Three families from SUSY confinement: an effective-field-theory route to scalar bootstrap and charged-fermion mass relations

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Abstract

We present a four-dimensional $\mathcal{N} = 1$ effective-field-theory realization of the scalar-bootstrap program previously formulated at the level of composite counting and representation theory. The earlier $SO(32)/SU(5)$ analysis singled out three generations and the scalar content $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$; the purpose of the present paper is to determine how much of that target can already be realized within ordinary low-energy supersymmetric dynamics. The Diophantine constraints are recalled only to define this target.

The mesonic sector is described by confined SU(3) SQCD with an O'Raifeartaigh deformation, while the lepton sector is organized by an analogous SU(2) construction. At the distinguished seed point $gv/m = \sqrt{3}$ the O'Raifeartaigh block yields the exact spectrum $(0, (2-\sqrt{3})m, (2+\sqrt{3})m)$ and two exact algebraic consequences: the structural ratio $(2+\sqrt{3})^2 \approx 13.93$ and the identity $\langle z_k^2 \rangle = z_0^2$ when the masses are written as $m_k = (z_0 + z_k)^2$. Starting from the single isospin input $m_u + m_d$ (equivalently m_π through GMOR), the seed and its conditional EFT extensions give $m_c = 1301\,\mathrm{MeV}$, $m_b = 4233\,\mathrm{MeV}$, $m_t = 171\,250\,\mathrm{MeV}$, $m_u/m_d = 0.453$, $m_\tau = 1776.97\,\mathrm{MeV}$, and the GST value of the Cabibbo angle. Within this framework, seven of the Standard Model's thirteen charged-fermion-sector parameters are fixed.

The scope of the claim is explicit. The symmetric flavor $\mathbf{15}$ required by the bootstrap is not an ordinary scalar qq composite in the color-$\bar{\mathbf{3}}$ channel, and the vacuum-selection and bloom mechanisms are not yet derived from first principles. The result is therefore a conditional but sharp EFT statement: once the distinguished seed configuration is selected, the low-energy supersymmetric theory generates a nontrivial and highly constrained chain of charged-fermion mass relations.

1 Introduction

The flavour problem has two logically distinct parts. One is *kinematic*: why are there three generations, and why do the scalar quantum numbers relevant to supersymmetric extensions of the Standard Model fall into such structured multiplets? The other is *dynamical*: if a compelling multiplet structure exists, can any part of it be realized in

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a controlled four-dimensional field theory rather than only at the level of representation theory or numerical pattern matching? The present paper is aimed at the second question.

The starting point is the scalar-bootstrap construction developed in [1]. There the scalar partners of Standard Model fermions were interpreted as composites built from quark–antiquark and quark–quark states, and a self-consistency requirement on the multiplicities of such composites singled out three generations together with an SU(5) flavour organization. In group-theoretic language, the relevant scalars fall into $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$, precisely the combination that appears in the adjoint of $SO(32)$ under the appropriate branching. That earlier result supplied a kinematic target. What it did not supply was a concrete low-energy supersymmetric dynamics capable of realizing any part of that target and translating it into mass relations.

This paper is written as the EFT follow-up to that precursor. Its novelty is not the Diophantine counting by itself, nor the bare observation that the scalar content organizes into $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$. Those results belong to the earlier paper and are recalled here only to define the target theory. The new question is narrower and, for publication purposes, more useful: how much of the scalar bootstrap survives when one demands an explicit four-dimensional $\mathcal{N} = 1$ supersymmetric effective description?

Our answer is that the mesonic sector survives in a remarkably rigid form. The quark sector admits a natural description in terms of confined SU(3) SQCD degrees of freedom in the Seiberg framework [3], supplemented by an O'Raifeartaigh deformation that generates a distinguished seed spectrum. At the seed point $gv/m = \sqrt{3}$, the fermion masses take the exact form $(0, (2 - \sqrt{3})m, (2 + \sqrt{3})m)$ and obey the algebraic identity $\langle z_k^2 \rangle = z_0^2$ when written as $m_k = (z_0 + z_k)^2$. This seed algebra then organizes the low-energy mass relations in both the quark and lepton sectors. In that precise sense, the paper adds dynamics and EFT structure to the bootstrap program rather than merely restating its arithmetic origin.

The resulting framework is strong enough to motivate the headline claim that seven of the Standard Model's thirteen charged-fermion-sector parameters are fixed, but it is important to be explicit about the status of that claim. Inside the EFT developed here, the parameter reduction is real: once the independent inputs are specified, the remaining masses and the Cabibbo angle follow by a chain of algebraic and dynamical relations. What is not yet established is that every step in that chain has been derived from a complete microscopic Lagrangian with no unresolved sector. Two obstructions remain visible and should not be hidden.

The first obstruction is kinematic and concerns the diquarks. The same bootstrap that selects the symmetric flavour $\mathbf{15}$ also collides with the Pauli theorem: an ordinary scalar qq state in the colour $\bar{\mathbf{3}}$ channel cannot carry symmetric flavour. This is not a detail of a particular ansatz but a structural mismatch between the composite interpretation and the representation required by the counting. The second obstruction is dynamical. The O'Raifeartaigh seed at $gv/m = \sqrt{3}$ is the point from which the cleanest mass relations follow, and the paper presents effective-potential ingredients that isolate this point and deform it in the phenomenologically correct direction; however, a first-principles derivation of the full vacuum-selection and bloom mechanisms remains incomplete. These are not peripheral caveats. They mark the exact boundary between the part of the program that is already a controlled EFT and the part that still requires additional ultraviolet or nonperturbative input.

This way of stating the result also clarifies the relation between the present paper and the companion Koide phenomenology note. The role of the present paper is not to re-litigate every empirical mass pattern. It is to identify the dynamical core that could underwrite such patterns: the confined-SQCD/O'Raifeartaigh seed, the exact seed algebra, the Seiberg seesaw structure, and the conditional extensions that propagate the seed to heavier quark and lepton masses. The empirical note may still be useful as a ledger of suggestive relations, but the publication-driving paper should stand or fall on the internal field-theoretic logic developed here.

The paper is therefore organized around three logical levels of claim. First come the exact statements internal to the EFT, such as the Diophantine uniqueness theorem and the seed algebra at $gv/m = \sqrt{3}$. Next come the conditional mass relations that follow once the distinguished seed is selected and extended by the bion and seesaw mechanisms. Finally come the speculative ultraviolet interpretations, including the $SO(32)$ string embedding, which are retained as outlook rather than used as foundations for the core results. Maintaining that separation is essential if the paper is to read as a controlled EFT construction rather than as a single package of theorem, phenomenology, and UV speculation.

The remainder of the paper follows that logic. Section 2 reviews the bootstrap counting and the uniqueness of $(N, r, s) = (3, 3, 2)$, but only to the extent needed to define the EFT target. Section 3 discusses the SU(5) flavour representations and makes the symmetric-**15** obstruction explicit. Section 4 develops the confined-SQCD/O'Raifeartaigh description and derives the exact seed identities. Section 5 builds the conditional heavy-quark mass chain. Section 6 shows how the same algebraic structure reappears in the lepton sector. Sections 7–10 collect electroweak, spectrum, and ultraviolet material, while Section 11 states the remaining open problems in a form that keeps the status of the framework transparent.

2 Bootstrap target (recap of the EPJC precursor)

This section recalls only the part of [1] that the EFT construction actually uses. The detailed composite census and its SO(32) embedding belong to that earlier paper; here they serve as boundary data. The low-energy theory needs just three consequences of the bootstrap:

1. the unique solution of the counting problem is $(N, r, s) = (3, 3, 2)$;
2. the participating light flavours organize into $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$ of $\mathrm{SU}(5)_f$;
3. one up-type flavour, identified phenomenologically with the top, remains outside the light composite sector.

The counting itself may be summarized in one line. If r light down-type quarks and s light up-type quarks are allowed to form composites, matching charged and neutral scalar composites to the fermionic degrees of freedom of N generations gives

$$rs = 2N, \qquad \frac{r(r+1)}{2} = 2N, \qquad r^2 + s^2 - 1 = 4N. \tag{1}$$

The unique positive-integer solution is

$$(N, r, s) = (3, 3, 2). \tag{2}$$

We will not repeat the detailed Diophantine proof here; the role of Eq. (2) in the present paper is simply to identify the EFT target and to justify why the confined block should be organized around three active generations but only five light participating flavours.

What matters dynamically is therefore not the full counting argument but the representation-theoretic target that survives it. The mesonic composites fill the flavour adjoint **24**, while the diquark sector is forced into a symmetric **15** together with its conjugate. This is exactly the point where the present paper begins: the mesonic **24** already admits a controlled supersymmetric EFT realization, whereas the symmetric **15** immediately runs into the Pauli obstruction reviewed in the next section.

3 SU(5) flavor and the diquark sector

3.1 Counting partners

In the full sBootstrap [1], the scalar partners of the Standard Model fermions include both mesons ($q\bar{q}$) and diquarks (qq). For five active flavors, the meson matrix gives $5 \times 5 = 25$ scalars. The diquarks $[q_i q_j]$ form a representation of $SU(5)_f$:

$$\mathbf{5} \otimes \mathbf{5} = \overline{\mathbf{10}}_A \oplus \mathbf{15}_S \,. \tag{3}$$

The symmetric $\mathbf{15}_S$ gives 15 diquarks including same-flavor states $[uu]$, $[dd]$, $\ldots$, while the antisymmetric $\overline{\mathbf{10}}_A$ gives 10 diquarks with $i \neq j$ only.

3.2 The symmetric representation

The sBootstrap SUSY generator acts asymmetrically on the two fermion sectors. When it acts on a *lepton*, it produces a meson: $\mathcal{Q} \cdot \ell \sim q\bar{q}$. Since the quark and antiquark are distinguishable, the meson lies in the full $\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} + \mathbf{1}$. There is no Pauli constraint, and the meson spectrum is perfectly reproduced.

When it acts on a *quark*, it produces a diquark: $\mathcal{Q} \cdot q_i \sim q_i q_j$. The bosonic SUSY generator applied to a single flavor index i naturally produces the **symmetric** product—the $\mathbf{15}_S$ representation, not the antisymmetric $\overline{\mathbf{10}}$. This is the representation identified in the $SO(32)$ decomposition [1]: the adjoint of $SO(32)$ under $SU(5)_f$ decomposes to include the $\mathbf{15} + \overline{\mathbf{15}}$ (diquarks) alongside the $\mathbf{1} + \mathbf{24}$ (mesons).

3.3 The Pauli theorem for scalar diquarks

The requirement of the symmetric **15** conflicts with Fermi statistics—not merely for $L = 0$ diquarks, but for *all* scalar diquarks in the color $\bar{3}$ channel, regardless of internal orbital structure.

Theorem. *For a spin-0 ($J = 0$) diquark in the color $\bar{3}$ (antisymmetric) representation, the flavor wave function is antisymmetric for* ***all*** *values of the orbital angular momentum L and spin S.*

Proof. $J = 0$ requires $L = S$ (since $|L - S| \leq J \leq L + S$). The exchange symmetry of the spin-orbital wave function is $(-1)^{L+S+1} = (-1)^{2L+1} = -1$ for all L. Combined with the color $\bar{3}$ factor (-1), the non-flavor product is $(-1)(-1) = +1$ (symmetric). Total antisymmetry then requires flavor to be antisymmetric: $\overline{\mathbf{10}}$. □

This result is purely kinematic—independent of dynamics, binding mechanisms, or spatial extent. The key cases:

L	S	Spin-orbital $(-1)^{2L+1}$	$\times$ Color	Flavor
0	0	-1	$\times(-1) = +1$	must be A
1	1	-1	$\times(-1) = +1$	must be A
2	2	-1	$\times(-1) = +1$	must be A

The theorem has a sharp consequence: **the sBootstrap diquark in the 15 cannot be a qq composite.** It must be either:

1. An algebraically generated object—the image of the Miyazawa supercharge [8, 9] acting on an antiquark, whose symmetry is determined by the superalgebra rather than by the statistics of constituent quarks; or
2. A fundamental string-theoretic object, as in the $SO(32)$ decomposition [1], where the **15** arises from the adjoint representation of the gauge group and the open-string diquark is not built from point-like quarks.

The contrast with the meson sector is instructive. When the SUSY generator acts on a lepton, it produces a meson ($q\bar{q}$): the quark and antiquark are distinguishable, no Pauli constraint applies, and the meson spectrum is reproduced without tension. When it acts on a quark, it produces a diquark in the "wrong" representation—and the theorem shows this cannot be fixed by any internal structure of a qq pair. This is an unresolved obstruction: the sBootstrap requires the symmetric **15**, but QCD composite diquarks cannot fill it.

4 Effective-field-theory realization

This section fixes the EFT used in the rest of the paper. The confined dynamical block is the light-flavor theory with $N_f = N_c = 3$. The O'Raifeartaigh deformation defines the seed spectrum. The heavier flavors (c, b) are introduced only later, as external flavor labels and Yukawa data, when the charged-fermion chain is assembled. Keeping these layers separate avoids conflating exact Seiberg results with wider flavor bookkeeping.

4.1 The Seiberg framework

The dynamical starting point is $\mathcal{N} = 1$ SQCD with gauge group $SU(3)_c$ and three light flavors of chiral superfields Q^i, $\bar{Q}_j$. Seiberg confinement [3] gives the low-energy degrees of freedom

$$M^i{}_j = Q^i \cdot \bar{Q}_j \tag{4}$$

for the mesons, together with baryons $B = \epsilon_{ijk} Q^i Q^j Q^k$ and $\bar{B}$. They satisfy the quantum-modified constraint

$$\det M - B\bar{B} = \Lambda^6 \,, \tag{5}$$

where Λ is the dynamical scale.

Each entry $M^i{}_j$ is a chiral superfield containing a complex scalar and a Weyl fermion. In the sBootstrap interpretation the scalar is the mesonic degree of freedom, while the fermionic partner supplies the leptonic side of the meson–lepton identification in the supersymmetric limit. The later $N_f = 5$ notation is therefore only flavor bookkeeping: the exact confining dynamics used here remain those of the light 3×3 block.

4.2 F-term breaking: the O'Raifeartaigh mechanism

The seed algebra is supplied by the standard O'Raifeartaigh superpotential [4]

$$W = f\,\Phi_0 + m\,\Phi_1\Phi_2 + g\,\Phi_0\Phi_1^2\,. \tag{6}$$

Its F-terms are

$$F^*_{\Phi_0} = f + g\,\phi_1^2\,, \tag{7}$$
$$F^*_{\Phi_1} = m\,\phi_2 + 2g\,\phi_0\phi_1\,, \tag{8}$$
$$F^*_{\Phi_2} = m\,\phi_1\,. \tag{9}$$

For $f \neq 0$ and $m \neq 0$, $F_{\Phi_2} = 0$ implies $\phi_1 = 0$, and then $F_{\Phi_0} = f \neq 0$, so supersymmetry is broken. Writing $\phi_0 = v$, the fermion mass matrix at the vacuum is

$$W_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2gv & m \\ 0 & m & 0 \end{pmatrix}, \tag{10}$$

with eigenvalues 0 and

$$m_\pm = \sqrt{g^2v^2 + m^2} \pm gv\,. \tag{11}$$

The seed condition $\sqrt{m_-}/\sqrt{m_+} = 2 - \sqrt{3}$ fixes the dimensionless ratio $t \equiv gv/m$ through $\sqrt{t^2+1} - t = 2 - \sqrt{3}$, hence

$$\frac{gv}{m} = \sqrt{3}\,. \tag{12}$$

At this point

$$m_\pm = (2 \pm \sqrt{3})\,m\,. \tag{13}$$

The rest of the construction uses this seed spectrum, together with the question of how it is deformed by the Kähler geometry and by the non-perturbative terms added below.

4.3 Abelian charges and the mass-charge identity

Write each fermion mass as the square of an abelian charge,

$$m_k = \mathfrak{z}_k^2, \qquad \mathfrak{z}_k = z_0 + z_k, \qquad k = 1, 2, 3, \tag{14}$$

with $z_0 \equiv \bar{\mathfrak{z}} = \frac{1}{3}\sum_k \mathfrak{z}_k$ and the tracelessness constraint $\sum_k z_k = 0$. Then the mass-charge identity $\langle z_k^2\rangle = z_0^2$ takes the form of a *charge-variance identity*: expanding

$$\frac{1}{3}\sum_k z_k^2 = \frac{1}{3}\sum_k \mathfrak{z}_k^2 - z_0^2 = \frac{\sum m_k}{3} - \frac{(\sum\sqrt{m_k})^2}{9}, \tag{15}$$

so that $\langle z_k^2\rangle = z_0^2$ is equivalent to $\frac{1}{3}\sum m_k = 2z_0^2$, which rearranges to

$$\boxed{\frac{\sum m_k}{\left(\sum\sqrt{m_k}\right)^2} = \frac{2}{3}}\,. \tag{16}$$

In statistical language the identity states that the population variance of the charges equals the squared mean: $\mathrm{Var}(\mathfrak{z}) = \bar{\mathfrak{z}}^2$.

Angular parametrisation. The constraint $\sum z_k = 0$ restricts (z_1, z_2, z_3) to a two-dimensional hyperplane in $\mathbb{R}^3$, parametrised as

$$z_k = r\cos\!\left(\delta + \tfrac{2\pi k}{3}\right), \qquad k = 1, 2, 3. \tag{17}$$

For any δ the tracelessness $\sum \cos(\delta+2\pi k/3) = 0$ is automatic, and the identity $\sum \cos^2(\delta + 2\pi k/3) = 3/2$ gives $\langle z_k^2 \rangle = r^2/2$. The charge-variance identity $\langle z_k^2 \rangle = z_0^2$ then fixes

$$r = \sqrt{2}\, z_0, \tag{18}$$

and the angle δ—the *bloom angle*—parametrises the family of mass triples at fixed vacuum charge z_0. The full charges read $\mathfrak{z}_k = z_0\big[1 + \sqrt{2}\cos(\delta + 2\pi k/3)\big]$.

The seed angle. At the O'Raifeartaigh seed, $m_1 = 0$ forces $\mathfrak{z}_1 = 0$, hence $z_1 = -z_0$. From (17),

$$r\cos\delta = -z_0 = -\frac{r}{\sqrt{2}} \quad\Longrightarrow\quad \cos\delta = -\frac{1}{\sqrt{2}}, \qquad \delta = \frac{3\pi}{4}. \tag{19}$$

Every O'Raifeartaigh seed sits at $\delta = 3\pi/4$.

Cartan-plane geometry. The mass triple defines a vector in the (λ_3, λ_8) Cartan plane of $SU(3)$: $\vec{r} = \sum_k \mathfrak{z}_k\, \vec{w}_k$, where $\vec{w}_k$ are the fundamental weights. Since $\sum \vec{w}_k = 0$ the constant part drops out, and a standard trigonometric reduction gives

$$\vec{r} = \frac{z_0\sqrt{6}}{2}\left(\sin(\delta + \tfrac{\pi}{3}),\ \cos(\delta + \tfrac{\pi}{3})\right), \qquad \theta = \frac{\pi}{6} - \delta\,, \tag{20}$$

where θ is the polar angle of $\vec{r}$. This is an exact identity for *all* δ and z_0: the bloom is a rigid rotation in the Cartan plane at fixed radius $R = z_0\sqrt{6}/2$, with no radial component. The remaining two eigenvalues satisfy $t^2 - 4mt + m^2 = 0$ with $z_0^2 = 2m/3$, giving $m_\pm = (2 \pm \sqrt{3})\, m$, which is the spectrum at $gv/m = \sqrt{3}$.

Determinant condition. Their product $m_+ m_- = m^2$ is the *determinant condition*: it equals $\det W_{ij}$ restricted to the massive 2×2 block. That this follows from $\langle z_k^2 \rangle = z_0^2$ is direct: at the seed, $z_2 + z_3 = z_0$ and $z_2 z_3 = -z_0^2/2$, so

$$m_+ m_- = \big[(z_0{+}z_2)(z_0{+}z_3)\big]^2 = \left[z_0^2 + z_0^2 - \tfrac{z_0^2}{2}\right]^2 = \tfrac{9z_0^4}{4} = m^2. \tag{21}$$

Conversely, requiring $m_+ m_- = m^2$ with $m_+ + m_- = 4m$ forces $\langle z_k^2 \rangle = z_0^2$. Thus the mass-charge identity at the seed is the determinant of the superpotential mass matrix.

Generality of the determinant condition. For a general O'Raifeartaigh superpotential $W = f\Phi_0 + \frac{1}{2} m_{kl}\Phi_k\Phi_l + \frac{1}{2}\Phi_0\, g_{kl}\Phi_k\Phi_l$, the massive fermion block is $M = m + v\, g$. Then $\det M = \det m$ for all v if and only if $m^{-1}g$ is *nilpotent*. For $n = 2$ this reduces to two conditions: $\mathrm{Tr}(\mathrm{adj}(m) \cdot g) = 0$ and $\det g = 0$. Both are satisfied automatically when m is antidiagonal and g has a single diagonal entry (as in the standard model), since $\mathrm{adj}(m) \cdot g$ is then strictly lower-triangular. The product $m_+ m_- = m^2$ is therefore not a numerical accident at $gv/m = \sqrt{3}$; it holds for *all* values of the pseudo-modulus.

Signed charges and the quark sector. The charge $\mathfrak{z}_k$ need not be positive: a negative sign $\mathfrak{z}_1 = -\sqrt{m_1}$ still gives $m_1 = \mathfrak{z}_1^2 > 0$ while changing the sign of the field's mass contribution in the superpotential.

Vacuum-charge doubling. The seed triple $(0, \sqrt{m_s}, \sqrt{m_c})$ has $v_0^{\text{seed}} = (\sqrt{m_s}+\sqrt{m_c})/3$. If $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$, substitution gives

$$v_0^{\text{bloom}} = \frac{-\sqrt{m_s} + \sqrt{m_c} + 3\sqrt{m_s} + \sqrt{m_c}}{3} = \frac{2(\sqrt{m_s} + \sqrt{m_c})}{3} = 2\, v_0^{\text{seed}}. \tag{22}$$

Conversely, $v_0^{\text{bloom}} = 2v_0^{\text{seed}}$ requires $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$, predicting $m_b = 4177\,\text{MeV}$ (PDG: 4180 ± 30, 0.1σ). The bion relation and the vacuum-charge doubling are algebraically equivalent statements: one is the other rewritten in the charge language.

The mass-charge identity is a theorem about the O'Raifeartaigh superpotential at $gv/m = \sqrt{3}$, not an empirical observation. The angular parametrisation, determinant condition, and vacuum-charge doubling are its corollaries.

4.4 Pseudo-modulus geometry and the missing stabilization mechanism

In the minimal O'Raifeartaigh model with canonical Kähler potential, the Coleman–Weinberg contribution is monotonically increasing and selects $\langle X \rangle = 0$. A negative quartic Kähler correction

$$K = |X|^2 + \frac{c}{\Lambda_K^2}\,|X|^4\,, \qquad c < 0 \tag{23}$$

modifies the tree-level potential to $V_{\text{tree}} = f^2/(1 + 4c\,v^2/\Lambda_K^2)$, which diverges at $v_{\text{pole}} = \Lambda_K/(2\sqrt{|c|})$ where the Kähler metric vanishes. The field space terminates at v_{pole}: the proper distance to the pole is finite, $\sigma_{\max} = \sqrt{3}\pi m/(4g)$.

Setting $v_{\text{pole}} = \sqrt{3}\,m/g$ gives the exact condition

$$c = -\frac{\Lambda_K^2}{12\,m^2}\,. \tag{24}$$

With $\Lambda_K = m$ this is $c = -1/12$, an algebraic result: $\Lambda_K/(2\sqrt{1/12}) = \sqrt{12}/2 = \sqrt{3}$.

This is precisely where the EFT stops being complete. Both V_{tree} and V_{CW} are monotonically increasing functions of v: the one-loop effective potential is minimized at $v = 0$, not at the pole. The pole therefore isolates the distinguished value $v = \sqrt{3}\,m/g$ geometrically but does not by itself stabilize the pseudo-modulus there. Reaching an actual vacuum at the seed requires an additional negative contribution to the effective potential near the pole. The three-instanton operator discussed later is a plausible candidate, but this paper does not derive the required minimum from first principles. All results that depend on the seed should therefore be read exactly as stated: exact once the seed is selected, not yet a theorem that the full potential selects it.

4.5 The O'Raifeartaigh structure in SQCD

The quantum constraint. In Seiberg's SQCD with $N_f = N_c = 3$, the confined degrees of freedom are the meson matrix $M^i{}_j = Q^i\bar{Q}_j/\Lambda$, the baryons $B = \epsilon_{ijk}Q^iQ^jQ^k/\Lambda^3$ and

$\bar{B} = \epsilon^{ijk}\bar{Q}_i\bar{Q}_j\bar{Q}_k/\Lambda^3$, subject to the quantum-modified constraint $\det M - B\bar{B} = \Lambda^6$. A Lagrange multiplier X enforces this:

$$W = X\big(\det M - B\bar{B} - \Lambda^6\big) + \sum_j m_j\, M^j{}_j\,. \tag{25}$$

The Seiberg seesaw. Setting $B = \bar{B} = 0$ and diagonal $M = \mathrm{diag}(M_1, M_2, M_3)$, the F-term equation $\partial W/\partial M^j{}_j = 0$ gives

$$m_j + X\,\frac{\det M}{M_j} = 0 \qquad (\text{no sum on } j)\,, \tag{26}$$

so that $M_j = C/m_j$ with $C = -X \det M$ flavor-independent. The constraint $\det M = \Lambda^6$ fixes $C = \Lambda^2(m_1 m_2 m_3)^{1/3}$. Heavy quarks produce light mesons: this is the Seiberg seesaw.

F-term SUSY breaking. Adding an O'Raifeartaigh deformation $\Delta W = f\,X + g\,X\,\phi^2/2$ (where ϕ is an off-diagonal meson fluctuation), the F-term for X becomes

$$F_X^* = \big(\det M - B\bar{B} - \Lambda^6\big) + f + \tfrac{1}{2} g\,\phi^2\,. \tag{27}$$

At the meta-stable vacuum where $\det M \approx \Lambda^6$ and $\phi = 0$, the constraint term vanishes but $F_X^* = f \neq 0$. Setting all F-terms to zero simultaneously would require $\det M - \Lambda^6 = -f$, contradicting the remaining F-term equations which drive $\phi \to 0$ and $\det M \to \Lambda^6$. Supersymmetry is therefore spontaneously broken with vacuum energy $V_0 = |f|^2$.

ISS parameter map. In the ISS magnetic dual with gauge group $SU(N_f - N_c)$, the rank condition provides an independent argument: $\tilde{q}\,q$ is an $N_f \times N_f$ matrix of rank $\tilde{N}_c = N_f - N_c < N_f$, so it cannot equal $\mu^2 \mathbb{1}_{N_f}$. The N_c frustrated F-terms give $V_0 = N_c|h\mu^2|^2$. The O'Raifeartaigh parameters map as $g \to h$, $m \to h\mu$, $v \to \langle X\rangle$, so the condition $gv/m = \sqrt{3}$ becomes $\langle X\rangle/\mu = \sqrt{3}$—the magnetic Yukawa h cancels.

4.6 The mass-charge identity is not an RG attractor

One might hope that the identity $\langle z_k^2\rangle = z_0^2$ is a dynamical attractor: that three masses evolving under SUSY-breaking dynamics flow toward the $\langle z_k^2\rangle = z_0^2$ surface. However, the one-loop soft-mass RG equations in $\mathcal{N} = 1$ SQCD with universal Casimirs and negligible Yukawas give $dm_i/dt = \gamma(M_g^2 - m_i)$, which drives all masses to a common value $m_i \to M_g^2$, yielding $\langle z_k^2\rangle/z_0^2 \to 0$—the *opposite* extreme. The algebraic preservation condition on the $\langle z_k^2\rangle = z_0^2$ surface is incompatible with any uniform attractor flow (by the Cauchy–Schwarz inequality).

The identity therefore does *not* arise from simple RG running. It must instead be a *boundary condition* on the superpotential—as in the O'Raifeartaigh realization, where $gv = \sqrt{3}\,m$ produces the exact seed. The condition is set at the scale of SUSY breaking and then preserved by QCD-invariant mass ratios.

4.7 The EFT ansatz

We now collect the ingredients actually used later. The notation is flavor-wide (u, d, s, c, b), but the confining dynamics remain in the light 3×3 block. The heavier flavors enter as external mass and Yukawa data attached to the same flavor bookkeeping. With that understanding, consider $\mathcal{N}=1$ SQCD with gauge group $SU(3)_c$ and $N_f=5$ flavors of chiral superfields Q^i, $\bar{Q}_j$ ($i,j=1\ldots5$, corresponding to u,d,s,c,b). The total Lagrangian is

$$\mathcal{L}=\int d^4\theta\, K+\left[\int d^2\theta\, W+\text{h.c.}\right]+\mathcal{L}_D+\mathcal{L}_{\text{soft}}\,, \tag{28}$$

with Kähler potential (canonical plus stabilization and bion corrections),

$$K=\underbrace{\mathrm{Tr}(M^\dagger M)+|X|^2+|B|^2+|\bar{B}|^2+|H_u|^2+|H_d|^2+|S|^2}_{K_{\text{can}}}\underbrace{-\frac{|X|^4}{12\,\mu^2}}_{K_{\text{stab}}}+\underbrace{\frac{\zeta^2}{\Lambda^2}\Big|\sum_{k=1}^{N_c}s_k\sqrt{\hat{m}_k}\Big|^2}_{K_{\text{bion}}}\,. \tag{29}$$

Here K_{stab} places the Kähler pole at the distinguished value $\langle X\rangle=\sqrt{3}\,\mu$ without by itself proving that the vacuum sits there; $\zeta\propto e^{-S_0/(2N_c)}$ is the monopole fugacity, $\hat{m}_k$ are the current quark masses, and $s_k=\pm1$ encode the seed-to-bloom phase.

The superpotential combines mass, constraint, and Yukawa terms:

$$\begin{aligned}W=&\underbrace{\mathrm{Tr}(\hat{m}\,M)}_{\text{chiral mass}}+\underbrace{X\big(\det M^{(3)}-B\bar{B}-\Lambda_{\text{eff}}^6\big)}_{\text{Seiberg constraint}}+\underbrace{c_3\,\frac{(\det M^{(3)})^3}{\Lambda_{\text{eff}}^{18}}}_{\text{3-instanton}}\\&+\underbrace{y_c\,H_u\,{M^c}_c+y_b\,H_d\,{M^b}_b+y_t\,H_u\,Q^t\bar{Q}_t}_{\text{Yukawa}}+\underbrace{\lambda_S\,S\,H_u\!\cdot\! H_d+\tfrac{\kappa}{3}\,S^3}_{\text{NMSSM}}\,.\end{aligned} \tag{30}$$

Here $M^{(3)}$ is the 3×3 light-flavor block (u,d,s) and Λ_{eff} is the dynamical scale of the confined $N_f=N_c=3$ theory.

The D-term Lagrangian includes an optional Fayet–Iliopoulos parameter ξ for $U(1)_{\text{em}}$:

$$\mathcal{L}_D=-\frac{e^2}{2}\left(\sum_i Q_i\,|\phi_i|^2+\xi\right)^2. \tag{31}$$

Finally, soft SUSY-breaking terms arise from a spurion $\langle F_S\rangle=f_\pi^2$:

$$\mathcal{L}_{\text{soft}}=-\tilde{m}^2\,\mathrm{Tr}(M^\dagger M)-\big(B_\mu\,\mathrm{Tr}(\hat{m}\,M)+A_y\big(y_c\,H_u\,{M^c}_c+y_b\,H_d\,{M^b}_b\big)+\text{h.c.}\big)\,,\quad \tilde{m}^2\sim f_\pi^2\,. \tag{32}$$

The soft scalar mass $\tilde{m}^2\sim f_\pi^2\approx(92\,\text{MeV})^2$ generates the universal meson–lepton splitting $m_{\text{meson}}^2-m_{\text{lepton}}^2=f_\pi^2$.

4.8 Domain of validity and restoration criterion

The ansatz contains five inputs that are used explicitly later: (i) the exact O'Raifeartaigh seed algebra once $gv/m=\sqrt{3}$ is imposed; (ii) the meson–lepton soft splitting scale f_π^2; (iii) a bion-inspired radial constraint whose minimum encodes $\sqrt{m_b}=3\sqrt{m_s}+\sqrt{m_c}$; (iv) the heavy-sector Yukawa couplings; and (v) a three-instanton term that can shape the bloom potential. It should therefore be read as an EFT synthesis: the confined block is under analytic control, whereas the seed-selection and bloom terms are modelled extensions.

This is also the right place to state the symmetry-restoring target. A completed sBootstrap should admit a limit in which three common spectral labels $A=1,2,3$ exist such that

$$m(Q_A)=m(L_A)=m(M_A)=m_A \tag{33}$$

in the limit $f, \tilde{m}^2, \varepsilon, \xi \to 0$. Here Q_A denotes the quark/diquark-side state, L_A the lepton, and M_A the meson scalar. The observed flavor states are then related to the restored basis by unitary rotations,

$$q_i = U^{(q)}_{iA} Q_A \,, \qquad \ell_i = U^{(\ell)}_{iA} L_A \,, \qquad \phi_i = U^{(m)}_{iA} M_A \,. \tag{34}$$

The present EFT already realizes the common seed eigenvalues $(0, m_-, m_+)$ in the meson–lepton sector. What is missing is the quark/diquark block with the same three eigenvalues. Because the symmetric **15** is still unresolved, restoration should at present be understood as a target criterion on the completed theory, not as an achieved part of the current construction.

4.9 The flavour-universal Yukawa identity

The Seiberg seesaw and the standard Yukawa coupling combine to produce a flavour-independent product. Each confined quark j has a Yukawa coupling $y_j = \sqrt{2}\, m_j / v_{\rm EW}$ (at $\tan\beta = 1$), while the seesaw fixes the meson VEV $\langle M_j \rangle = C/m_j$ with $C = \Lambda^2 \big(\prod_k m_k\big)^{1/N_f}$ flavour-independent. Their product is

$$y_j \, \langle M_j \rangle = \frac{\sqrt{2}\, m_j}{v_{\rm EW}} \, \frac{C}{m_j} = \frac{\sqrt{2}\, C}{v_{\rm EW}} \,, \tag{35}$$

which is the same for every confined flavour: the quark-mass dependence cancels exactly. Evaluating for the light block (u, d, s) with $C = 8.8 \times 10^5\,\mathrm{MeV}^2$:

$$y_j \, \langle M_j \rangle = \frac{\sqrt{2} \times 8.8 \times 10^5}{2.46 \times 10^5} = 5.07\,\mathrm{MeV} \qquad \text{(all confined flavours)}\,. \tag{36}$$

This identity has two consequences. First, it shows that the Yukawa couplings are *not free parameters*: they are determined by C and $v_{\rm EW}$, both of which are fixed by the Lagrangian. Second, because the Higgs–meson vertex $y_j \langle M_j \rangle$ is flavour-universal, the Glashow–Weinberg condition [24] for the absence of tree-level Higgs-mediated FCNCs is satisfied automatically. This is not a tuning; it is a structural consequence of the seesaw.

The two-doublet structure has a direct consequence for the mass triples. Since the quark mass triples obligatorily mix up-type and down-type quarks, the translation from masses to Yukawa couplings depends on $\tan\beta$. For the (c, b, t) triple, with c and t receiving mass from H_u and b from H_d:

$$\left.\frac{\sum m_k}{(\sum \sqrt{m_k})^2}\right|_{c,b,t} = \frac{y_c^2 \sin^2\!\beta + y_b^2 \cos^2\!\beta + y_t^2 \sin^2\!\beta}{(y_c \sin\beta + y_b \cos\beta + y_t \sin\beta)^2} \,, \tag{37}$$

which satisfies $\langle z_k^2 \rangle = z_0^2$ for the Yukawa eigenvalues only when $\sin\beta = \cos\beta$, i.e., $\tan\beta = 1$. Thus $\tan\beta = 1$ is the unique value at which the mass-charge identity on *masses* is equivalent to the same identity on *Yukawa couplings* for mixed up/down-type triples.

5 Conditional mass relations from the superpotential

This section applies the EFT in four steps. First, the seed ratio $m_+/m_- = (2+\sqrt{3})^2$ maps m_s to m_c. Second, the bion term maps (m_s, m_c) to m_b. Third, the Yukawa eigenvalue constraint at $\tan\beta = 1$ maps (m_c, m_b) to m_t. Fourth, the dual seesaw identity maps

(m_s, m_b) to m_d. The logic of the chain is therefore transparent: input, structural relation, output. The only remaining question is status. Some steps are exact once the seed is chosen; others also require the bion term, the seesaw interpretation, or the continuity from $\mathbb{R}^3 \times S^1$ to $\mathbb{R}^4$.

5.1 The O'Raifeartaigh mass ratio

The O'Raifeartaigh sector of the superpotential,

$$W_{\mathrm{OR}} = f\,\Phi_0 + m\,\Phi_1\Phi_2 + g\,\Phi_0\Phi_1^2\,, \tag{38}$$

has a metastable SUSY-breaking vacuum at $\phi_1 = 0$, with Φ_0 a pseudo-modulus. At the vacuum $gv/m = \sqrt{3}$, the fermion eigenvalues are $m_\pm = (2 \pm \sqrt{3})\,m$, and the mass ratio of the two nonzero fermion eigenvalues is a structural constant:

$$R_{\mathrm{OR}} = \frac{m_+}{m_-} = \frac{2+\sqrt{3}}{2-\sqrt{3}} = (2+\sqrt{3})^2 = 7 + 4\sqrt{3} \approx 13.928\,. \tag{39}$$

Application to the quark seed. The SQCD realisation of the O'Raifeartaigh mechanism identifies the light fermion eigenvalue with the confined strange quark: $m_- = m_s$. Then the heavy eigenvalue gives

$$m_c^{\mathrm{pred}} = R_{\mathrm{OR}} \times m_s = 13.928 \times 93.4\ \mathrm{MeV} = 1301\ \mathrm{MeV}\,. \tag{40}$$

Quantity	Value
m_c (predicted)	1301 MeV
m_c (PDG 2024)	1275 ± 25 MeV
Deviation	$+2.0\%$ $(+1.0\sigma)$

The prediction uses no fitted parameters beyond the single input $m_s = 93.4$ MeV ($\overline{\mathrm{MS}}$, 2 GeV).

Status. Exact once the seed point $gv/m = \sqrt{3}$ is taken as part of the EFT vacuum data.

5.2 The bion mass relation

The Kähler potential receives non-perturbative corrections from bion configurations—monopole–instanton pairs in the semiclassical analysis on $\mathbb{R}^3 \times S^1$ [6]. Each monopole-instanton carries a single fermionic zero mode per flavour; when lifted by the quark mass m_k, the amplitude acquires a factor $\sqrt{m_k}$. In the colour-flavour locked phase with $N_f = N_c$, each monopole couples to exactly one flavour, so the contributions enter *additively* in the superpotential: $W_{\mathrm{mon}} = \zeta\, e^{-S_0/N_c} \sum_a \sqrt{m_a}$. (For $N_f > N_c$ the mass dependence would be multiplicative, $\prod_k \sqrt{m_k}$, and the sum-of-square-roots structure would not arise.) The leading bion correction to the Kähler potential has the form

$$K_{\mathrm{bion}} = \frac{\zeta^2}{\Lambda^2} \left| \sum_{k=1}^{N_c} s_k \sqrt{\hat{m}_k} \right|^2 , \tag{41}$$

where $\zeta \propto e^{-S_0/(2N_c)}$ is the monopole fugacity, $\hat{m}_k$ are the current quark masses, and $s_k = \pm 1$ are signs determined by the monopole zero-mode structure.

Organising the bion amplitudes into seed-sector and bloom-sector channels:

$$S_{\text{seed}} = \sqrt{m_s} + \sqrt{m_c}\,, \qquad S_{\text{bloom}} = -\sqrt{m_s} + \sqrt{m_c} + \sqrt{m_b}\,, \tag{42}$$

the Kähler correction contains a term

$$\delta K \;\sim\; \lambda_1\,|S_{\text{seed}}|^2 + \lambda_2\,|S_{\text{bloom}} - 2\,S_{\text{seed}}|^2\,. \tag{43}$$

The second factor is a perfect square that vanishes at the minimum. Setting $S_{\text{bloom}} = 2\,S_{\text{seed}}$ gives $-\sqrt{m_s} + \sqrt{m_c} + \sqrt{m_b} = 2(\sqrt{m_s} + \sqrt{m_c})$, hence the v_0-doubling condition:

$$\boxed{\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}\,.} \tag{44}$$

The integer coefficient $3 = 2 + 1$: the factor 2 comes from z_0-doubling; the extra 1 from the sign flip ($\sqrt{m_s}$ enters the seed with $+1$ but the bloom with -1, adding $2\sqrt{m_s}$ to the right-hand side).

Evaluation. Using the predicted $m_c = 1301$ MeV:

$$\sqrt{m_b} = 3 \times \sqrt{93.4} + \sqrt{1301} = 3 \times 9.665 + 36.069 = 65.064\ \text{MeV}^{1/2}\,, \tag{45}$$

$$m_b^{\text{pred}} = 4233\ \text{MeV}\,. \tag{46}$$

If instead the PDG value $m_c = 1275$ MeV is used: $m_b = 4186$ MeV ($+0.2\sigma$ from PDG).

Input m_c	m_b (predicted)	PDG (4180 ± 30)	Pull
1301 MeV (from step 1)	4233 MeV	$+1.8\sigma$	
1275 MeV (PDG)	4186 MeV	$+0.2\sigma$	

The tighter prediction ($+0.15\%$, 0.2σ) uses the measured charm mass and tests the bion relation in isolation. The wider one ($+1.3\%$, 1.8σ) propagates the O'Raifeartaigh prediction and tests the chain.

The principal caveat is that this analysis applies on $\mathbb{R}^3 \times S^1$; connecting to $\mathbb{R}^4$ physics requires Ünsal's continuity conjecture [7], which posits that center-symmetric confinement at small S^1 is continuously connected to the large-S^1 confining vacuum.

5.3 The Yukawa eigenvalue constraint

The heavy quarks (c, b, t) couple to the electroweak sector through the Yukawa superpotential. At $\tan\beta = 1$ (required by the mixed up/down-type structure of the mass triples), the eigenvalue sum rule of the 3×3 mass matrix in the (c, b, t) sector imposes:

$$\boxed{\langle z_k^2 \rangle = z_0^2 \quad \text{for} \quad \mathfrak{z}_k = \sqrt{m_k}, \quad k \in \{c, b, t\}\,.} \tag{47}$$

This is the same structural identity that appears at the O'Raifeartaigh vacuum—an algebraic consequence of the superpotential, not a fitted parameter.

Solving for m_t. Let $x = \sqrt{m_t}$, $A = \sqrt{m_c} + \sqrt{m_b}$, $\Sigma = m_c + m_b$. The constraint $3(\Sigma + x^2) = 2(A + x)^2$ gives:

$$\sqrt{m_t} = 2(\sqrt{m_c} + \sqrt{m_b}) + \sqrt{3}\sqrt{m_c + m_b + 4\sqrt{m_c\, m_b}}\,. \tag{48}$$

From the self-consistent chain ($m_c = 1301$, $m_b = 4233$ MeV):

$$\begin{aligned} A &= \sqrt{1301} + \sqrt{4233} = 36.07 + 65.06 = 101.13\,, \\ \sqrt{m_t} &= 2 \times 101.13 + \sqrt{44762} = 202.26 + 211.57 = 413.83\,, \\ m_t^{\text{pred}} &= 171{,}250 \text{ MeV}\,. \end{aligned} \tag{49}$$

Quantity	Value
m_t (predicted, chain)	171,250 MeV
m_t (PDG pole)	$172{,}760 \pm 300$ MeV
Deviation	-0.87%

Scheme dependence. The chain input $m_s = 93.4\,\text{MeV}$ is $\overline{\text{MS}}$ at $2\,\text{GeV}$, while the PDG top mass $172.76 \pm 0.30\,\text{GeV}$ is the pole mass. The two differ by the QCD self-energy: $m_t^{\text{pole}}/\bar{m}_t(\bar{m}_t) \approx 1.06$ at two loops, giving $\bar{m}_t(\bar{m}_t) = 162.5 \pm 1.1\,\text{GeV}$. Converting the prediction to $\overline{\text{MS}}$ at the same order yields $\bar{m}_t(\bar{m}_t) \approx 161.3\,\text{GeV}$ (-1.1σ from PDG). The dominant systematic is the chain's mixed-scale nature—light quarks at $2\,\text{GeV}$, heavy quarks effectively at $m_q(m_q)$—which introduces an $\mathcal{O}(1\,\text{GeV})$ scheme ambiguity. The prediction is therefore stated as $m_t = 171.3 \pm 1.5\,\text{GeV}$, where the uncertainty reflects this scheme mismatch.

Status. Exact within the EFT once the mixed up/down Yukawa block is written at $an\beta = 1$; comparison with data inherits the scheme and scale ambiguity of the mixed quark-mass inputs.

5.4 The dual mass-charge identity from the Seiberg seesaw

The Seiberg seesaw (Section 4) maps quark masses m_j to meson VEVs $\langle M_j \rangle = C/m_j$, inverting the mass hierarchy. This inversion acts on the mass charges: if the direct charges are $\mathfrak{z}_k = \sqrt{m_k}$, the *dual charges* are $\xi_k \propto 1/\sqrt{m_k}$. Defining $\xi_0 = \frac{1}{3}\sum_k \xi_k$, the dual mass-charge identity is

$$\langle \xi_k^2 \rangle = \xi_0^2\,, \qquad \text{equivalently} \quad \frac{\sum_k 1/m_k}{\left(\sum_k 1/\sqrt{m_k}\right)^2} = \frac{2}{3}\,. \tag{50}$$

At the O'Raifeartaigh seed, one mass vanishes: $(m_1, m_2, m_3) = (0, m_-, m_+)$. In this limit the direct and dual identities coincide: with $\mathfrak{z}_1 = 0$,

$$\frac{m_- + m_+}{(\sqrt{m_-} + \sqrt{m_+})^2} = \frac{1/m_- + 1/m_+}{(1/\sqrt{m_-} + 1/\sqrt{m_+})^2}\,, \tag{51}$$

since both simplify to $(m_- + m_+)/(m_- + m_+ + 2\sqrt{m_- m_+})$ when the zero-mass entry is absent from all sums. At $gv/m = \sqrt{3}$ both equal $2/3$ exactly: the seed is *self-dual.*

After the bloom ($m_1 > 0$), the direct and dual ratios diverge. For the down-type quarks (d, s, b), the direct ratio $\langle \mathfrak{z}_k^2 \rangle / \mathfrak{z}_0^2 = 0.732$ is far from unity, while the dual ratio $\langle \xi_k^2 \rangle / \xi_0^2 = 0.665/0.667 = 0.997$ deviates by only 0.22%. The seesaw preserves the mass-charge identity much better than the direct masses do, because it maps the bloomed spectrum back toward the seed hierarchy $m_{\text{light}} \ll m_{\text{heavy}}$.

Setting $\langle \xi_k^2 \rangle = \xi_0^2$ exactly on (d, s, b), and taking m_s and m_b as inputs, determines m_d:

$$m_d^{\text{pred}} = 4.605\,\text{MeV} \qquad (\text{PDG: } 4.67 \pm 0.48\,\text{MeV}, \ \ -0.13\sigma)\,. \tag{52}$$

This prediction is a direct consequence of the Lagrangian: the seesaw $\langle M_j \rangle = C/m_j$ follows from the F-term equations $\partial W/\partial M_j = 0$ of the superpotential $W = \text{Tr}(\hat{m}\, M) + X(\det M - \Lambda^6)$. The dual mass-charge identity is therefore not an independent assumption but an algebraic property of the seesaw vacuum.

Status. Exact in the seesaw vacuum of the light block; its application to the physical (d, s, b) spectrum assumes that the bloomed down-type triple is still well described by the same dual map.

5.5 The complete mass chain

Five quark masses from one input:

$$\boxed{\begin{array}{l} m_u + m_d = 6.83 \text{ MeV} \quad \text{(input)} \\ \qquad\qquad \xrightarrow{\langle z_k^2 \rangle \approx z_0^2 \text{ on } (0, m_u + m_d, m_s)} m_s = 95.1 \text{ MeV} \quad [+1.8\%] \\ \qquad\qquad \xrightarrow{R_{\text{OR}} = 7 + 4\sqrt{3}} m_c = 1301 \text{ MeV} \quad [+2.0\%] \\ \qquad\qquad \xrightarrow{\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}} m_b = 4233 \text{ MeV} \quad [+1.3\%] \\ \qquad\qquad \xrightarrow{\langle z_k^2 \rangle = z_0^2 \text{ on } (c,b,t)} m_t = 171{,}250 \text{ MeV} \quad [-0.9\%] \\ \qquad\qquad \xrightarrow{\langle \xi_k^2 \rangle \approx \xi_0^2 \text{ on } (d,s,b)} m_d = 4.6 \text{ MeV} \quad [-1.5\%] \end{array}} \tag{53}$$

The chain spans a factor of 2.5×10^4 in mass (from $m_u + m_d$ to m_t), controlled by four structural constants of the Lagrangian: the isospin seed ratio, the O'Raifeartaigh mass ratio R_{OR}, the bion doubling (with coefficient $N_c = 3$), and the eigenvalue sum rule $\langle z_k^2 \rangle = z_0^2$; plus the dual mass-charge identity from the Seiberg seesaw. None of these is fitted; each is an algebraic consequence of the superpotential at its stabilised vacuum. The isospin and dual mass-charge identity are approximate ($\langle z_k^2 \rangle / z_0^2 \approx 1$, bloomed from the exact seed); the remaining three are exact.

6 The lepton sector

The lepton block is not introduced as an independent sector with its own microscopic derivation. The claim is narrower and more useful for EFT. If supermultiplet restoration identifies the fermionic components of the three meson superfields with the charged leptons in the symmetry-restored limit, then the same 3×3 O'Raifeartaigh algebra that appears in the confined meson block must reappear in the lepton block. What is transferred is the seed algebra, not a second copy of the full SQCD derivation.

6.1 Leptons as the second copy of the seed algebra

Write the lepton superpotential in the same algebraic form as the mesonic seed,

$$W_{\text{lepton}} = f\,X_L + m\,L_1L_3 + g\,X_LL_1^2\,. \tag{54}$$

At the metastable vacuum $\langle L_1\rangle = \langle L_3\rangle = 0$ and $\langle X_L\rangle = v$, the massive fermion block is

$$M_L(v) = \begin{pmatrix} 2gv & m \\ m & 0 \end{pmatrix}, \tag{55}$$

with eigenvalues

$$m_\pm = \sqrt{g^2v^2 + m^2} \pm gv\,. \tag{56}$$

At the distinguished point $t \equiv gv/m = \sqrt{3}$ this becomes

$$(0,\, m_-,\, m_+) = (0,\, (2-\sqrt{3})m,\, (2+\sqrt{3})m)\,. \tag{57}$$

The K"ahler geometry is inherited as well,

$$K_{\text{lepton}} = |X_L|^2\left(1 - \frac{|X_L|^2}{3\mu_K^2}\right) + |L_1|^2 + |L_2|^2 + |L_3|^2, \qquad \mu_K = \sqrt{2}\,m/g\,. \tag{58}$$

As in the mesonic block, the pole at $|X_L|^2 = 3\mu_K^2$ isolates the same special point in field space but does not by itself prove vacuum stabilization there. In the lepton sector, therefore, $t = \sqrt{3}$ is not an additional input: it is inherited from the common seed that would exist in the symmetry-restored limit.

6.2 Why the quadratic charge condition reappears

The relation

$$\langle z_k^2\rangle = z_0^2 \tag{59}$$

should not be read as an extra postulate once the block (55) is fixed at $t = \sqrt{3}$. It is simply the coordinate form of the two O'Raifeartaigh invariants

$$m_+ + m_- = 4m, \qquad m_+m_- = m^2, \tag{60}$$

with one additional zero mode. Indeed, for the seed triple $(0, m_-, m_+)$,

$$\frac{m_- + m_+}{(\sqrt{m_-} + \sqrt{m_+})^2} = \frac{4m}{(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 m} = \frac{2}{3}, \tag{61}$$

which is equivalent to (59). In other words, within the seed algebra the quadratic charge condition is forced by the same trace and determinant data that define the O'Raifeartaigh block.

It is useful to separate this statement from the choice of coordinates used to write a given mass triple. For any ordered triple X one may write

$$\sqrt{m_k^{(X)}} = x_0 + x_k, \qquad \sum_k x_k = 0, \tag{62}$$

and the corresponding Koide form is $\langle x_k^2 \rangle = x_0^2$. For a different triple Y one writes instead $\sqrt{m_k^{(Y)}} = y_0 + y_k$. The observables are the masses, not the affine coordinates (x_0, x_k) or (y_0, y_k). The same physical energy can therefore appear in two different tuples with

$$E_A = (x_0 + x_i)^2 = (y_0 + y_j)^2, \tag{63}$$

without requiring $x_0 + x_i = y_0 + y_j$ as coordinates. The charges are triple-dependent coordinates on the same seed orbit; only the eigenvalue E_A is physical. This is the precise sense in which the symmetry-restored limit may contain equal masses across quark, lepton, and meson sectors even though the associated charge decompositions differ from tuple to tuple.

6.3 Lepton mass relation

Once the charged leptons are identified with one copy of the seed algebra, the physical lepton triple should remain close to the seed because there is no colour sector and hence no bion-induced v_0 doubling. The observed lepton masses are therefore modeled as a small angular deformation of the seed at nearly fixed radius. Using pole masses,

$$\langle z_k^2 \rangle = z_0^2 \qquad \text{for} \qquad \mathfrak{z}_k = \sqrt{m_k}, \quad k \in \{e, \mu, \tau\}, \tag{64}$$

leads to the standard quadratic equation for m_τ. With m_e and m_μ as input one finds

$$m_\tau^{\text{pred}} = 1776.97 \text{ MeV}. \tag{65}$$

Quantity	Value
m_τ (predicted)	1776.97 MeV
m_τ (PDG 2024)	1776.86 ± 0.12 MeV
Deviation	$+0.006\%$ $(+0.9\sigma)$

The lepton block is therefore the cleanest test of the seed algebra: it uses pole masses, requires no threshold matching between quark sectors, and does not invoke the bion deformation.

Neutrino note. The field L_2 remains massless at tree level. A higher-dimensional coupling $L_1 L_2 L_3/\Lambda_L^3$ would induce a Coleman–Weinberg mass of order

$$m_{L_2} \sim \frac{g^2 m}{4\pi} \left(\frac{m}{\Lambda_L} \right)^6, \tag{66}$$

placing the pseudo-modulus in the atmospheric window for $\Lambda_L \sim 9$–12 GeV if $g = O(1)$. This is only a scale estimate; the electroweak quantum numbers of the state are not fixed by the present EFT.

6.4 Parameter counting for the lepton block

The lepton block uses one dimensional scale and one angular deformation:

Parameter	Role	Count
m_{OR}	overall seed scale	1
δ_{bloom}	departure from the seed angle	1
$gv/m = \sqrt{3}$	inherited seed condition	0

Thus any two charged-lepton masses determine the third. What remains open is not the algebra of the lepton block but its microscopic identification with the meson supermultiplets of the confined theory.

7 Electroweak consequences and extensions

The material in this section lies one step beyond the light-flavour EFT core. It is included because the same framework points naturally toward these structures, but the logical status is weaker than in Sections 4 and 5. The useful question here is not whether these relations are exact, but whether the EFT strongly prefers them over nearby alternatives.

7.1 Higgs sector at $\tan\beta = 1$

At $\tan\beta = 1$ the MSSM D-term quartic vanishes on the D-flat direction, so the tree-level light Higgs mass is zero. A viable 125 GeV Higgs then requires an additional quartic. In an NMSSM completion with

$$W_H = \lambda_S S H_u \cdot H_d, \tag{67}$$

one finds at $\sin 2\beta = 1$

$$m_h^2 = \frac{\lambda_S^2 v^2}{2} \qquad \Rightarrow \qquad \lambda_S \simeq 0.72 \text{ for } m_h \simeq 125 \text{ GeV}. \tag{68}$$

The Seiberg multiplier X cannot play the role of S: because $[X] = \text{mass}^{-3}$, the operator $XH_u \cdot H_d$ would require a highly irrelevant dimensionful coupling and cannot reproduce the needed quartic at the mesonic scale. Within the present framework the economical conclusion is that the Higgs singlet, if used, must be elementary rather than part of the confined block.

Quantity	Value
m_h (tree-level, $\lambda_S = 0.72$)	125.35 GeV
m_h (PDG 2024)	125.25 ± 0.17 GeV
Pull	0.6σ

7.2 Cabibbo mixing from the inverse hierarchy

The clean electroweak consequence of the seesaw vacuum is the 1–2 mixing angle. For a nearest-neighbour texture,

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}, \tag{69}$$

with $|A| \ll |B| \ll C$, one obtains

$$m_1 \approx \frac{|A|^2 C}{|B|^2}, \qquad m_2 \approx \frac{|B|^2}{C}, \qquad m_3 \approx C, \tag{70}$$

and therefore the GST relation

$$\sin\theta_C \approx \sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{4.67}{93.4}} = 0.2236\,. \tag{71}$$

Numerically this agrees well with the PDG value,

Method	$\sin\theta_C$	θ_C
$\sqrt{m_d/m_s}$ (GST)	0.2236	12.92°
$\lvert V_{us}\rvert$ (PDG 2024)	0.2243 ± 0.0008	12.96°
Deviation	-0.3%	(-0.9σ)

Within the present EFT this should be read as the first entry of the mixing problem, not as its full solution. The light-sector inverse hierarchy naturally produces the GST form, so the Cabibbo angle is the piece of CKM structure that already sits close to the seed algebra. What is still missing is the dynamical origin of the misalignment between the up-type and down-type flavour bases once the heavy sector is included.

A useful clue is that the tuple organization itself seems to prefer different generation orderings in different sectors. In the numerical triples that motivate the present construction, the natural pairings do not line up with the standard (u, c, t) and (d, s, b) order simultaneously; one is led instead to an ordering pattern of the form (ts), (ub), (cd) when the tuples are read as spectral data. That observation is not yet a derivation of CKM mixing, but it points to the right kind of mechanism: mixing should arise from the mismatch between the ordering that diagonalizes the seed-derived tuples and the ordering selected by electroweak doublets. Put differently, the Cabibbo angle is likely the first visible relic of a more general left-handed basis misalignment.

For that reason the honest status of CKM in the present paper is the following. The framework gives a plausible origin for V_{us} through the GST texture, and it suggests that nontrivial generation ordering is the correct clue for the rest of the CKM matrix. It does not yet derive V_{cb}, V_{ub}, or the CKM phase. Achieving that would require a controlled heavy-sector completion in which the same tuple algebra generates distinct up- and down-sector unitary rotations.

A useful way to formulate the missing step is to write

$$V_{\rm CKM} = U^\dagger_{u_L} U_{d_L}\,. \tag{72}$$

The present construction essentially controls one eigenvalue problem at a time: it gives a seed algebra, a hierarchy, and a preferred tuple ordering. What it does not yet provide is a simultaneous dynamical construction of the two left-handed rotations U_{u_L} and U_{d_L} in the same electroweak basis. That is why the observed clue about reordered tuples is interesting. It suggests that the nontrivial CKM structure is not an extra decorative ingredient but a necessary consequence of fitting the same spectral data into two inequivalent orderings.

From this perspective the present paper already contains a testable research program for CKM. One should build the heavy-sector completion so that the up and down mass matrices inherit the same seed spectrum but with different admissible orderings, then ask whether the mismatch reproduces the observed hierarchy $|V_{us}| \gg |V_{cb}| \gg |V_{ub}|$. The current paper stops at the first step of that ladder. What it contributes is the reason to believe that the ladder is there at all.

8 Complete particle spectrum and the restoration problem

This section collects the spectrum generated by the EFT and separates it from the stronger statement one would like to prove, namely full supermultiplet restoration across quark, lepton, and meson sectors. The current EFT achieves the meson and lepton copies of the seed algebra explicitly. The quark/diquark copy remains incomplete because of the symmetric **15** obstruction.

8.1 Seed spectrum and soft deformation

At general $t = gv/m$ and $y = gf/m^2$ the scalar mass matrices of the O'Raifeartaigh block are

$$\frac{\mathcal{M}_\sigma^2}{m^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4t^2{+}2y{+}1 & 2t \\ 0 & 2t & 1 \end{pmatrix}, \qquad \frac{\mathcal{M}_\pi^2}{m^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4t^2{-}2y{+}1 & 2t \\ 0 & 2t & 1 \end{pmatrix}. \tag{73}$$

The B-term $B_{11} = 2gf$ is the only source of the σ–π splitting. At $t = \sqrt{3}$ the fermion eigenvalues are $(0, m_-, m_+)$ with $m_\pm = (2 \pm \sqrt{3})m$, while in the SUSY limit $y \to 0$ the scalar partners are doubly degenerate at the same masses. The supertrace vanishes identically for all t and y before the universal soft meson mass $\tilde{m}^2 = f_\pi^2$ is added.

Stage	Fermions	Scalars	STr
0. Full SUSY ($y = 0$)	$(0,\, m_-,\, m_+)$	$\sigma = \pi = (0,\, m_-^2,\, m_+^2)$; exact pairs	0
1. F-breaking ($f \neq 0$)	unchanged	B-term splits: $\pi_k^2 < m_k^2 < \sigma_k^2$	0
2. CW lifting	unchanged	only the pseudo-modulus multiplet is lifted	0
3. Bion deformation	radial deformation of the seed triple	scalars track the shifted fermion scales	0
4. Angular bloom	δ selected	no new supertrace contribution	0
5. Soft ($\tilde{m}^2 = f_\pi^2$)	unchanged	all mesonic scalars shift by f_π^2	$18\, f_\pi^2$

Table 1: Spectrum flow from the supersymmetric seed to the physical mesonic vacuum. The first four stages preserve the boson–fermion supertrace. A nonzero supertrace appears only after the universal soft meson mass is added.

The point of Table 1 is structural. The seed algebra fixes the three energy levels. The later deformations move the system in controlled ways: first by splitting scalar partners, then by lifting the pseudo-modulus, then by radial and angular deformations of the fermion triple, and finally by a common soft scalar shift.

8.2 Confined mesons, mesinos, and baryons

At the Seiberg vacuum with $\tilde{m}^2 \sim f_\pi^2$ the confined light sector contains:

- twelve off-diagonal real scalar mesons with mass $\sqrt{2}\, f_\pi \approx 130$ MeV;

- six diagonal real scalar modes with mass f_π;
- three off-diagonal mesino Dirac pairs with masses controlled by the inverted hierarchy $|X_0|\langle M_k\rangle \propto 1/m_k$;
- a baryon and antibaryon multiplet with $m_B \simeq \Lambda \simeq 300$ MeV.

The important point is not the exact numerical values of every confined state but the block structure: off-diagonal mesons do not mix through the seesaw vacuum, and the inverse hierarchy resides in the fermionic confined sector. That is the mechanism by which quark ratios re-enter the low-energy mesonic EFT.

For reference, the three off-diagonal mesino Dirac masses are

Pair	Spectator	Dirac mass
$\psi(M^u_d, M^d_u)$	s	1.1×10^{-5} MeV
$\psi(M^u_s, M^s_u)$	d	2.3×10^{-4} MeV
$\psi(M^d_s, M^s_d)$	u	4.9×10^{-4} MeV

so the lightest spectator quark corresponds to the heaviest mesino pair, as expected from the seesaw inversion.

8.3 Restoration criterion and tuple-dependent coordinates

The "symmetry-restoring" limit should be stated as a criterion, not as a list of numerical near-coincidences. Let E_A $(A = 0, 1, 2)$ denote the three seed energy levels. Full restoration would mean that there exist sector-dependent unitary rotations U_Q, U_L, and U_M such that

$$M_Q = U_Q \,\mathrm{diag}(E_0, E_1, E_2)\, U_Q^\dagger, \qquad M_L = U_L \,\mathrm{diag}(E_0, E_1, E_2)\, U_L^\dagger, \qquad M_M = U_M \,\mathrm{diag}(E_0, E_1, E_2)\, U_M^\dagger. \tag{74}$$

In that limit the sectors share the same three energies even if they use different tuple coordinates to describe them.

Concretely, for the quark, lepton, and meson triples one may write

$$E_A = (x_0 + x_A)^2 = (y_0 + y_A)^2 = (z_0 + z_A)^2, \tag{75}$$

with separate traceless conditions on x_A, y_A, and z_A. The affine data (x_0, x_A), (y_0, y_A), and (z_0, z_A) are not themselves observables; they are sector-dependent coordinates on the same energy triple. This is why a single energy can arise from different "charges" in different tuples without contradiction. What the EFT must reproduce dynamically is the common spectrum $\{E_A\}$, not equality of the affine decompositions.

This point matters especially for the mesonic sector. Once the scalar O'Raifeartaigh eigenstates are identified with confined mesonic channels, the same affine-square parametrization predicts mesonic seed and bloomed tuples, not only leptonic or quark ones. The companion phenomenology note gives the explicit examples $(0, \pi, D_s)$ and $(-\pi, D_s, B)$ [2]. Their role here is not primarily to add one more numerical coincidence, but to mark which observed mesons are being guided toward the bosonic slots of the restored effective supermultiplets. In that sense the meson tuples are part of the restoration criterion itself: they exhibit the bosonic realization of the same common spectral labels that must also appear in the lepton and quark images.

A compact way to summarize the situation is:

Sector	Algebraic seed	Physical interpretation	Status
Mesons	explicit	confined $q\bar{q}$ block	realized
Leptons	explicit copy	meson–lepton image	realized conditionally
Quark/diquark sector	target only	requires symmetric **15**	open

The restoration problem is therefore not to discover one more numerical coincidence. It is to upgrade the third row of this table from "target only" to "explicit" without destroying the first two. Any successful completion must preserve the common eigenvalue structure while explaining why the quark copy is both composite and compatible with Fermi statistics.

The present paper realizes Eq. (74) only partially. The meson and lepton blocks share the same O'Raifeartaigh seed algebra. The third copy, in the quark/diquark sector, is blocked by the unresolved symmetric **15**. For that reason supermultiplet restoration is presently a criterion for completion of the program rather than a theorem.

8.4 Heavy sector and parameter budget

The heavy charged-fermion chain produced by the EFT is summarized by

Relation	Predicts	From
Isospin seed: $m_s \approx (2+\sqrt{3})^2(m_u+m_d)$	m_s from m_u+m_d	bloomed seed relation
O'Raifeartaigh: $m_c/m_s = (2+\sqrt{3})^2$	m_c from m_s	seed at $gv/m = \sqrt{3}$
Bion: $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$	m_b from m_s, m_c	radial deformation
Yukawa: $\langle z_k^2\rangle = z_0^2$ on (c, b, t)	m_t from m_c, m_b	heavy-sector Yukawa block
Dual: $\langle \xi_k^2\rangle = \xi_0^2$ on (d, s, b)	m_d from m_s, m_b	Seiberg seesaw
Lepton: $\langle z_k^2\rangle = z_0^2$ on (e, μ, τ)	m_τ from m_e, m_μ	lepton O'Raifeartaigh block
GST: $\sin\theta_C = \sqrt{m_d/m_s}$	θ_{12} from m_d, m_s	inverse hierarchy
Remaining free:	m_u, m_e, m_μ, θ_{23}, θ_{13}, δ_{CP} (6 parameters)	

Renormalisation scale. The quark masses appearing in the mass-charge triples are $\overline{\mathrm{MS}}$ masses at the PDG conventional scales: $m_{u,d,s}$ at $\mu = 2$ GeV, $m_c(m_c)$, $m_b(m_b)$, and $m_t(m_t)$. Pure QCD running rescales all masses in a given triple by a common factor, so ratios of the form

$$\frac{\sum m_k}{(\sum \sqrt{m_k})^2} \tag{76}$$

are exactly invariant under one-loop QCD evolution. The observed departures of the quark triples from the exact seed value $2/3$ therefore measure bloom away from the seed rather than ordinary QCD running. The preferred scale convention for the cross-threshold triples remains the natural one for the EFT: each heavy quark evaluated at its own mass threshold.

Quantity	Predicted	PDG 2024	Source
m_s	95.1 MeV	93.4 ± 8.6 (0.2σ)	Isospin seed
m_c	1,301 MeV	$1{,}275 \pm 25$ (1.0σ)	O'Raifeartaigh: $gv/m = \sqrt{3}$
m_b	4,233 MeV	$4{,}180 \pm 30$ (1.8σ)	Bion Kähler
m_t	171.3 ± 1.5 GeV	172.76 ± 0.30	Yukawa eigenvalue constraint
m_d	4.6 MeV	4.67 ± 0.48 (0.1σ)	Dual mass-charge identity
m_u/m_d	0.453	0.46 ± 0.05 (0.2σ)	Self-consistency
m_τ	1776.97 MeV	1776.86 ± 0.12 (0.9σ)	Lepton mass-charge identity
$\sin^2\theta_W$	0.22310	0.22320 ± 0.00026 (0.4σ)	Casimir equation
V_{us}	0.2236	0.2243 ± 0.0008 (0.9σ)	GST relation
m_h	125.35 GeV	125.25 ± 0.17 (0.6σ)	NMSSM: $\lambda_S v/\sqrt{2}$
$\tan\beta$	1	— (not directly measured)	Mixed-triple structure
$\mathrm{STr}[M^2]$	$18\, f_\pi^2$	— (not directly measured)	Soft mass only

Table 2: Numerical output of the EFT framework. The table deliberately mixes three logical levels: exact seed consequences once $gv/m = \sqrt{3}$ is imposed, conditional relations that also use the bion or seesaw deformations, and more exploratory electroweak consequences. All quark predictions follow from a single input $m_u + m_d = 6.83\,\mathrm{MeV}$ (equivalently m_π via GMOR); the isospin seed gives m_s, the O'Raifeartaigh chain gives m_c, the bion gives m_b, and the dual mass-charge identity gives m_d (hence m_u). Cross-threshold masses are $\overline{\mathrm{MS}}$ at $\mu = m_q$. The lepton prediction uses m_e and m_μ as inputs.

9 Summary of EFT results, assumptions, and parameter reduction

The table is best read as a map rather than as a homogeneous block of predictions. The entries for m_c and the mass-charge seed identities belong to the most rigid part of the construction. The entries for m_b, m_d, and m_t require progressively more of the EFT scaffold: radial bion structure, dual-charge consistency, and the heavy-sector Yukawa constraint. The Higgs line sits one step further out, because it depends on the choice of electroweak completion. This hierarchy is exactly what underwrites the statement that the framework fixes seven charged-fermion-sector parameters: the reduction is real, but it is a reduction *within the EFT* rather than a theorem that every Standard Model parameter has been absorbed into a unique microscopic completion.

10 Ultraviolet constraints suggested by the $SO(32)$ embedding

This section is kept only to the extent that it helps the EFT. The seed algebra, the O'Raifeartaigh spectrum, and the charged-fermion mass chain do not require a string embedding. The ultraviolet discussion earns its place only where it does one of three things: it explains why the flavour target $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$ recurs, it motivates the triplication structure behind the low-energy bookkeeping, or it suggests specific operators and basis changes that may control the unfinished EFT parameters.

Read in that way, the string material is not part of the proof of the low-energy claims. It is a source of constraints and clues. In particular, the Type I $SO(32)$ adjoint already contains the meson and diquark representations singled out in the earlier scalar-bootstrap work, the $T^6/\mathbb{Z}_3$ orbifold supplies a natural origin for three chiral copies, and the orbifold geometry suggests why the EFT repeatedly organises itself around a distinguished $\mathbb{Z}_3$-compatible structure.

10.1 Open strings and Chan-Paton labels

Type I open superstrings provide a concrete ultraviolet setting in which the microscopic degrees of freedom are labelled by endpoint indices. Each open string state $|i, j\rangle$ carries one Chan-Paton label at each end and can be viewed as a matrix unit acting on the space of string endpoints. In that language, gauge bosons arise from the massless vector mode tensored with the Chan-Paton algebra, and the orientifold projection keeps precisely the combinations compatible with the unoriented worldsheet. Anomaly cancellation then fixes the ten-dimensional gauge group to $SO(32)$ [27, 28]. The point of recalling this standard construction is not to rederive string theory here, but to note that the endpoint labels furnish the kind of "pre-composite" indices one would want if low-energy states are to carry nontrivial flavour and colour labels before any constituent-quark picture is imposed.

This viewpoint matters for the symmetric $\mathbf{15}$. The Pauli theorem of Section 3 rules out that representation as an ordinary scalar qq composite in the two-quark Hilbert space. A string embedding can only help if the low-energy $\mathbf{15}$ is not the same object made merely larger, but a different microscopic state whose flavour and colour quantum numbers descend from Chan-Paton and worldsheet structure rather than from the exchange symmetry of two pointlike quarks. In that sense the UV loophole is not "extended size" by itself, but a change of microscopic category.

10.2 The $SO(32)$ adjoint contains the EFT target

The 32-dimensional real fundamental of $SO(32)$ decomposes under $SU(16) \times U(1)_A$ as

$$\mathbf{32} = \mathbf{16}_{+1} \oplus \overline{\mathbf{16}}_{-1} \, . \tag{77}$$

Taking the complex $\mathbf{16}$ further under $SU(5) \times SU(3) \times U(1)_B$ gives

$$\mathbf{16} = (\mathbf{5}, \mathbf{3})_{+1} \oplus (\mathbf{1}, \mathbf{1})_{-15} \, . \tag{78}$$

The 15 entries $(\mathbf{5}, \mathbf{3})$ are the five participating quark flavours in three colours, while the singlet can be identified with the top quark. In this sense the earlier scalar-bootstrap counting and the string construction point to the same bookkeeping block,

$$5 \times 3 + 1 = 16 \, . \tag{79}$$

The real payoff comes from the adjoint $\mathbf{496} = \wedge^2(\mathbf{32})$. Under $SU(5) \times SU(3)$ it contains the distinguished pieces $(\mathbf{24}, \mathbf{1})$ and $(\mathbf{15}, \overline{\mathbf{3}}) \oplus (\overline{\mathbf{15}}, \mathbf{3})$, namely the meson and diquark representations that define the EFT target. The full decomposition is displayed in Table 3; what matters for the present paper is that the ultraviolet representation content is not guessed after the fact but already sits inside the $SO(32)$ adjoint.

The same decomposition also clarifies the status of the symmetric flavour representation. The identity

$$\wedge^2(V \otimes W) = \left[S^2(V) \otimes \wedge^2(W)\right] \oplus \left[\wedge^2(V) \otimes S^2(W)\right] \tag{80}$$

shows that the flavour-symmetric $\mathbf{15} = S^2(\mathbf{5})$ is naturally paired with the colour-antisymmetric $\overline{\mathbf{3}} = \wedge^2(\mathbf{3})$. In other words, the UV theory wants precisely the representation that the ordinary two-quark scalar picture fails to realise. That tension is one of the main reasons to keep the string perspective in the paper at all.

Sector	$SU(5)\times SU(3)$	dim	q_A	q_B	Origin
255$\oplus$**1**	$(\mathbf{24},\mathbf{8})$	192	0	0	adj($SU(15)$)
	$\boldsymbol{(24,1)}$	**24**	0	0	**mesons**
	$(\mathbf{1},\mathbf{8})$	8	0	0	adj($SU(3)$)
	$(\mathbf{5},\mathbf{3})+(\bar{\mathbf{5}},\bar{\mathbf{3}})$	30	0	±16	cross-terms
	$2\times(\mathbf{1},\mathbf{1})$	2	0	0	$U(1)_{A,B}$
120	$\boldsymbol{(15,\bar{3})}$	**45**	$+2$	$+2$	**diquarks**
	$(\overline{\mathbf{10}},\mathbf{6})$	60	$+2$	$+2$	$\wedge^2(\mathbf{5})\otimes S^2(\mathbf{3})$
	$(\mathbf{5},\mathbf{3})$	15	$+2$	-14	cross-term
$\overline{\mathbf{120}}$	$\boldsymbol{(\overline{15},3)}$	**45**	-2	-2	$\overline{\textbf{diquarks}}$
	$(\mathbf{10},\bar{\mathbf{6}})$	60	-2	-2	conjugate
	$(\bar{\mathbf{5}},\bar{\mathbf{3}})$	15	-2	$+14$	conjugate
	Total	**496**			

Table 3: Decomposition of the $SO(32)$ adjoint under $SU(5)_{\text{flavour}}\times SU(3)_{\text{colour}}$. The boldface entries are the EFT target states: mesons $(\mathbf{24},\mathbf{1})$ and diquarks $(\mathbf{15},\bar{\mathbf{3}})\oplus(\overline{\mathbf{15}},\mathbf{3})$.

10.3 The $\mathbb{Z}_3$ orbifold and three copies

The ultraviolet structure becomes useful for the EFT again when the extra six dimensions are compactified. A $T^6/\mathbb{Z}_3$ orbifold is the simplest choice that preserves $\mathcal{N}=1$ supersymmetry in four dimensions while producing a triplication compatible with the three-copy bookkeeping already present in the low-energy spectrum. The $\mathbb{Z}_3$ action on the three complex coordinates,

$$\theta:(z_1,z_2,z_3)\longrightarrow(\omega z_1,\omega z_2,\omega z_3)\,,\qquad \omega=e^{2\pi i/3}\,,\tag{81}$$

is the unique symmetric twist of this type that preserves exactly one four-dimensional supercharge.

Embedding the same $\mathbb{Z}_3$ action in Chan-Paton space with $\mathrm{Tr}(\gamma_\theta)=1$ leads to the unique multiplicities

$$n_0=6\,,\qquad n_1=n_2=5\,.\tag{82}$$

The corresponding partition of the neutral block yields the gauge factor

$$G=SU(5)_{\text{flavour}}\times SU(3)_{\text{colour}}\times U(1)^2\,.\tag{83}$$

The associated chiral spectrum contains three copies of the relevant matter content,

$$3\times\Big[(\mathbf{10},\mathbf{1})+(\bar{\mathbf{5}},\mathbf{3})+(\mathbf{5},\bar{\mathbf{3}})\Big]\,.\tag{84}$$

For the purposes of the EFT, the essential lesson is modest but useful: the ultraviolet construction naturally produces the same triplication and the same $SU(5)_f\times SU(3)_c$ bookkeeping that the low-energy framework needs.

At the strict orbifold point, however, the projection gives the antisymmetric **10** rather than the symmetric **15**. The value of the UV section is therefore not that it solves the diquark problem already, but that it localises it sharply: the desired symmetric state is not generic across the moduli space but would require a deformation of the orientifold/orbifold data. That is already actionable information for future model building.

10.4 UV clues for the distinguished EFT parameters

Two low-energy structures invite a geometric or instantonic reinterpretation: the special ratio $gv/m = \sqrt{3}$ that organises the seed algebra, and the higher-harmonic deformation that would have to control the bloom angle. Neither is derived here from first principles, but the string embedding does suggest where one should look.

First, the $\mathbb{Z}_3$ orbifold requires each two-torus to be compatible with a threefold rotation, which picks out the $SU(3)$ root lattice. Its complex structure has

$$\tau = e^{i\pi/3}, \qquad \mathrm{Im}(\tau) = \frac{\sqrt{3}}{2}. \tag{85}$$

This does not prove the vacuum condition $gv/m = \sqrt{3}$, but it does show that the same number appears naturally in the unique geometry underlying the triplication. The right claim is therefore not that the Lagrangian already forces the vacuum through the orbifold, but that the UV geometry singles out the same irrational constant that the EFT seed treats as distinguished.

Second, the three-instanton operator

$$W_3 = c_3 \, \frac{(\det M^{(3)})^3}{\Lambda^{18}} \tag{86}$$

fits naturally with the product structure of $T^6 = (T^2)^3$. The exact identity

$$\prod_{k=1}^{3} \left[1 + \sqrt{2}\cos(\delta + 2\pi k/3)\right] = -\tfrac{1}{2} + \tfrac{\cos 3\delta}{\sqrt{2}} \tag{87}$$

shows that the determinant already knows about the $\mathbb{Z}_3$ harmonic of one torus, while the cube of the determinant is precisely the sort of operator one would expect if the three tori contribute multiplicatively. This is not yet a derivation of the bloom potential, but it is a concrete UV hint about why the EFT should organise its angular dependence in multiples of 3δ.

11 Open problems

The program is now sharp enough that its unresolved points can be stated without vagueness. They are not miscellaneous loose ends. They fall into two principal classes. One is structural: how the symmetric flavour **15** is to be realized despite the Pauli obstruction on ordinary scalar diquarks. The other is dynamical: how the distinguished seed and its subsequent bloom are to be selected by the full effective potential. The subsections below keep those two problems separate, because they are the two obstacles that currently prevent the framework from being promoted from a conditional EFT to a finished microscopic theory.

11.1 The diquark sector

The Pauli theorem of Section 3 proves that no scalar, colour-$\bar{\mathbf{3}}$ two-quark state can transform as a symmetric flavour **15**. The obstruction is therefore not a detail of the binding force; it is a statement about the ordinary two-quark Hilbert space. This is why the low-energy **15** cannot simply be interpreted as a more tightly bound version of an ordinary scalar diquark.

The most conservative reading is that the EFT is signalling the need for a different microscopic object. In the ultraviolet language of Section 10, the candidate would be an open-string state with the same $SU(5)_f \times SU(3)_c$ quantum numbers, whose symmetry properties are fixed by Chan-Paton and worldsheet data rather than by exchange symmetry of two constituent quarks. In the algebraic language of the bootstrap, the same state can be viewed as the missing bosonic partner that the low-energy supercharge wants to exist even though the point-particle diquark construction cannot supply it.

What is still missing is an explicit four-dimensional realization of that object. At the strict $T^6/\mathbb{Z}_3$ orbifold point one obtains the antisymmetric **10** rather than the symmetric **15**, so the string picture does not yet solve the problem outright. What it does do is sharpen the target: one needs a deformation of the orientifold/orbifold data, or another UV completion with the same low-energy representation content, in which the symmetric **15** appears as a genuine state rather than as a forbidden qq composite. The exotic $+4/3$ states remain the clearest observable consequence of that sector.

11.2 The bloom mechanism

The mass-charge identity constrains but does not uniquely fix the angular parameter δ that controls how the seed develops into a full triple. The O'Raifeartaigh vacuum fixes the seed ratio $gv/m = \sqrt{3}$ but leaves δ undetermined.

The lowest-dimension R-breaking deformation that affects the spectrum is the bilinear $\Delta W = \epsilon\, \Phi_0 \Phi_2$, which forces $\langle \Phi_1 \rangle \neq 0$ and mixes the goldstino with the massive sector. (Cubic deformations in Φ_1, Φ_2 alone vanish at the vacuum and are inert.) To first order in $\varepsilon = \epsilon/m$, all eigenvalue shifts vanish, but the bloom angle shifts at $O(\varepsilon)$ because $\delta\delta \sim \sqrt{m_0} \sim \varepsilon$:

$$\delta\delta = 3^{5/4}\, |\varepsilon| \approx 3.95\, |\varepsilon| \,.$$

The direction is *toward* the physical quark spectrum ($\delta > 3\pi/4$). The determinant condition $m_+ m_- = m^2$ is broken at $O(\varepsilon^2)$ with fractional deviation $49\, \varepsilon^2$. The seed is therefore an unstable fixed point under any R-breaking, and the instability points in the right direction. The bloom magnitude, however, requires a nonperturbative mechanism.

A natural candidate for a dynamical origin is the instanton structure of SQCD: for $N_f = N_c = 3$, instantons break $U(1)_{R+2A} \to \mathbb{Z}_{18}$, which contains $\mathbb{Z}_9$ as a subgroup. The three-instanton operator $W_3 = c_3 (\det M)^3/\Lambda^{18}$ generates a δ-dependent potential. To see the harmonic content, note the exact identity

$$\prod_{k=1}^{3} \left[1 + \sqrt{2} \cos(\delta + 2\pi k/3)\right] = -\tfrac{1}{2} + \tfrac{\cos 3\delta}{\sqrt{2}}\,, \tag{88}$$

so $\det M = z_0^6\, f(\delta)^2$ with $f = -1/2 + \cos(3\delta)/\sqrt{2}$. The potential $|W_3|^2 \propto f(\delta)^{12}$ contains all harmonics $\cos(3n\delta)$ for $n = 0, \ldots, 6$; the $\cos(9\delta)$ term (the unique $\mathbb{Z}_9$ harmonic) carries only 34% of the amplitude of the dominant $\cos(3\delta)$ mode. The effective potential therefore has six minima per 2π cycle, not nine.

The bion Kähler potential $\delta K \sim |v_0 - 2v_0^{\text{seed}}|^2$ constrains the overall scale z_0 (radial direction in the Cartan plane), while the three-instanton F-term potential constrains the angle δ. For the physical $(-s, c, b)$ triple, the bloom angle $\delta_{\text{phys}} = 159°$ lies $24°$ past the seed and $21°$ before the nearest instanton well at $180°$, at a *maximum* of the combined effective potential $V_{\text{eff}} = -|f|^{2p} + r\,(S-3)^2$ for all powers p and positive r. The physical point is therefore a *saddle*: stable in the radial direction (bion minimum), unstable in the angular direction (instanton-driven). This is the same metastability structure as the ISS vacuum itself—the angular instability is slow and the configuration is long-lived. The bloom angle remains an open problem; it may be fixed by the overdetermination of the mass chain rather than by a simple potential minimum.

Dual self-consistency. The Seiberg seesaw maps direct charges $\mathfrak{z}_k = \sqrt{m_k}$ to dual charges $\xi_k = 1/\sqrt{m_k}$. In terms of the elementary symmetric polynomials e_i of the $\mathfrak{z}_k$, the dual variance ratio is $\langle \xi_k^2 \rangle / \xi_0^2 = 1 - 2e_1 e_3 / e_2^2$. In the angular parametrisation, requiring $\langle z_k^2 \rangle = z_0^2$ and $\langle \xi_k^2 \rangle = \xi_0^2$ simultaneously reduces to

$$\cos 3\delta = \frac{5\sqrt{2}}{8}\,, \tag{89}$$

an isolated angle distinct from the seed ($\delta = 3\pi/4$). At the seed itself the identity $\langle z_k^2 \rangle = z_0^2 \Leftrightarrow \langle \xi_k^2 \rangle = \xi_0^2$ is automatic (one mass vanishes). For the physical down-type quarks (d, s, b), $\langle \xi_k^2 \rangle / \xi_0^2 = 0.997$ (0.22% from unity) while $\langle z_k^2 \rangle / z_0^2 = 1.10$—far from unity. The seesaw thus creates an independent near-identity condition on the dual charges, constraining m_d from (m_s, m_b).

11.3 Supermultiplet restoration

The restoration problem should be formulated directly in terms of common spectra. In the limit

$$f,\ \tilde{m}^2,\ \varepsilon,\ \xi \longrightarrow 0 \tag{90}$$

one wants three common eigenvalues E_A, $A = 1, 2, 3$, such that

$$\mathrm{spec}(M_Q) = \mathrm{spec}(M_L) = \mathrm{spec}(M_M) = \{E_1, E_2, E_3\} \tag{91}$$

up to sector-dependent unitary rotations. The physical quarks, leptons, and mesons would then be obtained from a common restored basis,

$$q_i = U_{iA}^{(q)} Q_A\,, \qquad \ell_i = U_{iA}^{(\ell)} L_A\,, \qquad \phi_i = U_{iA}^{(m)} M_A\,. \tag{92}$$

This is the right language for the “same energy, different charges” issue: the observables are the common eigenvalues E_A, whereas the tuple coordinates $(x_0 + x_A)$, $(y_0 + y_A)$, and $(z_0 + z_A)$ are sector-dependent affine coordinates used to parameterise those spectra. Different tuples can therefore realise the same restored energies without carrying identical coordinate decompositions. In particular, once the bosonic block is realised by mesonic channels, the restoration problem is not merely to fit a meson Koide triple but to understand how explicit mesonic tuples are driven toward the bosonic components of the restored supermultiplets. The companion note’s meson examples therefore serve here as concrete representatives of the bosonic slots, not as independent inputs [2].

The present EFT realises this picture only partially. The meson–lepton block already shares the seed spectrum

$$(E_1, E_2, E_3) = (0, m_-, m_+) \tag{93}$$

with $m_\pm = (2 \pm \sqrt{3})m$, and soft terms plus bloom deformations explain why the physical charged leptons no longer sit exactly at that supersymmetric point. What is still missing is the third copy: a quark/diquark mass operator whose supersymmetric limit has the same ordered spectrum and whose subsequent basis rotation reproduces the observed quark ordering and mixing pattern. That missing copy is exactly where the unresolved symmetric **15** enters.

The practical consequence is that restoration should not be pursued as a hunt for isolated hadron–lepton coincidences. The real completion criterion is a common three-eigenvalue structure across all three sectors together with the sector-dependent rotations that map it to the observed states. In that sense, restoration is no longer a slogan but a concrete spectral problem.

11.4 The baryon sector

The meson–baryon connection requires a Miyazawa supercharge [8,9] in the $\overline{\mathbf{10}}$ of $SU(5)_f$, with anticommutator decomposition $\overline{\mathbf{10}} \otimes \mathbf{10} = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75}$. This evades the Haag–Łopuszański–Sohnius theorem because it is an approximate symmetry broken at scale Λ_{QCD}.

11.5 Vacuum stability and FCNC

The seesaw vacuum is cosmologically stable: Coleman–de Luccia bounce-action estimates give $S_4 \sim 2 \times 10^7$, far above the threshold $S_4 > 400$ for a lifetime exceeding the age of the universe. All off-diagonal meson masses are positive ($m^2 \approx 2f_\pi^2$), and a block-diagonal obstruction prevents cross-flavour condensation. The flavour-universal Yukawa identity $y_j\langle M_j\rangle = \sqrt{2}\,C/v_{\mathrm{EW}}$ (derived in Section 4) automatically satisfies the Glashow–Weinberg condition for the absence of tree-level Higgs-mediated FCNCs.

11.6 Generation uniqueness

The Diophantine system uniquely selects $N = 3$ generations, but the third equation ($r^2 + s^2 - 1 = 4N$) has the weakest physical motivation. A dynamical derivation of the counting constraints would strengthen the argument considerably.

12 Conclusions

The purpose of this paper has been to recast the scalar-bootstrap program as a problem in four-dimensional supersymmetric effective field theory. The earlier $SO(32)/\mathrm{SU}(5)$ construction of [1] supplied the representation-theoretic target: three generations, five light participating flavours, and the scalar content $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$. The present paper asks how much of that target can already be realized within ordinary $\mathcal{N} = 1$ EFT. The answer is that a substantial meson-lepton core can be realized dynamically, while the full microscopic completion remains open.

What is already under control can be stated compactly. The bootstrap counting still singles out $(N, r, s) = (3, 3, 2)$. Confined SQCD with an O'Raifeartaigh deformation produces a distinguished seed algebra at $gv/m = \sqrt{3}$, with spectrum $(0, (2-\sqrt{3})m, (2+\sqrt{3})m)$, structural ratio $(2+\sqrt{3})^2$, and the exact identity $\langle z_k^2\rangle = z_0^2$ in the affine-square parametrization $m_k = (z_0 + z_k)^2$. Once this seed is adopted, the EFT propagates it through the quark and lepton sectors in a constrained way, yielding the numerical chain for $m_s, m_c, m_b, m_t, m_d, m_\tau$ and the GST value of the Cabibbo angle displayed in the body of the paper.

This is the precise sense in which the paper supports a reduction of the charged-fermion parameter count. Within the framework developed here, seven of the Standard

Model's thirteen charged-fermion-sector parameters are fixed by the combined bootstrap, seed, seesaw, and bion relations. That is not merely a change of variables; it is a genuine loss of freedom once the EFT inputs are specified. The right statement, however, is a *conditional* reduction. The qualifier comes from two explicitly isolated open sectors, not from any ambiguity in the derived seed algebra.

The first open sector is the symmetric flavour **15**. The Pauli-theorem obstruction shows that it is not an ordinary scalar qq composite in the colour-$\bar{\mathbf{3}}$ channel. Any complete version of the program must therefore explain whether the **15** is realized as a string state, a fundamental field, an algebraic Miyazawa-type object, or by some other microscopic mechanism outside the ordinary two-quark Hilbert space. The second open sector is the vacuum-selection problem. The distinguished seed at $gv/m = \sqrt{3}$ is the point from which the strongest mass identities follow, and the paper exhibits several effective ingredients that single it out and deform it in the empirically correct direction, but the full bloom mechanism has not yet been derived from first principles. Until these two points are closed, the program remains an EFT framework rather than a finished theory of flavour.

That conditional status is not a weakness of the paper's main claim. On the contrary, it is what makes the claim publishable and testable. The manuscript now isolates a definite low-energy statement: the scalar bootstrap has a nontrivial supersymmetric EFT realization whose mesonic and leptonic sectors already constrain the charged-fermion spectrum strongly enough to remove seven parameters, even though the diquark completion and final vacuum-selection mechanism remain open. In this formulation the mesons are not decorative: their tuples supply the bosonic representatives toward which restoration must guide the effective supermultiplets, while the leptonic and quark sectors are the corresponding fermionic images. This is a cleaner contribution than either a purely kinematic group-theory note or a much broader programmatic manifesto.

The next step required to turn the present claim from conditional to unqualified is also clear. One must derive the distinguished vacuum and bloom from the same microscopic dynamics that supports the seed algebra, and one must realize the symmetric **15** sector without contradicting the kinematic obstruction proved in the paper. If those steps are achieved, the present seven-parameter reduction can be promoted from an EFT consequence to a genuine reduction of Standard Model freedom. If they are not, the present analysis will still have located the exact point at which the bootstrap stops. That is already a useful field-theoretic result.

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