The top quark does not form light composites and plays no role in the flavor group; in the sBootstrap language it is the "elephant," while the five participating quarks are the "turtles."

The two-body composites decompose under SU(5) as follows. Meson-like (quark–antiquark) composites transform as

$$\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}\,, \tag{2}$$

and diquark (quark–quark) composites as

$$\bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \mathbf{15} \oplus \mathbf{10}\,. \tag{3}$$

The identification is:

- **Sleptons $\leftrightarrow$ pseudoscalar mesons**: the **24** contains the scalar partners of charged leptons, identified with the pseudoscalar meson nonet (three generations of charged sleptons $\leftrightarrow$ three families of charged mesons).
- **Squarks $\leftrightarrow$ diquarks**: the **15** and $\overline{\mathbf{15}}$ contain scalar partners of quarks, identified with diquark states.
- **Sneutrinos $\leftrightarrow$ neutral pseudoscalars**: the singlet **1** accommodates the neutral sector (η, η').

The **15** also produces three exotic diquarks of electric charge $\pm 4/3$, one per generation, from the $(1, 3)$ component of the SU(5) decomposition [**?**].

B. Unique closure at $N = 3$

The identification above is not merely a relabeling. The requirement that the composite scalars reproduce exactly the sfermion content of an N-generation supersymmetric standard model imposes

$$2N = k_u\, k_d\,, \tag{4}$$

$$2N = \frac{k_d(k_d+1)}{2}\,, \tag{5}$$

$$4N = k_u^2 + k_d^2 - 1\,, \tag{6}$$

where k_u and k_d are the numbers of up-type and down-type turtle quarks [**?** **?**]. The unique solution is

$$N = 3, \qquad k_u = 2, \qquad k_d = 3\,. \tag{7}$$

This is the "turtles all the way down" postulate: the composites of the turtles must be the superpartners of the turtles themselves, and this self-consistency condition uniquely requires three generations with five turtle quarks.

C. The SO(32) embedding and mass formula

The five-flavor SU(5) sits naturally inside the SO(32) group of Type I superstrings [**?**]. The adjoint of SO(32) decomposes under $\mathrm{SU}(5) \times \mathrm{SU}(3)_c$ as

$$496 = (\mathbf{24}, \mathbf{1}) \oplus (\mathbf{15}, \bar{\mathbf{3}}) \oplus (\overline{\mathbf{15}}, \mathbf{3}) \oplus \cdots\,, \tag{8}$$

where the displayed terms contain exactly the scalars of a three-generation SUSY model.

The classical energy of a pair composite with preon charges q_a, q_b is

$$E(q_a, q_b) = (q_a + q_b)^2\, K_\Omega\,, \tag{9}$$

where K_Ω is a common kinematic factor. For three generations, the abelian charges z_k $(k = 1, 2, 3)$ and a central charge z_0 obey two conditions:

$$z_1 + z_2 + z_3 = 0\,, \tag{10}$$

$$z_0^2 = \frac{z_1^2 + z_2^2 + z_3^2}{3}\,. \tag{11}$$

The masses of the three generations are then

$$m_k = (z_k + z_0)^2\,. \tag{12}$$

The SU(3) Cartan parametrization of the traceless charges is

$$\begin{aligned} z_1 &= \tfrac{1}{2}\cos\alpha + \tfrac{1}{2\sqrt{3}}\sin\alpha\,, \\ z_2 &= -\tfrac{1}{\sqrt{3}}\sin\alpha\,, \\ z_3 &= -\tfrac{1}{2}\cos\alpha + \tfrac{1}{2\sqrt{3}}\sin\alpha\,, \end{aligned} \tag{13}$$

with $z_0 = \pm 1/\sqrt{6}$. For charged leptons, the angle $\alpha = 0.7458\,\mathrm{rad}$ reproduces the experimental masses [**?**].

III. ALGEBRAIC EQUIVALENCE WITH KOIDE

A. Proof that $Q = 3/2$

I now show that the conditions (**??**) and (**??**) imply the Koide relation $Q = 3/2$ for the masses (**??**).

Numerator. Taking square roots of the masses (all positive by construction):

$$\sqrt{m_k} = z_k + z_0\,, \tag{14}$$

so

$$\sum_{k=1}^{3} \sqrt{m_k} = (z_1 + z_2 + z_3) + 3z_0 = 0 + 3z_0 = 3z_0\,, \tag{15}$$

using the tracelessness condition (**??**). Hence

$$\Big(\sum_k \sqrt{m_k}\Big)^2 = 9\,z_0^2\,. \tag{16}$$

Denominator. Expanding the masses:

$$\begin{aligned}\sum_{k=1}^{3} m_k &= \sum_k (z_k + z_0)^2 \\ &= \sum_k z_k^2 + 2z_0 \sum_k z_k + 3z_0^2 \\ &= \sum_k z_k^2 + 0 + 3z_0^2\,.\end{aligned} \tag{17}$$

Applying the normalization condition (**??**), $\sum_k z_k^2 = 3z_0^2$, yields

$$\sum_k m_k = 3z_0^2 + 3z_0^2 = 6z_0^2\,. \tag{18}$$

Ratio. Dividing (**??**) by (**??**):

$$\boxed{Q = \frac{9z_0^2}{6z_0^2} = \frac{3}{2}\,.} \tag{19}$$

This holds for any value of $z_0 \neq 0$ and any angle in the parametrization (**??**). The Koide relation is not an additional constraint imposed on the sBootstrap mass formula—it is an algebraic *consequence* of the tracelessness and normalization conditions, Eqs. (**??**) and (**??**).

B. Connection to Koide's original condition

In Koide's 1981 model, the charged lepton mass matrix is written as $M_l = (U + V)^2$, where $U \propto \mathbf{1}$ is the central part and V is traceless, with the constraint $\mathrm{Tr}[U^2] = \mathrm{Tr}[V^2]$ [**?**]. In the sBootstrap parametrization, $U = z_0\,\mathbf{1}$ and $V = \mathrm{diag}(z_1, z_2, z_3)$, so

$$\mathrm{Tr}[U^2] = 3z_0^2\,, \qquad \mathrm{Tr}[V^2] = z_1^2 + z_2^2 + z_3^2\,. \tag{20}$$

The normalization condition (**??**) is therefore identical to Koide's constraint:

$$z_0^2 = \frac{\sum_k z_k^2}{3} \quad\Longleftrightarrow\quad \mathrm{Tr}[U^2] = \mathrm{Tr}[V^2]\,. \tag{21}$$

The sBootstrap does not introduce a new mass formula. It provides a physical context—composite scalar normalization within a SUSY framework—for the same algebraic condition that Koide identified in 1981.

C. Angle correspondence

The Koide chain literature uses the Z_3 phase parametrization

$$q_k = \sqrt{M_0}\left(1 + \sqrt{2}\,\cos\!\left(\frac{2\pi k}{3} + \delta\right)\right), \quad k = 1, 2, 3\,, \tag{22}$$

with masses $m_k = q_k^2$ and $\sqrt{M_0}$ setting the overall scale [**? ?**]. Comparing with the sBootstrap masses $m_k = (z_k + z_0)^2$, the identification is

$$z_0 = \sqrt{M_0}\,, \qquad z_k = \sqrt{2\,M_0}\,\cos\!\left(\frac{2\pi k}{3} + \delta\right). \tag{23}$$

These z_k satisfy (**??**) identically (the sum of three equally spaced cosines vanishes), and (**??**) by construction (the sum of their squares equals $3M_0/2\cdot 2 = 3M_0$, while $3z_0^2 = 3M_0$, recalling the normalization $z_0 = \pm 1/\sqrt{6}$ in the unit-norm convention vs. $z_0 = \sqrt{M_0}$ in physical units).

The coordinate transformation between the SU(3) Cartan basis (**??**) and the Z_3 phase basis (**??**) is a rotation by $\pi/6$:

$$\alpha_{\text{Cartan}} = \delta_{Z_3} + \frac{\pi}{6}\,. \tag{24}$$

This can be verified by writing out both parametrizations and matching the three charge components, using $\cos(\theta + \pi/6) = (\sqrt{3}\cos\theta - \sin\theta)/2$.

Numerical check. For charged leptons:

$$\begin{aligned} &\text{SO(32) paper:} && \alpha = 0.7458 \text{ rad} = 42.73^\circ\,, \\ &\text{Koide chain:} && \delta = 0.2222 \text{ rad} = 12.73^\circ\,, \\ &\text{Difference:} && \alpha - \delta = 30.00^\circ = \frac{\pi}{6}\,. \end{aligned} \tag{25}$$

The correspondence is exact. The two parametrizations describe the same one-parameter family of mass spectra, rotated by $\pi/6$ in the charge space on the SU(3) Cartan torus. The value $\delta = 2/9$ rad was first identified in Brannen's circulant mass matrix construction [**?**], which is mathematically identical to the Z_3 parametrization.

IV. SECTOR SELECTIVITY FROM THE SBOOTSTRAP

The sBootstrap identification of sfermions with specific composites provides a structural mechanism for the observed selectivity of Koide-type relations. I analyze each sector in turn.

A. Charged leptons

Charged leptons are the fundamental fermions of the model. Their scalar superpartners (sleptons) are identified with the charged pseudoscalar mesons living in the **24** of SU(5). The Koide condition $\mathrm{Tr}[U^2] = \mathrm{Tr}[V^2]$, Eq. (**??**), is interpreted as a SUSY condition on the (lepton, slepton = meson) multiplet: it constrains the mass matrix of both the fermion and its scalar partner simultaneously. That the charged lepton masses satisfy $Q = 3/2$ to 10^{-5} precision is, in this picture, a consequence of the SUSY condition being (approximately) preserved.

Sumino [**?**] has shown how a $\mathrm{U}(3)\times\mathrm{O}(3)$ family gauge symmetry can protect the Koide relation against QED radiative corrections at the pole-mass level. The sBootstrap framework is compatible with this mechanism: the family gauge symmetry would operate within the lepton sector, while the sBootstrap provides the structural context linking leptons to mesons.

B. Pseudoscalar mesons

Pseudoscalar mesons ($J^P = 0^-$) are the lightest spin-0 $q\bar{q}$ composites. In the sBootstrap, they *are* the sleptons: the **24** of SU(5) from the $\mathbf{5}\otimes\bar{\mathbf{5}}$ decomposition. The Koide condition on sleptons translates directly to a constraint on pseudoscalar meson mass triples.

The numerical evidence confirms this: several pseudoscalar meson triples satisfy $Q \approx 3/2$ with a sign-flipped pion [**?**], as shown in Table **??**. The best is $(-\pi^0, D_s, \eta_c)$ with $|Q - 3/2| = 6 \times 10^{-4}$. The sBootstrap explains *why* these tuples exist: pseudoscalar mesons show near-Koide behavior because they are the sfermions, subject to the same SUSY-origin constraint as the leptons.

The match is "near" rather than exact because SUSY is not an exact symmetry at hadronic scales: QCD dynamics, chiral symmetry breaking, and quark mass differences all contribute to deviations from the $\mathrm{Tr}[U^2] = \mathrm{Tr}[V^2]$ condition. The sBootstrap predicts that the deviations should be controlled by the ratio of QCD effects to the slepton mass scale.

C. Vector mesons

Vector mesons ($J^P = 1^-$) are spin-1 $q\bar{q}$ states. They are *not* sfermions. In any supersymmetric framework, the scalar partners of fermions carry spin 0. The sBootstrap SUSY operates exclusively on scalar composites: only spin-0 mesons (pseudoscalar and, potentially, scalar) qualify as sfermions. No Koide-type constraint applies to vectors.

The data confirm this: the best vector meson triple achieves only $Q \approx 1.31$, far from 3/2.

D. Baryons

Baryons are qqq (three-body) composites. The sBootstrap SUSY operates on $q\bar{q}$ (meson) and qq (diquark) composites—both two-body. Three-body states fall outside the $\mathbf{5}\otimes\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}\otimes\bar{\mathbf{5}}$ decompositions. No Koide constraint applies.

Numerically, the best baryon triple gives $Q \approx 1.19$.

E. The quark chain

The five turtle quarks (u, c, d, s, b) are the fundamental objects of the SU(5) flavor group. The Koide chain $(0, s, c) \to (-s, c, b) \to (c, b, t)$ [**?**] connects successive quark triples, with each link satisfying $Q \approx 3/2$. The SU(5) structure is *compatible* with this chain: the five turtles span exactly the quarks appearing in the chain, and the top (the elephant) serves as the chain endpoint.

However, the factor-of-three angle relation $\delta_{scb} \approx 3\,\delta_{e\mu\tau}$ [**? ?**] is not derived from the SU(5) turtle structure alone. The sBootstrap explains *which* quarks participate (the five turtles) but not *why* the chain angles are related to the lepton angle by a factor of three.

F. Inverse down-type quarks: an honest negative

The tuple $K(1/m_d, 1/m_s, 1/m_b)$ gives $Q = 1.503$, while $K(1/m_e, 1/m_\mu, 1/m_\tau)$ gives $Q = 1.17$ and $K(1/m_u, 1/m_c, 1/m_t)$ gives $Q = 1.09$. Only the down-type inverse approaches Koide.

The sBootstrap provides a *structural reason* for why (d, s, b) is distinguished among all quark triples: the closure condition (**??**) requires exactly $k_d = 3$ down-type turtles, and the Standard Model contains exactly three down-type quarks. The bootstrap equations do not select this triple by fitting it to the data; they select it algebraically through the self-consistency of the composite spectrum.

However, the inverse-mass Koide relation itself is *not* derived from the sBootstrap. Three distinct arguments show this:

1. **The Z_3 parametrization does not close under inversion.** The inverse charges $w_k = 1/q_k = 1/[\sqrt{M_0}(1 + \sqrt{2}\cos(2\pi k/3 + \delta))]$ are not of the Z_3 form $A(1+\sqrt{2}\cos(2\pi k/3+\delta'))$ for any constants A and δ'. The Koide condition $Q = 3/2$ is automatic for direct masses in the Z_3 parametrization but *not* for inverse masses.

2. **No Cartan algebra involution exists** that maps $m_k \to 1/m_k$ within the SO(32) mass formula. The

masses $m_k = (z_k + z_0)^2$ are quadratic in the charges; the inverse $1/m_k = 1/(z_k + z_0)^2$ is not related to any linear transformation of the Cartan components.

3. **The diquark channel provides no constraint.** Down-type diquarks in the $(\mathbf{6}, \mathbf{1})$ component of $\mathbf{15}$ have masses $E(q_a, q_b) = (q_a + q_b)^2 K_\Omega$ that depend on *sums* of charges. These sum constraints do not impose conditions on *individual* inverse masses $1/m_k$.

There is a curious numerical coincidence. The discriminant of the inverse-Koide quadratic at $\delta = 15°$ evaluates to $\Delta_{\mathrm{II}} = 9$, and $\sqrt{\Delta_{\mathrm{II}}} = 3 = k_d$. Whether this connection between the Koide discriminant and the turtle count is accidental or points to a deeper structure is an open question. I note that no representation-theoretic quantity of SU(5) (dimensions, Casimir invariants, Dynkin indices of the $\mathbf{24}$ or $\mathbf{15}$) produces the factor 3 in a natural combination; the only quantity that equals 3 is k_d itself.

The status of the inverse down-type tuple is therefore: *structurally selected triple, unexplained mass relation.*

G. Sector comparison

Table **??** summarizes the sector selectivity analysis. The sBootstrap provides a complete structural explanation for four of six sectors and partial structural compatibility for the remaining two. There are zero contradictions with the empirical data.

V. THE NON-HADRONIC BRIDGE

The central structural contribution of the sBootstrap to the Koide problem is a five-link algebraic bridge connecting two apparently unrelated observations: the charged lepton Koide relation ($Q = 3/2$ exact) and the near-Koide behavior of pseudoscalar meson triples. I present each link explicitly, with its algebraic content and confidence assessment.

A. The five-link bridge

1. **Link 1: Lepton Koide = trace constraint.** The lepton mass matrix $M_l = (U + V)^2$ with $\mathrm{Tr}[U^2] = \mathrm{Tr}[V^2]$ is algebraically identical to the sBootstrap conditions $z_1 + z_2 + z_3 = 0$, $z_0^2 = \sum z_k^2/3$. This constrains the mass spectrum to a one-parameter Z_3 orbit, with the Koide condition $Q = 3/2$ as an automatic consequence (Sec. **??**).

2. **Link 2: Sleptons = mesons in the 24 of SU(5).** With five turtles $\{u, c, d, s, b\}$ and one elephant $\{t\}$, the SU(5) flavor products $\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}$ identify $q\bar{q}$ mesons with sleptons. This is a representation-theoretic identification, not a fit [**?**].

3. **Link 3: Shared charge structure transfers the constraint.** The *same* set of charges $\{z_k\}$ from the SU(3) Cartan subalgebra determines both lepton masses $m_k = (z_k + z_0)^2$ and meson pair energies $E(q_a, q_b) = (q_a + q_b)^2 K_\Omega$ [**?**]. The Koide condition constraining the lepton sector transfers to the meson sector through these shared charges. The transfer is not exact because QCD corrections modify the meson masses, producing the observed $|Q - 3/2| \sim 10^{-4}$–10^{-3} deviations.

4. **Link 4: Spin-selection and bilinear exclusion.** Sfermions are spin-0 scalars. This selects pseudoscalar mesons ($J^P = 0^-$, the lightest spin-0 $q\bar{q}$ states) and excludes vector mesons ($J^P = 1^-$, spin-1) by an algebraic selection rule. Baryons (qqq) are trilinear composites that do not appear in the bilinear $\mathbf{5} \otimes \bar{\mathbf{5}}$ products and are excluded on separate grounds.

5. **Link 5: Compatibility with the chiral/heavy-light selector.** The sBootstrap's spin-0 selection provides a structural explanation for the chiral and heavy-light selectors identified in purely hadronic analyses: chiral symmetry breaking affects pseudoscalars (which are the Goldstone sector) differently from vectors, and heavier turtle quarks (c, b) have larger charges that place their meson composites closer to the SUSY mass formula regime. The pion sign flip arises from chiral symmetry breaking and is a QCD correction, not an sBootstrap prediction.

The bridge is non-hadronic: it connects leptons to mesons through supersymmetry, not through QCD. Previous attempts to connect the lepton Koide relation to hadronic physics relied on purely hadronic mechanisms (chiral symmetry, heavy-quark symmetry, confinement-scale dynamics). These can explain why certain meson triples approach Koide within the hadronic sector, but they cannot explain the connection *to* the lepton relation, since leptons are not hadrons.

The hierarchy of precision—leptons closer to exact Koide than mesons—is naturally explained: Sumino's family gauge symmetry [**?**] can protect the lepton side of the bridge against QED radiative corrections, while the meson side receives QCD corrections of order $\Lambda_{\mathrm{QCD}}/m_{\mathrm{meson}}$ that are not cancelled.

B. What the bridge does not explain

The sBootstrap bridge has clear limitations:

1. It does not derive the Koide angle δ. For each sector, δ remains a free parameter determined by

TABLE I. Sector selectivity: empirical Koide status and sBootstrap explanation. The Koide ratio Q is computed from PDG 2024 masses in the $m_q(m_q)$ convention for quarks and pole masses for leptons [?]. The sBootstrap prediction column indicates whether the framework selects or excludes each sector and why.

Sector	Best tuple	Q	$\lvert Q-3/2\rvert$	Selected?	sBootstrap reason
Charged leptons	(e,μ,τ)	1.50000	2×10^{-5}	Yes	SUSY condition on (lepton, meson) multiplet
Pseudoscalar mesons	$(-\pi^0, D_s, \eta_c)$	1.5006	6×10^{-4}	Yes	Mesons *are* the sleptons (spin-0, $q\bar{q}$)
Quark chain	$(0, s, c)$	1.505	5×10^{-3}	Partial	Compatible with SU(5) turtles; angle ratio not derived
Inverse down-type	$K(1/d, 1/s, 1/b)$	1.503	3×10^{-3}	Partial	(d,s,b) selected as $\mathrm{SU}(3)_D$; inversion not derived
Vector mesons	$(-\rho, B_s^*, \Upsilon)$	1.31	0.19	No	Wrong spin ($J=1$, not sfermions)
Baryons	$(-p, \Sigma_b^-, \Omega_b^-)$	1.19	0.31	No	Wrong composite structure (3-body, not 2-body)

the mass data. For leptons, $\delta_{e\mu\tau} = 12.73°$; for the quark chain, $\delta_{scb} \approx 3\times 12.73° \approx 38°$.

2. It does not derive the ratio $\delta_{scb}/\delta_{e\mu\tau} \approx 3$, which would require understanding the $\mathrm{SU}(5)\to\mathrm{SU}(3)\times\mathrm{SU}(2)$ breaking pattern at a dynamical level.

3. It does not derive the inverse down-type relation $Q(1/m_d, 1/m_s, 1/m_b) \approx 3/2$: the Z_3 parametrization does not close under inversion (Sec. **??**).

4. It does not explain *which* specific meson triples satisfy $Q \approx 3/2$: the selection of tuples like $(-\pi^0, D_s, \eta_c)$ requires the mass formula with a specific angle, which the sBootstrap does not fix.

5. It does not predict specific mass values. The overall scale M_0 and the angle δ are both determined from data, not from first principles.

VI. DISCUSSION

A. Assessment: partial structural insight

The milestone verdict for the sBootstrap contribution to the Koide program is *partial structural insight with a genuine non-hadronic bridge*. More precisely, the sBootstrap provides four things that no other framework offers simultaneously:

1. A genuine non-hadronic bridge from leptons to mesons, via the slepton = meson identification in the **24** of SU(5).

2. A structural explanation for sector selectivity: spin-0 two-body composites are selected, vectors and baryons excluded.

3. An algebraic identification of the sBootstrap mass formula with the Koide condition, showing they are not merely compatible but identical (Sec. **??**).

4. A generation-counting argument: $N = 3$ uniquely from the composite self-consistency conditions (**??**)–(**??**).

These results constitute a structural insight, not a dynamical derivation. The sBootstrap explains *which* sectors should show Koide behavior (and why), but does not explain *how* the specific mass values arise. The Koide angle δ encodes the mass spectrum within each sector, and the sBootstrap leaves it as a free parameter.

B. Consistency with prior exclusions

An earlier systematic analysis of the Koide landscape established five exclusion claims: no universal Koide mechanism; vectors fail; baryons fail; Yukawaon/Sumino falsified as universal mechanism; hadronic-only embedding survives as the best framework [?]. The sBootstrap is consistent with all five. It does not claim universal Koide (it claims *selective* Koide). It predicts vector and baryon failure from spin-selection and composite structure. It is independent of the Yukawaon mechanism but compatible with Sumino's gauge protection as an IR mechanism. And it extends the hadronic-only embedding by providing the non-hadronic bridge that was previously absent.

C. Community context

A systematic evaluation of 43 ideas from extended Physics Forums discussions on Koide-type relations [? ?] found that 70% are either incorporated in the sBootstrap framework or independently support it, with zero contradictions. Three findings deserve special mention:

- Brannen's circulant mass matrix with $\delta = 2/9$ [?] is mathematically identical to the Z_3/Cartan parametrization used here, derived independently from Clifford algebra preons. The two parametrizations differ by the $\pi/6$ rotation of Eq. (**??**).

- The pion-mass ratio identity $m_{\pi^+}/m_{\pi^0} = 1 + \sqrt{m_\mu/m_Z}$ holds to 7 ppm with PDG 2024 masses. This relation connects a hadronic mass splitting to an electroweak/lepton quantity and is consistent with the sBootstrap's pion-as-slepton identification, but is not derived from it.

- Masiero and Veneziano's demonstration [?] that one generation of leptons arises from SQCD with $N_c = N_f = 3$ provides the strongest published support for the sBootstrap's mesino = lepton identification, predating the sBootstrap by two decades.

D. Honest negatives

Three important results are *not* achieved by the sBootstrap:

1. **The inverse down-type relation is not derived.** The turtles classify (d, s, b) as a distinguished triple, but the Z_3 parametrization does not close under mass inversion (Sec. ??). The factor-of-three coincidence $\sqrt{\Delta_{\rm II}} = k_d$ remains unexplained.

2. **The $3\times$ angle ratio is not derived.** No ratio of SU(5) representation invariants (dimensions, Casimirs, Dynkin indices of **24** vs. **15**) produces the factor 3. The only quantity equal to 3 is k_d itself.

3. **No specific mass values are predicted.** The angle δ is free per sector, so the framework cannot predict any individual mass. Given any three masses satisfying $Q = 3/2$, the framework can accommodate them by choosing an appropriate δ.

E. Predictions

The sBootstrap makes three testable predictions regarding Koide-type relations:

1. **Spin-selection.** The sBootstrap predicts that Koide-type mass structure appears *only* in spin-0 meson sectors (pseudoscalar and potentially scalar mesons, such as f_0, a_0, K_0^*) and *not* in spin-1 (vectors) or three-body composites (baryons). The sfermion identification is the mechanism: only spin-0 $q\bar{q}$ composites qualify as scalar partners. This prediction is structural and does not depend on the free parameter δ. Any vector meson triple with $Q \approx 3/2$ would falsify it.

2. $N = 3$ **uniqueness and chain termination.** The closure conditions (??)–(??) have no solution for $N = 4$. The Koide chain terminates at the top quark (the elephant): there is no fourth-generation link. Any discovery of a fourth generation of quarks, or a Koide chain continuing beyond the top, would falsify the sBootstrap's generation-counting argument.

3. **Exotic $\pm 4/3$ diquarks.** The $(1, 3)$ component of the **15** produces three exotic diquarks of charge $\pm 4/3$ (the uu, uc, cc states). If observed, they would form their own Koide-type triple. This is a conditional prediction: naive contact-interaction estimates give $Q = 2.0$–2.8 for these states, far from $3/2$, but the diquark mass dynamics in the **15** may differ from the **24** where Koide is established.

The structural predictions (spin-selection and $N = 3$) are strongly falsifiable: they make definite yes/no claims that do not depend on δ. The diquark prediction is conditional on the existence of states not yet observed. A deferred prediction—the neutrino Koide relation, requiring the right-handed neutrinos that the sBootstrap demands for internal consistency—becomes testable when absolute neutrino masses are measured.

F. Comparison with other approaches

Yukawaon models.—Koide's yukawaon framework [?] uses $\mathrm{U}(3) \times S_3$ family symmetry to derive mass matrices whose vacuum expectation values reproduce the Koide relation. This is a more dynamical approach than the sBootstrap, but it applies sector by sector and does not address the pseudoscalar/vector selectivity. The sBootstrap and yukawaon approaches are not mutually exclusive; the yukawaon could provide the dynamical mechanism that the sBootstrap's structural framework leaves unspecified.

Superconformal light-front holographic QCD.—The Brodsky-de Teramond-Dosch program [?] establishes a hadronic supersymmetry connecting meson and baryon Regge trajectories through a superconformal algebra. This framework naturally distinguishes pseudoscalar from vector mesons and could potentially explain the meson Koide selectivity at a QCD level. However, it has not been directly connected to Koide-type sum rules, and it does not provide a bridge to the lepton sector. A synthesis of the sBootstrap (lepton–meson bridge) with LFHQCD (hadronic mass patterns) is an interesting direction for future work.

Z_3 parametrization.—Zenczykowski's systematic use of the Z_3 angle parametrization [? ? ?] provided the algebraic foundation on which the chain structure was recognized. The sBootstrap adds the physical interpretation of the Cartan angle (Eq. (??)) and the generation-counting context, but the phenomenological content of the Z_3 parametrization is independent of the sBootstrap and predates it. The discrete S_3 symmetry underlying the parametrization was already implicit in the Harari-Haut-Weyers quark mass ansatz [?].

G. Open questions

Several questions remain open for future work:

- What dynamical mechanism fixes the Koide angle δ? The angle determines the mass spectrum within each sector and is the key quantity not derived by the sBootstrap. Possibilities include the Sumino gauge sector [?] or QCD vacuum dynamics.

- What explains the chain ratio $\delta_{scb} \approx 3\,\delta_{e\mu\tau}$? The factor of three might emerge from the SU(5) $\to$ SU(3) $\times$ SU(2) breaking pattern, but no combination of Casimir invariants or representation dimensions produces it. Brannen's α-correction formula $\delta = 2/9 - 4\pi\alpha'/3^{12}$, which matches the lepton angle to 10 digits [?], suggests a perturbative origin for the angle that remains to be derived.

- Can the inverse down-type relation be derived from a dual structure in the diquark channel? The $\bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \mathbf{15} \oplus \mathbf{10}$ decomposition contains the diquark sector; the present analysis shows that sum constraints on diquark masses do not constrain individual inverse masses (Sec. **??**).

- How do Yukawa corrections to quark mass running affect the Koide angles? At one-loop QCD, mass ratios (and hence Koide angles) are exactly RG-invariant [? ?]. Yukawa contributions, dominated by the top, are flavor-dependent and could shift δ_{scb} and δ_w at the percent level.

VII. CONCLUSIONS

The sBootstrap provides a partial structural explanation for the observed sector selectivity of Koide-type mass relations—partial because the Koide angle δ remains free, but structural because the selectivity pattern is derived from algebraic and representation-theoretic arguments rather than fitted to data.

Through the sfermion = hadron identification via SU(5) flavor symmetry, the framework selects spin-0 two-body composites (pseudoscalar mesons) as the hadronic sector where Koide-type behavior is expected, while excluding vectors (wrong spin) and baryons (wrong composite structure). The algebraic proof that the sBootstrap mass formula implies $Q = 3/2$, Eqs. (**??**)–(**??**), establishes that the two frameworks are not merely compatible but identical, with the angle correspondence $\alpha_{\text{Cartan}} = \delta_{Z_3} + \pi/6$.

The five-link non-hadronic bridge—trace constraint, slepton = meson identification, shared charge transfer, spin-selection, and chiral/heavy-light compatibility—connects the lepton Koide relation to hadronic mass patterns through SUSY rather than QCD. The framework is consistent with all six particle sectors examined: four explained (leptons, pseudoscalar mesons, vectors, baryons), two compatible (quark chain, inverse down-type), zero contradictions. It makes three testable predictions: spin-selection, $N = 3$ uniqueness, and the $\pm 4/3$ diquark sector.

What the sBootstrap does not provide is equally important. The Koide angle δ remains a free parameter, so no individual mass is predicted. The chain ratio $\delta_{scb}/\delta_{e\mu\tau} \approx 3$ is not derived; no ratio of SU(5) invariants produces it. The inverse down-type relation is structurally motivated (the turtles select (d, s, b)) but not proven: the Z_3 parametrization does not close under mass inversion, and the coincidence $\Delta_{\text{II}} = 9 = k_d^2$ has no known mechanism. The framework offers a structural map, not a dynamical theory. The derivation of δ from first principles remains the central open problem in the Koide program.

ACKNOWLEDGMENTS

I thank the participants of the Physics Forums community for discussions on Koide-type relations, especially C. Brannen for the circulant parametrization and H. de Velde for the anomalous moment coincidences. I thank M. Porter for identifying the Masiero-Veneziano and Seiberg duality connections. This work was supported by the BIFI research infrastructure at the Universidad de Zaragoza.

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