The Koide formula and quark-lepton mass relations: evidence and statistical assessment

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Abstract

We compile and statistically evaluate a family of mass relations among Standard Model fermions built on the Koide ratio $Q(m_1, m_2, m_3) \equiv (m_1 + m_2 + m_3)/(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2$. The original charged-lepton relation $Q(m_e, m_\mu, m_\tau) = 2/3$ holds to 0.001% (0.91σ in m_τ); a Monte Carlo look-elsewhere analysis over all fermion triples gives 2.8σ. Extending the formula to signed mass charges $\mathfrak{z}_k = \pm\sqrt{m_k}$, we find the quark triples $(-m_s, m_c, m_b)$ and (m_c, m_b, m_t) satisfy $Q \approx 2/3$ to 1.2% and 0.4% respectively, both obligatorily mixing up- and down-type quarks. A seed triple $(0, m_s, m_c)$ with the O'Raifeartaigh mass ratio $\sqrt{m_s}/\sqrt{m_c} = 2 - \sqrt{3}$ is confirmed to 1.2%. The vacuum-charge doubling relation $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ predicts $m_b = 4177$ MeV (0.1σ from PDG). The Koide phase parameter satisfies $\delta_0 \bmod 2\pi/3 \approx 2/9$ to 33 ppm (3.1σ with look-elsewhere, 4.5σ specific). A meson triple $(-\pi, D_s, B)$ extends $Q \approx 2/3$ to composite hadrons at 0.1%. The dual Koide $Q(1/m_d, 1/m_s, 1/m_b) = 0.665$ (0.22% from 2/3) suggests a Seiberg-dual structure. The de Vries angle $R = (\sqrt{19} - 3)(\sqrt{19} - \sqrt{3})/16 = 0.22310\ldots$ matches the on-shell $\sin^2\theta_W$ to 0.13σ, and the Gatto–Sartori–Tonin relation $\sin\theta_C = \sqrt{m_d/m_s}$ holds to 0.3%. Taken individually, several of these relations have modest significance; taken together, their interconnections—shared algebraic structure, overlapping mass triplets, and common Koide phase—point to a single organizing principle. The companion paper [I] constructs the $\mathcal{N} = 1$ Lagrangian that realizes this principle via an O'Raifeartaigh superpotential with structural invariant $Q = 2/3$.

1 Introduction

1.1 The Koide formula

In 1981, Koide derived a relation among the charged lepton masses from a composite model with S_3 permutation symmetry acting on preon constituents [?]. Stripped of its model-dependent origin, the relation states that the ratio

$$\boxed{Q(m_1, m_2, m_3) \equiv \frac{m_1 + m_2 + m_3}{\left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right)^2}} \tag{1}$$

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equals 2/3 when evaluated on the electron, muon, and tau masses. With current PDG values [?],

$$Q(m_e,\, m_\mu,\, m_\tau) \;=\; 0.666661 \pm 0.000007\,, \tag{2}$$

where the uncertainty is dominated by $\delta m_\tau = 0.12$ MeV. If the formula is exact, it predicts $m_\tau = 1776.97$ MeV, consistent with the experimental value 1776.86 ± 0.12 MeV to 0.91σ.

Foot reformulated this as a geometric condition: the vector of square roots of masses makes a 45° angle with the democratic vector $(1, 1, 1)$ [?]. Equivalently, in the angular parametrization

$$\sqrt{m_k} \;=\; \sqrt{M_0}\,\Big(1 + \sqrt{2}\,\cos\!\Big(\tfrac{2\pi k}{3} + \delta_0\Big)\Big)\,, \qquad k = 0, 1, 2\,, \tag{3}$$

the condition $Q = 2/3$ is satisfied identically for any M_0 and δ_0, and the three masses are determined by two parameters. The phase for charged leptons is $\delta_0 = 0.2222$ (in natural units of $2\pi/3$).

The precision of the lepton relation—better than one part in 10^5—has prompted four decades of theoretical attempts to explain it, ranging from discrete symmetries (S_3, A_4) acting on flavon fields [?, ?] to radiative mechanisms that protect the formula against renormalization group running [?]. Yet no consensus model exists. The formula has been called an "unexplained coincidence" and a "tantalizing hint" in roughly equal measure.

1.2 The skeptic's objection: look-elsewhere and scheme dependence

Two objections are standard.

Look-elsewhere. With six quarks and three charged leptons forming $\binom{9}{3} = 84$ triples (or more, allowing signed square roots), a near-hit at $Q = 2/3$ somewhere in the data is not implausible by chance. We address this head-on. A Monte Carlo simulation drawing nine masses log-uniformly over the range [0.1, 200,000] MeV and scanning all 84 triples times 8 sign assignments finds that a match as precise as the charged leptons occurs in only 0.24% of random mass spectra. This is a look-elsewhere corrected significance of 2.8σ. Not overwhelming—but the lepton Koide is only the first entry in a longer list.

Renormalization scheme. The Koide formula is stated in terms of pole masses (or $\overline{\text{MS}}$ masses evaluated at the mass itself, $m_q(m_q)$). Running to the GUT scale shifts Q by a few percent for quarks and by less than 0.1% for leptons [?, ?]. This is sometimes used to argue that the relation is an accident of the infrared. But it is equally true that many well-established mass relations—the Gell-Mann–Okubo formula, the Gatto–Sartori–Tonin (GST) relation—are statements about low-energy masses, not running couplings at a unification scale. If the Koide relation originates from boundary conditions on an effective superpotential (as argued in [I]), then the infrared is exactly where it should hold.

1.3 Beyond the leptons: a web of mass relations

The real case for the Koide formula rests not on a single relation but on a web of interlocking observations. In 2011, Rodejohann and Zhang [?] noted that the triple (m_c, m_b, m_t) satisfies $Q = 0.6693$, within 0.4% of 2/3. This was surprising because it mixes quark generations and charge sectors. In [?], one of us showed that allowing signed square roots reveals a further triple $(-m_s, m_c, m_b)$ with $Q = 0.675$ (1.2% from 2/3), and that these triples chain together: the output of one is the input of the next, covering five of the six quark masses.

The present paper collects all known mass relations of this type—ten observations spanning charged leptons, quarks, and pseudoscalar mesons—and evaluates each one statistically. The observations, in order of discovery, are:

1. **The charged-lepton Koide** [?]: $Q(m_e, m_\mu, m_\tau) = 2/3$ to 0.001%. Significance: 0.91σ in m_τ; 2.8σ after look-elsewhere.
2. **The quark seed** [?]: $Q(0,\, m_s,\, m_c) \approx 2/3$, meaning $\sqrt{m_s}/\sqrt{m_c} \approx 2-\sqrt{3}$. Observed ratio 0.271, theoretical 0.268; deviation 1.2%.
3. **The signed quark triple** [?]: $Q(-m_s,\, m_c,\, m_b) = 0.675$, deviation 1.24% from 2/3.
4. **The heavy triple** [?]: $Q(m_c,\, m_b,\, m_t) = 0.6693$, deviation 0.40%.
5. **The meson triple** [?]: $Q(-m_\pi,\, m_{D_s},\, m_B) = 0.6674$, deviation 0.10%.
6. **v_0-doubling** (this work): $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ to 0.07%, predicting $m_b = 4177$ MeV (PDG: 4180 ± 30 MeV, 0.1σ).
7. **The phase relation** [?]: $\delta_0 \bmod 2\pi/3 \approx 2/9$ to 33 ppm. Significance: 3.1σ with look-elsewhere over 128 simple fractions; 4.5σ for the specific value 2/9.
8. **The dual Koide** (this work): $Q(1/m_d,\, 1/m_s,\, 1/m_b) = 0.665$, 0.22% from 2/3. Suggests a Seiberg-dual seesaw structure $M_j \propto 1/m_j$.
9. **The cyclic echo** (this work): Iterating the Koide equation as $t \to c \to s \to \text{stall} \to c$ produces an algebraically exact cycle (agreement to machine epsilon).
10. **The second scaling fails** (this work): $M_2 = 9M_0$, $\delta_2 = 9\delta_0$ does *not* produce (c, b, t) or any physical triple. The pattern stops at one step. This negative result constrains the mechanism.

Two additional relations, while not Koide triples, belong to the same family:

- **The de Vries angle** [?]: $R = (\sqrt{19} - 3)(\sqrt{19} - \sqrt{3})/16 = 0.22310\ldots$ matches $\sin^2\theta_W$(on-shell) to 0.13σ in M_W.
- **The GST/Oakes relation** [?]: $\sin\theta_C = \sqrt{m_d/m_s} = 0.224$, matching $|V_{us}| = 0.2243$ to 0.3%.

1.4 What this paper adds

Previous discussions of these relations appear scattered across individual papers, conference proceedings, and blog posts. No systematic statistical assessment exists. This paper provides three things:

First, a self-contained compilation of all the observations, with current PDG values and error bars, in a single reference. We state every number with its uncertainty and pull, not just a percentage deviation.

Second, a rigorous look-elsewhere analysis. For each observation we define the trial space—how many similar relations could have been examined—and compute corrected p-values. We are explicit about which observations are significant in isolation (2.8σ for the lepton Koide, 3.1σ for the phase relation) and which derive their significance only from the structural context (v_0-doubling, dual Koide).

Third, an argument that the observations are not independent. The quark seed, the signed triple, the heavy triple, and the v_0-doubling relation share three of the same masses (m_s, m_c, m_b) and are linked by a common algebraic structure. An exhaustive scan of all $\binom{6}{3} = 20$ quark triples reveals that *every* triple close to $Q = 2/3$ obligatorily mixes up-type and down-type quarks [?]; pure same-charge triples (d, s, b) and (u, c, t) give $Q = 0.73$ and $Q = 0.85$ respectively, both far from 2/3. This is not a selection effect: while 90% of random triples from six quarks are mixed by combinatorics, the 100% rate among Koide-satisfying triples is a structural feature pointing toward CKM-type mixing.

The companion paper [I] constructs the $\mathcal{N} = 1$ supersymmetric Lagrangian that produces these relations. The present paper is deliberately agnostic about the Lagrangian. We state the empirical relations, assess their statistical weight, and leave the dynamical interpretation to [I]. The question this paper asks is precise: *given the data, is there evidence for a common algebraic structure in the fermion mass spectrum, or can the observations be dismissed as look-elsewhere artifacts?*

1.5 Conventions and mass inputs

Throughout this paper, quark masses refer to $\overline{\text{MS}}$ values at the self-energy scale $m_q(m_q)$ for c and b, and the pole mass for the top quark: $m_s = 93.4 \pm 5.8$ MeV, $m_c = 1270 \pm 20$ MeV, $m_b = 4180 \pm 30$ MeV, $m_t = 172.76 \pm 0.30$ GeV [?]. Light quark masses are $m_u = 2.16 \pm 0.49$ MeV, $m_d = 4.67 \pm 0.48$ MeV. Lepton masses are pole masses: $m_e = 0.51100$ MeV, $m_\mu = 105.658$ MeV, $m_\tau = 1776.86 \pm 0.12$ MeV. Meson masses are PDG averages: $m_\pi = 139.57$ MeV, $m_{D_s} = 1968.35$ MeV, $m_B = 5279.34$ MeV.

We use natural units $c = \hbar = 1$. The Koide ratio is denoted Q. We write $\mathfrak{z}_k = \pm\sqrt{m_k}$ for signed mass charges, with the sign conventions specified for each triple. Pulls are quoted as $(x_{\text{pred}} - x_{\text{obs}})/\sigma_{\text{obs}}$ in units of σ.

1.6 Outline

Section 2 reviews the Koide parametrization and the seed structure. Section 3 presents the quark chain: the seed, the signed triple, and the heavy triple. Section 4 discusses the v_0-doubling and the overlap prediction. Section 5 extends the analysis to pseudoscalar mesons. Section 6 covers the heavy triple (c, b, t). Section 7 presents mass relations and iterative chains. Section 8 discusses the de Vries angle and the Casimir equation. Section 9 gives the full statistical assessment. Section 10 summarizes the conclusions and the connection to [I].

2 The Koide parametrization and seed structure

2.1 Charges, not masses

The fundamental variable is the signed mass charge $\mathfrak{z} = z_0 + z_i \in \mathbb{R}$, with the physical mass $m = \mathfrak{z}^2 \geq 0$.

2.2 Derivation of $Q = 2/3$

For three charges $\mathfrak{z}_k = z_0 + z_k$ with z_0 the mean ($\sum z_k = 0$):

$$Q = \frac{\sum \mathfrak{z}_k^2}{(\sum \mathfrak{z}_k)^2} = \frac{3z_0^2 + \sum z_k^2}{9z_0^2} = \frac{1}{3} + \frac{\langle z_k^2 \rangle}{3z_0^2}\,. \tag{4}$$

Therefore:

$$\boxed{Q = \frac{2}{3} \quad \Longleftrightarrow \quad \langle z_k^2 \rangle = z_0^2\,.} \tag{5}$$

The two conditions are:

1. **Zero-sum perturbations:** $z_1 + z_2 + z_3 = 0$ (definition of z_0 as the mean charge).
2. **Energy balance:** The mean energy of the perturbations equals the energy of the unperturbed charge: $\langle z_k^2 \rangle = z_0^2$.

Condition 2 is the physics. It says the splitting scale matches the base scale—the coefficient of variation equals unity: $\sigma(\mathfrak{z})/\bar{\mathfrak{z}} = 1$. (Dimensionally, $\mathfrak{z}$ has units $\mathrm{MeV}^{1/2}$, so z_0^2 has units MeV—a mass scale, not an energy density. The "energy balance" terminology is metaphorical.) This is neither perturbative ($\sigma \ll \bar{\mathfrak{z}}$, giving $Q \to 1/3$, all equal) nor dominant ($\sigma \gg \bar{\mathfrak{z}}$, giving $Q \to 1$, one mass dominates). The Koide value $2/3$ is the equipartition point.

2.3 Why one mass can vanish

If $\langle z_k^2 \rangle = z_0^2$, a perturbation z_k can reach $|z_k| \sim z_0$, making $\mathfrak{z}_k = z_0 + z_k \approx 0$. A vanishing mass is the *natural extreme* of energy balance, not fine-tuning.

2.4 Seed triples: the $(0, a, b)$ structure

For a triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ with one massless member, the Koide ratio becomes:

$$Q = \frac{\mathfrak{z}_a^2 + \mathfrak{z}_b^2}{(\mathfrak{z}_a + \mathfrak{z}_b)^2}\,. \tag{6}$$

Setting $Q = 2/3$ and writing $r = \mathfrak{z}_a/\mathfrak{z}_b$ (with $r < 1$):

$$3(r^2 + 1) = 2(r + 1)^2 \quad \Longrightarrow \quad r^2 - 4r + 1 = 0\,, \tag{7}$$

giving:

$$\boxed{r = 2 - \sqrt{3} \approx 0.26795\,.} \tag{8}$$

Equivalently, in mass space: $m_{\text{light}}/m_{\text{heavy}} = (2 - \sqrt{3})^2 = 7 - 4\sqrt{3} \approx 0.07180$.

This is a *universal* prediction: any Koide triple with one massless member has the same charge ratio between the two massive members, regardless of their absolute scale.

2.5 The quark seed: $(0, s, c)$

The strange and charm quark charges give:

$$\frac{\sqrt{m_s}}{\sqrt{m_c}} = \frac{9.67}{35.64} = 0.2712\,, \qquad r_{\text{Koide}} = 0.2679\,, \qquad \text{deviation: } 1.2\%\,. \tag{9}$$

This triple has $Q = 0.6644$, deviating from $2/3$ by only 0.35%. Its Koide scale is $v_0^2 = (\mathfrak{z}_s + \mathfrak{z}_c)^2/9 \approx 228\,\mathrm{MeV}$.

Scale ambiguity in the quark seed. Throughout this paper, quark masses follow PDG conventions: $m_s(2\,\text{GeV}) = 93.4\,\text{MeV}$, $m_c(m_c) = 1.27\,\text{GeV}$, $m_b(m_b) = 4.18\,\text{GeV}$, $m_t = 172.76\,\text{GeV}$ (pole mass). This is a mixed convention, with the light quark evaluated at 2 GeV and heavier quarks at their own scale. QCD mass ratios are RG invariants at leading order (the anomalous dimension γ_m is flavor-universal), so Q is scheme-independent. However, the seed ratio $\sqrt{m_s}/\sqrt{m_c}$ uses masses at *different* scales: at a common $\overline{\text{MS}}$ scale the ratio shifts to ~ 0.29, worsening the seed match from 1.2% to $\sim 8\%$. This is a significant limitation of the quark seed claim. The meson seed, which uses unambiguous physical (pole) masses, is free of this scheme issue and provides the cleaner test of the seed hypothesis.

A historical note: Harari, Haut, and Weyers [?] observed a quark mass ratio pattern that, in the language of the present paper, corresponds to an approximate seed triple—$(0, d, s)$—at a time when the up quark mass was still thought to be possibly zero. With current masses, $\sqrt{m_d}/\sqrt{m_s} = 0.224$, deviating from $2 - \sqrt{3}$ by 16%—too large for a good Koide triple. But if m_s were $\sim 150\,\text{MeV}$ and $m_d \sim 7\,\text{MeV}$ (values common in that era), the ratio improves considerably. The seed idea was already present, ahead of its time.

2.6 The meson seed: $(0, \pi, D_s)$

The pion and D_s meson charges give:

$$\frac{\sqrt{m_\pi}}{\sqrt{m_{D_s}}} = \frac{11.81}{44.37} = 0.2663\,, \qquad r_{\text{Koide}} = 0.2679\,, \qquad \text{deviation: } 0.6\%\,. \tag{10}$$

This gives $Q = 0.6679$, deviating by 0.18%. It is the **meson counterpart** of the quark seed $(0, s, c)$—both involve the strange-charm sector. Indeed, $D_s = c\bar{s}$ is the meson that directly contains the s and c quarks.

2.7 The lepton triple as an "almost-seed"

The charged lepton triple (e, μ, τ) has $\mathfrak{z}_e = 0.715\,\text{MeV}^{1/2}$, which is small but nonzero. If m_e were zero, we would need $\sqrt{m_\mu}/\sqrt{m_\tau} = 2 - \sqrt{3} = 0.2679$; the actual value is 0.2439, deviating by 9%. So the leptons are *not* an exact seed—the electron, while very light, is not massless. But the triple is clearly the same family: the near-critical Koide phase puts one member close to zero.

2.8 Scale coincidence

The v_0^2 values of the seed triples cluster remarkably:

Triple	v_0^2 (MeV)	v_0 (MeV$^{1/2}$)
(e, μ, τ)	314	17.72
$(0, \pi, D_s)$	351	18.73
$(0, s, c)$	228	15.10

The lepton and meson seed triples share a Koide scale around 310–350 MeV, squarely in the QCD scale range. This is the sBootstrap connection made quantitative: **the lepton triple and the meson seed triple operate at the same scale.**

For the lepton triple, the coincidence is numerically sharp:

$$M_0 \approx \frac{f_\pi^2}{27}, \tag{11}$$

to 0.3%: the right-hand side gives 314.9 MeV versus the fitted $M_0 = 313.8\,\text{MeV}$. Equivalently, $v_0 \approx f_\pi/(3\sqrt{3})$. If this identification is exact and $\delta_0 \bmod 2\pi/3 = 2/9$ (the three-instanton value), all three charged lepton masses follow from f_π alone, each predicted to $\sim$0.3%.

3 The quark chain

3.1 From seed to full triple

The seed triples grow into full triples upon symmetry breaking:

Sector	Seed triple	Full triple	v_0^2 (full)
Quarks	$(0, s, c)$	$(-s, c, b)$	913 MeV
Mesons	$(0, \pi, D_s)$	$(-\pi, D_s, B)$	1230 MeV
Leptons	$(0 \approx e, \mu, \tau)$	(e, μ, τ)	314 MeV

In each case:

- The seed has a massless (or near-massless) member, two massive members in the ratio $2 - \sqrt{3}$, and $Q \approx 2/3$.
- The full triple has **no** massless member: the zero has been lifted (to m_e, or the sign-flipped strange/pion mass). The b quark and B meson are always present as flavor degrees of freedom; what the breaking does is give them their *distinctive masses*—the specific values that complete the Koide triple. Before breaking, the b-direction sits at the flat direction ($\mathfrak{z} = 0$); after, it acquires $\mathfrak{z}_b = 64.65\,\text{MeV}^{1/2}$, the value selected by the energy balance with c and the sign-flipped s. The bloom is not particle creation but *mass generation*.

The "tale of two tuples" is this: **each sector presents first as a seed with a massless member, then "blooms" into a full triple when SUSY breaks.** The massless member is the necessary initial condition; the breaking lifts it and generates the full spectrum.

3.2 Precision summary

Triple	Q	Dev. from 2/3	v_0^2 (MeV)	δ (rad)
		Seed triples		
$(0, \pi, D_s)$	0.6679	0.18%	351	$\approx 3\pi/4$
$(0, s, c)$	0.6644	0.35%	228	$\approx 3\pi/4$
$(0 \approx e, \mu, \tau)$	0.6667	0.001%	314	2.317
		Full triples		
(e, μ, τ)	0.66666	0.001%	314	2.317
$(-\pi, D_s, B)$	0.6674	0.10%	1230	2.806
$(-s, c, b)$	0.6750	1.24%	913	2.744
(c, b, t)	0.6695	0.42%	29 580	2.163

The Koide phases δ cluster in the range 2.2–2.8 rad (126°–161°), all displaced from the seed value $3\pi/4 \approx 2.356$. The lepton triple barely moved from its seed; the quark and meson triples bloomed further. All phases lie within one $\mathbb{Z}_3$ fundamental domain ($[0, 2\pi/3]$ modulo $2\pi/3$).

The lepton triple is unique: seed and full triple are nearly identical, because the electron mass is small enough to serve as the "zero" while being nonzero enough to make Q extremely precise. The $(-s, c, b)$ triple has the worst deviation (1.24%); this is expected because the bloom is a nonperturbative process that does not exactly preserve Q. The v_0-doubling constrains m_b independently (to 0.07% via $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$), so the 1.24% Koide deviation is the residual after the more fundamental v_0-constraint is satisfied.

3.3 Orthogonality of seed and full triples

Each Koide triple defines a perturbation vector $\vec{z} = (z_1, z_2, z_3)$ constrained to the plane $\sum z_k = 0$ with length $|\vec{z}|^2 = 3z_0^2$. The angle between seed and full triples measures how much the breaking rotates the spectrum in charge space:

Pair of triples	Angle
$(0, s, c)$ vs. $(-s, c, b)$	22°
$(0, \pi, D_s)$ vs. $(-\pi, D_s, B)$	26°
(e, μ, τ) vs. $(0, \mu, \tau)$	0.8°
$(0, s, c)$ vs. $(0, \pi, D_s)$	0.3°

Two results are notable. First, the quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ are **nearly parallel** (0.3°): the meson seed is the composite image of the quark seed, with the GOR map preserving direction while changing scale. Second, the lepton triple is essentially its own seed (0.8°)—the bloom has barely begun.

4 v_0-doubling and the overlap prediction

4.1 The vacuum-charge doubling

The vacuum charge $v_0 = \frac{1}{3}\sum \mathfrak{z}_k$ of the quark seed is $v_0^{(s)} = (\sqrt{m_s}+\sqrt{m_c})/3 = 15.10\,\mathrm{MeV}^{1/2}$. For the full triple, $v_0^{(f)} = (-\sqrt{m_s} + \sqrt{m_c} + \sqrt{m_b})/3 = 30.21\,\mathrm{MeV}^{1/2}$. Their ratio is

$$v_0^{(f)}/v_0^{(s)} = 2.0005 \approx 2\,. \tag{12}$$

If the bloom exactly doubles the vacuum charge, then $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$, giving $m_b = 4{,}177\,\mathrm{MeV}$—matching the PDG value $4{,}180 \pm 30\,\mathrm{MeV}$ to 0.1σ (0.06σ including propagated input uncertainties). This relation is independent of the Koide condition: it constrains m_b from (m_s, m_c) alone, and is more precise (0.07%) than the overlapping-triples prediction (0.5%).

4.2 Tension with the exact seed

A tension exists: the exact seed Koide gives $m_c = 1{,}301\,\mathrm{MeV}$ (not the PDG 1,270), and applying the v_0-doubling to this exact seed value gives $m_b = 4{,}233\,\mathrm{MeV}$ (1.8σ from PDG). The v_0-doubling works *with the physical* m_c, not the exact seed value—so these two constraints are mutually inconsistent at the stated precision. The resolution may be that the seed Koide fixes the ratio m_c/m_s only approximately (the 2.4% charm deviation being a higher-order correction to the $gv/m = \sqrt{3}$ condition), while the v_0-doubling constrains m_b directly from the physical masses. In this reading, the seed condition is the approximate one and the v_0-doubling the more fundamental. This hierarchy is not derived; it is imposed by the data.

4.3 The overlapping-triples prediction

Requiring $Q = 2/3$ on both $(-s, c, b)$ and (c, b, t) simultaneously with inputs (m_s, m_t) gives $m_b = 4159\,\mathrm{MeV}$ (0.5% from PDG) and $m_c = 1369\,\mathrm{MeV}$ (7.8% from PDG). The v_0-doubling is more precise for m_b (0.07% vs. 0.5%), while the overlapping-triples method additionally constrains m_c.

4.4 The exact seed and what breaks it

The seed configuration $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ requires *two* independent conditions:

1. One member at $\mathfrak{z} = 0$ exactly (the flat direction).
2. The ratio of the massive members equals $2 - \sqrt{3}$:

$$\frac{\sqrt{m_a}}{\sqrt{m_b}} = 2 - \sqrt{3} \approx 0.26795\,. \tag{13}$$

These are logically distinct. The chiral limit ($m_u = m_d = 0$, $\langle \bar{q}q \rangle \neq 0$) enforces condition (1) for the meson sector: the pion is an exact Goldstone boson, $m_\pi = 0$, and the B-meson direction sits at the flat direction before acquiring mass. But the chiral limit says *nothing* about condition (2)—it does not constrain $\sqrt{m_{D_s}}/\sqrt{m_B}$ or $\sqrt{m_s}/\sqrt{m_c}$ to equal $2 - \sqrt{3}$.

The physical deviations from the exact ratio are small but measurable:

Seed	Exact ratio	Physical ratio	Deviation
$(0, \pi, D_s)$	0.2680	$\sqrt{m_\pi}/\sqrt{m_{D_s}} = 0.2663$	-0.6%
$(0, s, c)$	0.2680	$\sqrt{m_s}/\sqrt{m_c} = 0.2712$	$+1.2\%$
$(0 \approx e, \mu, \tau)$	0.2680	$\sqrt{m_\mu}/\sqrt{m_\tau} = 0.2439$	-9.0%

The meson seed is closest to the exact ratio (0.6% deviation). The quark seed deviates by 1.2%. The lepton "seed" has the worst ratio match (-9%), which is consistent with the observation that the lepton triple is *not* a true seed: $m_e \neq 0$, and the electron was never at the flat direction. The leptons are an "almost-seed" whose excellent $Q = 0.66666$ arises not from the seed structure but from the smallness of m_e, which makes the two conditions approximately degenerate.

5 The meson Koide triple: $(-\pi, D_s, B)$

With the pion charge negative, the triple $(-\pi, D_s, B)$ satisfies:

$$Q(-\pi, D_s, B) = \frac{m_\pi + m_{D_s} + m_B}{(-\sqrt{m_\pi} + \sqrt{m_{D_s}} + \sqrt{m_B})^2} = 0.6674\,, \tag{14}$$

deviating from 2/3 by 0.10%—the second most precise Koide triple known. The energy balance:

$$\frac{\langle z_k^2 \rangle}{z_0^2} = 1.002\,. \tag{15}$$

For comparison, the "standard" all-positive quark triples (d, s, b) and (u, c, t) deviate from $Q = 2/3$ by 9.7% and 27% respectively at $\overline{\text{MS}}$ masses. **The meson triple is more precise than any quark triple**, including (c, b, t) (0.42%).

The pion's negative charge has physical meaning: as a pseudo-Goldstone boson, the pion would be massless ($\mathfrak{z} = 0$) if chiral symmetry were exact. It sits just below zero ($\mathfrak{z}_\pi = -11.81$), approaching zero from the negative side as $m_q \to 0$. The sign of $\mathfrak{z}$ encodes Goldstone nature: pseudo-Goldstone bosons sit on the negative side of the critical point, having crossed through zero from positive territory during chiral symmetry breaking.

Look-elsewhere correction. Eleven pseudoscalar mesons (π, K, η, η', D, D_s, B, B_s, B_c, η_c, η_b) yield $\binom{11}{3} = 165$ triples, times four independent sign patterns, giving 660 combinations. Scanning all 660, only $(-\pi, D_s, B)$ and the closely related $(-\pi, D_s, B_s)$ (deviation 0.14%) lie within 0.2% of $Q = 2/3$. A Monte Carlo with 10^7 random triples drawn from $[100, 10{,}000]$ MeV gives $p_{\text{single}} = 2.1 \times 10^{-4}$ for a match at 0.10%; correcting for 660 trials yields $p_{\text{LEE}} = 0.13$ (1.1σ). The meson Koide triple is therefore *not* statistically significant on its own, unlike the lepton triple (2.8σ after LEE). Its interest lies in the structural pattern: all six closest hits share the sign assignment $(-\pi, D_{(s)}, B_{(s,c)})$, i.e. a negative pion paired with charm-containing and bottom-containing mesons—the combination predicted by the $SU(5)$ flavor structure.

Extended scan: gauge and Higgs bosons. Enlarging the scan to include the W (80.38 GeV), Z^0 (91.19 GeV), and Higgs (125.25 GeV) bosons alongside the 14 pseudoscalar mesons adds 476 new triples (times sign patterns and both $Q(m)$ and $Q(1/m)$—some 7800 trials in total). The dual ratio $Q(1/m) \equiv \sum 1/m_k / \left(\sum 1/\sqrt{m_k}\right)^2$ is of interest because the Seiberg seesaw maps $m_j \to 1/m_j$.

Results: The best hit involving a gauge or Higgs boson is $Q(1/m_{\pi^0},\, 1/m_{\eta_c},\, 1/m_W) = 0.6661$ (0.08% from 2/3, all-positive signs), but exact $Q = 2/3$ requires $m_W = 82.6$ GeV (164σ from PDG)—excluded. Among $Q(m)$ triples, (π^0, B_c, H) reaches 0.15% and ($D^\pm$, D^0, H) reaches 0.21%, both all-positive; but again exact $Q = 2/3$ requires $m_H \approx 126.5$–126.9 GeV (7–10σ from PDG). With ~ 7800 trials, roughly six hits at the 0.1% level are expected by chance. No significant Koide relation exists among the electroweak bosons or between bosons and mesons beyond what the look-elsewhere correction explains.

6 The heavy triple: (c, b, t)

6.1 A tuple without mesons

The triple (c, b, t) with all positive signs gives $Q = 0.6695$, deviating by 0.42%. Its Koide scale $v_0^2 \approx 29.6$ GeV is far above the QCD scale.

This triple has **no meson counterpart**: the top quark decays weakly ($t \to Wb$, lifetime $\sim 5 \times 10^{-25}$ s) before the strong interaction can bind it ($\tau_{\text{QCD}} \sim 1/\Lambda_{\text{QCD}} \sim 10^{-24}$ s). No T-meson ($t\bar{q}$) exists. The meson matrix $M^i{}_j$ extends only to b-flavored states; the top sector lives purely at the quark level.

6.2 The (c, b, t) triple has no seed

Unlike the $(0, s, c)$ and $(0, \pi, D_s)$ seeds, there is no obvious $(0, x, y)$ seed for the (c, b, t) triple. No pair of quarks in this sector satisfies $\sqrt{m_{\text{light}}}/\sqrt{m_{\text{heavy}}} \approx 2 - \sqrt{3}$: one has $\sqrt{m_c}/\sqrt{m_t} = 0.086$ (too small) and $\sqrt{m_c}/\sqrt{m_b} = 0.551$ (too large).

This means (c, b, t) is *not* a "bloomed seed" in the same sense as $(-s, c, b)$. It may represent a fundamentally different structure—a triple that was never seeded by a massless member but instead emerged fully formed at a higher breaking scale, perhaps connected to electroweak symmetry breaking rather than to chiral symmetry breaking.

6.3 Interpretations

The (c, b, t) triple ($Q = 0.6695$, $v_0^2 = 29.6\,\text{GeV}$) has neither a seed nor a meson shadow. It operates at a scale far above Λ_{QCD}, suggesting its origin is different:

1. It may be governed by the electroweak Higgs mechanism rather than the chiral condensate. Since $m_i = y_i\, v_{\text{EW}}/\sqrt{2}$, the Koide condition $Q = 2/3$ translates into a constraint on the Yukawa couplings: $\sum y_i/(\sum \sqrt{y_i})^2 = 2/3$. With $y_t \approx 1$ fixed by the top mass, this determines y_b/y_t and y_c/y_t through the Koide parametrization.

2. It may represent a Volkov–Akulov realization at the electroweak scale: if there is a second SUSY breaking at $f' \sim v_{\text{EW}}$, the (c, b, t) triple would be its analog of the (π, μ) system.

3. Or it may be a purely kinematic accident—the quark masses happen to satisfy the Koide condition without a dynamical reason. The 0.42% precision makes this unlikely but not impossible.

6.4 CKM mixing and the Koide triples

A striking feature of the quark Koide triples is that they *obligatorily mix up-type and down-type quarks*: $(-s, c, b)$ contains one up-type (c) and two down-type (s, b), while (c, b, t) contains two up-type (c, t) and one down-type (b). An exhaustive scan of all $\binom{6}{3} = 20$ quark triples with all sign choices confirms that *every* triple close to $Q = 2/3$ is mixed—neither the pure down-type (d, s, b) nor the pure up-type (u, c, t) comes close ($Q = 0.73$ and $Q = 0.85$ respectively). This mixing of mass eigenvalues from different Yukawa sectors is puzzling in the Standard Model, where up-type and down-type masses are eigenvalues of independent matrices Y_u and Y_d. It is natural, however, in an $SU(5)$ flavor-symmetric framework where all five quarks are treated democratically.

The Koide constraints on the two overlapping triples, together with the seed condition on $(0, s, c)$, determine (s, c, b) from a single input (m_t). The unconstrained masses are m_u and m_d—the first generation. The Gatto–Sartori–Tonin relation [?] $|V_{us}| \approx \sqrt{m_d/m_s}$ connects precisely these free parameters to the Cabibbo angle ($|V_{us}| = 0.224$ vs. PDG 0.2243), suggesting that the CKM matrix encodes the residual freedom left undetermined by the Koide structure.

7 Mass relations and iterative chains

The preceding sections established the Koide energy-balance condition $Q = 2/3$ and identified the triples that satisfy it. In this section we quantify how special the lepton triple really is, introduce the angular parametrization that reveals a hidden rational structure in the Koide phase, and trace an iterative chain that connects the heavy quarks to the strange sector. We report both successes and failures: the scaling rule that generates the quark triple from the lepton triple works once and then stops, and the iterative chain stalls at a mass too light for any known fermion. These negative results constrain the scope of the pattern.

7.1 The lepton Koide formula: how special?

Using PDG 2024 values $m_e = 0.51100\,\mathrm{MeV}$, $m_\mu = 105.658\,\mathrm{MeV}$, $m_\tau = 1776.86 \pm 0.12\,\mathrm{MeV}$:

$$Q(e,\mu,\tau) = \frac{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2}{m_e + m_\mu + m_\tau} = 1.500014 \pm 0.000015\,, \tag{16}$$

where $Q = (\sum \sqrt{m})^2 / \sum m$ is the reciprocal of the convention in §2: the Koide condition is $Q = 3/2$ here, equivalently $\sum m/(\sum \sqrt{m})^2 = 2/3$ there. We use the 3/2 convention in this section to match the most common form in the Koide literature. The deviation from 3/2 is $|Q - 3/2| = 1.38 \times 10^{-5}$, corresponding to 0.91σ given the uncertainty dominated by δm_τ.

Look-elsewhere effect. The Standard Model contains 9 charged fermions (3 leptons, 6 quarks). An exhaustive scan of all $\binom{9}{3} = 84$ mass triplets yields the following ranking.

Rank	Triplet	$\lvert Q - 3/2\rvert$
1	(e, μ, τ)	1.38×10^{-5}
2	(c, b, t)	6.2×10^{-3}

The gap between rank 1 and rank 2 is a factor of 450.

To assess the statistical weight of the lepton result, we performed a Monte Carlo study: 10,000 random mass sets of 9 fermions drawn log-uniform on $[0.1, 200{,}000]\,\mathrm{MeV}$ were scanned for the best triplet. The fraction with $|Q - 3/2|_{\min} \le 1.38 \times 10^{-5}$ gives a look-elsewhere-corrected significance of $\mathbf{2.8}\boldsymbol{\sigma}$. This is suggestive but not conclusive: the lepton Koide formula is unlikely to be a pure accident, but the evidence does not reach the discovery threshold.

Extended scan with mesons. Enlarging the pool to 30 particles (9 fundamental fermions, W, Z, H, f_π, v_{EW}, and 16 pseudoscalar and vector mesons) with all 2^3 sign combinations per triple yields $\binom{30}{3} = 4060$ unique triples. The lepton triple remains rank 1 ($|Q - 3/2| = 1.4 \times 10^{-5}$, 0.001%), but (c, b, t) drops to rank 113 (0.42%) and $(-s, c, b)$ to rank 324 (1.24%): 112 meson-containing triples are closer to $Q = 3/2$ than (c, b, t). Twenty-one triples fall within 0.1% of 3/2, versus ~ 12 expected from a uniform distribution—a ratio of 1.7, not statistically remarkable. *Only (e, μ, τ) involves three independent fundamental parameters; the significance of quark Koide triples is diluted by the composite-meson pool.*

7.2 The Koide angle parametrization

Any triple satisfying $Q = 3/2$ exactly can be written [?]

$$\sqrt{m_k} = \sqrt{M_0}\left(1 + \sqrt{2}\cos\left(\frac{2\pi k}{3} + \delta_0\right)\right), \qquad k = 0, 1, 2\,, \tag{17}$$

where $M_0 > 0$ is the scale parameter and $\delta_0 \in [0, 2\pi)$ is the Koide phase. This is not an approximation: any three non-negative numbers whose Q-ratio equals 3/2 are exactly parametrized by some (M_0, δ_0).

Fitting to the charged lepton masses:

$$M_0 = 313.84\,\mathrm{MeV}\,, \qquad \delta_0 = 2.3166\ \mathrm{rad}\,. \tag{18}$$

The scale $M_0 \approx 314\,\mathrm{MeV}$ is the Koide scale v_0^2 of §2, now identified as the mass parameter of the angular decomposition.

The phase is close to 2/9. Unlike $Q = 2/3$, which is the algebraic consequence of the energy-balance condition (§2), the phase δ_0 is a free parameter that the energy balance does not fix. Its value is an empirical datum, not a derived quantity. The parametrization has a $2\pi/3$ periodicity in δ_0 (permuting the three masses). The physically meaningful quantity is the reduced phase

$$\delta_0 \bmod \frac{2\pi}{3} = 0.22222\ \mathrm{rad}\,. \tag{19}$$

This is strikingly close to the fraction $2/9 = 0.22222\ldots$:

$$\delta_0 \bmod \frac{2\pi}{3} - \frac{2}{9} = +7.4 \times 10^{-6}\,, \tag{20}$$

a residual of 33 ppm relative to 2/9 (using PDG 2024 central masses; within the m_τ uncertainty of 0.12 MeV, the residual vanishes at $m_\tau \approx 1{,}776.97\,\mathrm{MeV}$, i.e. 0.9σ above central).

Statistical significance of the rational approximation. There are 266 reduced fractions p/q with $q \leq 20$ and $0 < p/q < 2\pi/3$. The probability that a random phase (uniform on $[0, 2\pi/3)$) falls within 7.4×10^{-6} of *any* of these 266 fractions gives a look-elsewhere-corrected significance of $\mathbf{3.1\sigma}$. Evaluated specifically against the fraction 2/9 alone (i.e., without look-elsewhere correction), the significance is $\mathbf{4.5\sigma}$. The deviation from 2/9 is 0.9σ in the m_τ uncertainty, consistent with an exact match.

7.3 The scaling rule: $M_1 = 3M_0$, $\delta_1 = 3\delta_0$

A natural question is whether other Koide triples can be obtained from the lepton parameters by a simple scaling. The ansatz

$$M_1 = 3\,M_0 = 941.5\,\mathrm{MeV}\,, \qquad \delta_1 = 3\,\delta_0 = 6.9497\ \mathrm{rad}\,, \tag{21}$$

inserted into the Koide parametrization, predicts a quark triple. The following table compares predictions to PDG values.

Quark	Predicted (MeV)	PDG (MeV)	Deviation
s	92.3	93.4	1.2%
c	1360	1270	7.1%
b	4197	4180	0.4%

The agreement for s and b is at the percent level; charm shows a larger (7.1%) discrepancy. For the physical masses, $M_0(-s, c, b)/M_0(e, \mu, \tau) = 2.91$, not exactly 3—the $\times 3$ rule is approximate. By contrast, the vacuum-charge doubling (§4) predicts m_b to 0.1σ. The predicted triple satisfies the Koide condition with a negative square root for the strange quark:

$$Q(-\sqrt{m_s},\ \sqrt{m_c},\ \sqrt{m_b}) = \frac{(-\sqrt{m_s} + \sqrt{m_c} + \sqrt{m_b})^2}{m_s + m_c + m_b} = \frac{3}{2}\,, \tag{22}$$

holding to machine precision (residual 2.2×10^{-16}), since the masses are generated from the parametrization which enforces $Q = 3/2$ identically.

Renormalization caveat. The PDG quark masses used in the comparison are quoted at different scales: $m_s(2\,\text{GeV})$, $m_c(m_c)$, $m_b(m_b)$, and m_t as the pole mass. The predicted masses emerge at a single internal scale set by M_1. Running all masses to a common $\overline{\text{MS}}$ scale would change the individual deviations but not the structural point: the $\times 3$ scaling produces a recognizable quark triple. QCD mass *ratios* are RG invariants at leading order, so the Koide condition itself is scheme-independent, but the comparison of absolute masses to PDG values carries an $O(\alpha_s)$ ambiguity.

7.4 The second scaling fails

Iterating the scaling rule:

$$M_2 = 9\,M_0 = 2825\,\text{MeV}\,, \qquad \delta_2 = 9\,\delta_0 = 20.849\ \text{rad}\,, \tag{23}$$

gives predicted masses $(478,\ 92.3,\ 16{,}377)\,\text{MeV}$. Only the middle value echoes m_s. The lightest (478 MeV) does not match any known quark mass, and the heaviest (16.4 GeV) falls between m_b and m_t with no candidate. No SM triplet accommodates these values.

The scaling stops at one step. This is a negative result that constrains any attempt to build a recursive tower of Koide triples: the multiplicative rule $M \to 3M$, $\delta \to 3\delta$ is not a general principle but a one-off relation between the lepton and quark sectors. Whether this single success is accidental or reflects a deeper connection (perhaps related to the three colors of QCD, or to the three generations) remains open.

7.5 Iterative chain: descending from t and b

An alternative approach to connecting the quark masses is purely algebraic: given two known masses, solve the Koide equation $Q = 3/2$ for the third. Starting from the top and bottom quarks:

Step 1: $Q(m_b, m_?, m_t) = 3/2$. The Koide equation with (m_b, m_t) known is quadratic in $\sqrt{m_?}$, yielding four branches (two signs for $\sqrt{m_?}$, two roots of each quadratic). The solutions are:

$$m_? = \{1357,\ 60{,}280,\ 1.34 \times 10^6,\ 3.55 \times 10^6\}\,\text{MeV}\,. \tag{24}$$

Only the first branch gives a physically viable mass: $m_? = 1357\,\mathrm{MeV}$, which we identify with the charm quark (PDG: $m_c(m_c) = 1270\,\mathrm{MeV}$, deviation 6.8%).

Step 2: $Q(m_c, m_?, m_b) = 3/2$**.** With $m_c = 1357\,\mathrm{MeV}$ (the chain value, not the PDG value) and $m_b = 4180\,\mathrm{MeV}$:

$$m_? = 92.17\,\mathrm{MeV}\,, \tag{25}$$

which we identify with the strange quark. The negative square root emerges naturally from the algebra—the solution selects the branch with $\sqrt{m_?} < 0$ in the signed-charge framework, consistent with the $(-s, c, b)$ structure of §3.

Step 3: the chain stalls. Continuing from (m_s, m_c) with $m_s = 92.17\,\mathrm{MeV}$:

$$m_? = 0.035\,\mathrm{MeV}\,. \tag{26}$$

This is far too light for either u (2.16 MeV) or d (4.67 MeV). The chain does not reach the first-generation quarks; it stalls at a mass two orders of magnitude below m_u.

Step 4: cyclic echo. The stalled mass provides an algebraic consistency check. Taking $m_{\mathrm{stall}} = 0.035\,\mathrm{MeV}$ and $m_s = 92.17\,\mathrm{MeV}$, and solving $Q(m_{\mathrm{stall}}, m_?, m_s) = 3/2$:

$$m_? = 1357\,\mathrm{MeV} \quad \text{(to machine epsilon)}\,. \tag{27}$$

The chain returns to its starting point: $t \to c \to s \to \text{stall} \to c \to \cdots$. This is not a numerical coincidence but an algebraic necessity: the Koide condition, combined with the specific masses of b and t, defines a closed orbit in (m_1, m_2, m_3) space. The cycle is:

$$\cdots \to (b,t) \xrightarrow{Q=3/2} c \to (c,b) \xrightarrow{Q=3/2} s$$
$$\to (s,c) \xrightarrow{Q=3/2} \text{stall} \to (\text{stall}, s) \xrightarrow{Q=3/2} c \to \cdots$$

The chain captures c, b, s but misses u, d, and the leptons entirely. The first generation is outside the algebraic orbit seeded by the third generation.

7.6 Cross-checks and master comparison

Two independent routes lead to the quark masses: the lepton-seeded scaling rule and the top-bottom iterative chain. The following table compares all predictions.

Quark	Chain A (from t, b)	Chain B (from leptons)	PDG	Status
s	92.2	92.3	93.4	1–1.2% deviation
c	1357	1360	1270	7–7.1% deviation
b	(input)	4197	4180	0.4% deviation
t	(input)	—	172 760	—
stall	0.035	—	—	no SM match
M_2 set	—	(478, 92, 16377)	—	no SM match

The convergence of the two chains on $m_s \approx 92\,\mathrm{MeV}$ and $m_c \approx 1357$–$1360\,\mathrm{MeV}$ is the central positive result. Both routes, starting from completely different inputs (lepton masses vs. heavy quark masses), produce the same strange and charm masses to within 0.2% of each other. Against PDG values, the strange quark prediction is excellent ($\sim 1\%$), and the bottom quark prediction is even better (0.4%); the charm prediction shows a persistent $\sim 7\%$ overshoot.

The negative results are equally informative:

- The $\times 3$ scaling does not iterate: the $\times 9$ step produces no viable triple.
- The iterative chain does not reach the first generation: it stalls at $0.035\,\mathrm{MeV}$, two orders of magnitude below m_u.
- The chain is algebraically cyclic: it loops through (c, s, stall) indefinitely, never escaping to new masses.

These failures delineate the boundary of the Koide pattern: it connects the second and third quark generations (and, via the scaling rule, the leptons), but does not extend to the first generation or beyond the Standard Model spectrum. Whatever dynamics underlies the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ operates within a limited domain.

The isospin seed. The chain above fails to reach the first generation through individual quark masses, but the isospin combination $m_u + m_d = 6.83\,\mathrm{MeV}$ provides the missing link. The seed Koide ratio

$$Q(0,\, \sqrt{m_u + m_d},\, \sqrt{m_s}) = 0.6649 \tag{28}$$

deviates from 2/3 by only 0.27%—comparable to the $(0, s, c)$ seed at 0.35%. Since $Q(0, a, b) = 2/3$ requires $b/a = (2 + \sqrt{3})^2$, the exact seed predicts $m_s/(m_u + m_d) = (2+\sqrt{3})^2 = 13.93$, giving

$$m_s = (2 + \sqrt{3})^2\, (m_u + m_d) = 95.1\,\mathrm{MeV}\,, \tag{29}$$

a 0.36σ prediction from the PDG value $93.4 \pm 4.8\,\mathrm{MeV}$. The lattice average $m_s/m_{ud} = 27.23 \pm 0.10$ (FLAG 2021, $N_f = 2{+}1{+}1$) measures this ratio far more precisely than individual quark masses; since $m_{ud} \equiv (m_u{+}m_d)/2$, the prediction $2(2{+}\sqrt{3})^2 = 27.86$ exceeds the lattice value by 6.3σ. This deviation—2.3% in the ratio, 0.27% in Q—is the bloom: the physical spectrum is shifted from the exact seed $Q = 2/3$, just as $(-s, c, b)$ deviates by 1.2% and (c, b, t) by 0.4% in Q. Combined with the m_s chain ($m_s \to m_c \to m_b \to m_t$), the entire heavy quark spectrum descends from the single isospin parameter $m_u{+}m_d$—the same combination that enters the Gell-Mann–Oakes–Renner relation $m_\pi^2 = (m_u + m_d)\,|\langle \bar{q}q \rangle|/f_\pi^2$. Since $m_s/(m_u{+}m_d)$ is an RG invariant (all three masses share the same anomalous dimension), the isospin seed is scheme-independent.

The isospin seed does not contradict the chain's failure to reach the first generation: it reaches the *sum* $m_u + m_d$ but not m_u and m_d individually. The individual splitting $m_d - m_u$ requires separate input (connected to the Cabibbo angle via the GST relation).

7.7 The dual Koide

At the Seiberg vacuum with $B = \bar{B} = 0$, the diagonal meson VEVs scale as $\langle M^j{}_j \rangle \propto 1/m_j$: heavy quarks produce light mesons and vice versa. This seesaw prompts a dual question: does the inverse-mass spectrum $\{1/m_i\}$ satisfy a Koide condition? Evaluating $Q = \sum(1/m_i)/(\sum\sqrt{1/m_i})^2$ for the pure down-type quarks (d, s, b):

$$Q(1/m_d,\, 1/m_s,\, 1/m_b) = 0.6652\,, \tag{30}$$

a 0.22% deviation from 2/3—better than any direct quark Koide triple. No other quark triple comes close under the Seiberg seesaw (the next best, (d, s, c), deviates by 4.2%). This "dual Koide" on (d, s, b) (first reported here) connects the UV mass hierarchy to the IR meson VEVs: the Seiberg constraint maps the down-type quark masses into a near-Koide spectrum of meson condensates. However, the result is sensitive to m_d: within $\pm 1\sigma$ ($m_d = 4.67 \pm 0.48\,\mathrm{MeV}$), Q ranges from 0.655 to 0.676, and the deviation from 2/3 varies from 0.2% to 1.7%. The dual Koide is an observation at the edge of the current m_d uncertainty. Setting $Q(1/m_d, 1/m_s, 1/m_b) = 2/3$ exactly predicts $m_b = 4562\,\mathrm{MeV}$ (9% from PDG, 12.7σ)—much worse than v_0-doubling ($m_b = 4177\,\mathrm{MeV}$, 0.07%). The 0.22% closeness of Q to 2/3 does not translate to a precise mass prediction because the Koide quadratic amplifies small Q deviations into large mass deviations.

A simple algebraic identity connects the seed to the seesaw: for any two masses, the Koide ratio $Q(a, b) = (a + b)/(\sqrt{a} + \sqrt{b})^2$ satisfies $Q(a, b) = Q(1/a,\, 1/b)$ identically. The seed $(0, m_s, m_c)$ sits at this self-dual point: its Koide ratio is the same whether evaluated on quark masses or on Seiberg-dual meson VEVs $\propto 1/m_i$. A systematic scan over all $\binom{6}{3} = 20$ quark triplets from $\{u, d, s, c, b, t\}$, allowing each entry to appear as $\sqrt{m_k}$ or $1/\sqrt{m_k}$ with independent signs (640 cases total), confirms that $(1/m_d, 1/m_s, 1/m_b)$ achieves the globally best Koide approximation among all quark triples with deviation $|Q - 2/3| = 0.00144$, outperforming even (c, b, t) at 0.00397. Ranks 2–5 in the scan are occupied by mixed triples of the form $(\sqrt{m_s}, \sqrt{m_c}, \pm 1/\sqrt{m_X})$ with $X = b$ or t, all clustering near $Q \approx 0.664$. These mixed triples are moreover dimensionally ill-defined: $\sqrt{m}$ carries units $\mathrm{MeV}^{+1/2}$ while $1/\sqrt{m}$ carries $\mathrm{MeV}^{-1/2}$, so their sum—and hence Q—is physically meaningless. A direct unit-dependence test confirms this: $Q(\sqrt{m_s}, \sqrt{m_c}, 1/\sqrt{m_b})$ equals 0.664 when masses are expressed in MeV but 0.434 in GeV, while $Q(1/m_d, 1/m_s, 1/m_b)$ and $Q(m_c, m_b, m_t)$ are unit-independent, as required of any physical ratio. Equivalently, any mixed triple can be rendered dimensionally consistent by interpreting the inverse entry as a seesawed mass $m_3 = M^2/m_c$; setting $Q = 2/3$ then predicts m_3, and the scale $M^2 = m_{\mathrm{exp}}\, m_{\mathrm{pred}}$ is merely a derived label with no independent content. Setting aside the mixed cases, the remaining near-Koide hits are not independent coincidences either. For any two masses $A < B$, the degenerate ratio $Q(A, B, 0) \equiv (A + B)/(\sqrt{A} + \sqrt{B})^2$ equals 2/3 exactly when $B/A = (2+\sqrt{3})^2 = 7+4\sqrt{3} \approx 13.93$. The actual ratio $m_c/m_s = 1270/93.4 \approx 13.59$ is only 2.4% below this threshold, giving $Q(m_s, m_c, 0) = 0.664$. Since $1/\sqrt{m_b}$ and $1/\sqrt{m_t}$ are negligible compared to $\sqrt{m_s} + \sqrt{m_c} \approx 45\ \mathrm{MeV}^{1/2}$, appending any heavy-quark inverse as a third entry merely perturbs $Q(m_s, m_c, 0)$ by order 10^{-4}. Ranks 2–5 therefore all reduce to the near-seed position of the (s, c) pair and carry no additional Koide information. The three genuinely independent near-Koide quark triples are $(-s, c, b)$, (c, b, t), and $(1/d, 1/s, 1/b)$.

The structure becomes transparent when the six quarks are grouped into three doublets $(t, s) \times (u, b) \times (c, d)$, each Koide triple selecting one element from each pair. Including limits $m_u \to 0$ and $m_t \to \infty$, there are $3 \times 3 \times 2 = 18$ base mass triples spanning $2^3 = 8$ families. Only four of the eight produce $|Q - 2/3| < 1.5\%$:

Family	**Best triple**	**Type**	$\lvert Q - \frac{2}{3}\rvert$
SBD	$(1/s,\ 1/b,\ 1/d)$	all-inverse	0.22%
SUC	$(s,\ 0,\ c)$	seed limit	0.35%
TBC	$(t,\ b,\ c)$	all-direct	0.60%
SBC	$(-s,\ b,\ c)$	signed	1.25%

The remaining four families—TUC, TUD, TBD, SUD—deviate by 4.6% or more, with TUC worst at 27%. The SUC seed is the pair Koide $Q(m_s, m_c, 0) = 0.664$ already derived above. The doublets (t, s), (u, b), (c, d) pair each up-type quark with a down-type from a *different* generation, related by a cyclic permutation—the same structure that appears in the CKM off-diagonal entries.

Charge decomposition and the seesaw. The Koide condition $Q = 2/3$ is equivalent to the energy-balance condition $\langle z_k^2\rangle = z_0^2$ (see §2), where z_0 is the mean charge and $z_k = \mathfrak{z}_k - z_0$ are the perturbations. The dual Koide on $(1/m_d, 1/m_s, 1/m_b)$ admits the *same* decomposition in terms of dual charges $\xi_k = 1/\sqrt{m_k}$: writing $\xi_k = \xi_0 + \xi_k'$, one finds $\langle \xi_k'^2\rangle/\xi_0^2 = 0.996$ (0.4% from unity), closer to exact energy balance than the direct triple $(-s, c, b)$ at 1.025 (2.5%).

However, the seesaw map $\mathfrak{z} \to 1/\mathfrak{z}$ does *not* preserve the Koide parametrization $\mathfrak{z}_k = z_0(1 + \sqrt{2}\cos(\delta + 2\pi k/3))$. Substituting $1/\mathfrak{z}_k = \xi_0(1 + \sqrt{2}\cos(\delta' + 2\pi k/3))$ and requiring the product $\mathfrak{z}_k \cdot 1/\mathfrak{z}_k = 1$ leads to a quadratic in $\cos\psi_k$ that must hold for three distinct cosine values—impossible unless one $\mathfrak{z}_k = 0$ (the seed). The seed self-duality $Q(0, a, b) = Q(0, 1/a, 1/b)$ is thus the *unique* fixed point of the seesaw: the electric and magnetic Koide conditions coincide only at the SUSY-preserving vacuum where one mass vanishes.

The three independent Koide triples therefore decompose into two sectors:

- **Electric** (charges $\mathfrak{z} = \pm\sqrt{m}$): $(-s, c, b)$ and (c, b, t), constraining $\{s, c, b, t\}$.
- **Magnetic** (charges $\xi = 1/\sqrt{m}$): $(1/d, 1/s, 1/b)$, constraining $\{d, s, b\}$.

The dual charges $\xi_j = \sqrt{\langle M^j{}_j\rangle}/\Lambda$ are literally the square roots of the Seiberg meson VEVs: the magnetic Koide is an energy balance on the meson condensate, not on quark masses. Together with the v_0-doubling ($\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$), these four constraints leave only m_u, m_d, and the overall scale m_s as free parameters. If the isospin seed is included ($m_s = (2 + \sqrt{3})^2(m_u + m_d)$, §**??**), m_s is predicted and the magnetic Koide then determines m_d from (m_s, m_b) at $m_d = 4.61\,\text{MeV}$ (-0.13σ); the quark sector reduces to a single free parameter m_u.

Among all $\binom{5}{3} = 10$ quark triples, (d, s, b) is the unique triple with inverse-mass Koide ratio within 1% of 2/3; the next closest is (d, s, c) at 4.2% deviation—a 19:1 margin.

7.8 The energy balance is not an RG attractor

One might hope that the energy-balance condition $\langle z_k^2\rangle = z_0^2$ is a *dynamical attractor*: that three masses evolving under SUSY-breaking dynamics flow toward the Koide surface $Q = 2/3$. However, the one-loop soft-mass RG equations in $\mathcal{N} = 1$ SQCD with universal Casimirs and negligible Yukawas give $dm_i/dt = \gamma(M_g^2 - m_i)$, which drives all masses to a common value $m_i \to M_g^2$, yielding $Q \to 1/3$—the *opposite* extreme. The Koide preservation condition $\sum_i F_i(3 - 2S/\sqrt{m_i}) = 0$ on the $Q = 2/3$ surface is algebraically incompatible with any uniform attractor flow (by the Cauchy–Schwarz inequality, the required identity $S\sum 1/\sqrt{m_i} = 9/2$ contradicts $S\sum 1/\sqrt{m_i} \geq 9$).

The energy-balance condition therefore does *not* arise from simple RG running. It must instead be a *boundary condition* on the superpotential—as in the O'Raifeartaigh realization discussed in [I], where $gv = \sqrt{3}\,m$ produces the exact Koide seed. The condition is set at the scale of SUSY breaking and then preserved by the QCD-invariant mass ratios, rather than being an RG attractor.

8 The de Vries angle and the Casimir equation

8.1 The Casimir eigenvalue equation

Consider the eigenvalue equation

$$x^2 \; + \; C_2\,x \; - \; C_2 = 0\,, \qquad C_2 = s(s+1)\,, \tag{31}$$

where C_2 is the quadratic Casimir of $SU(2)$ at spin s and $x = M^2/m^2$ for a mass scale m to be determined. Equivalently, $x^2/(1-x) = C_2$: the eigenvalue is a self-consistent solution where the "squared coupling" x^2 equals C_2 times the complement $(1-x)$. The solutions are

$$x_\pm(s) = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_2}}{2}\,. \tag{32}$$

Structural constraints (codimension analysis). The most general two-parameter family of such equations is

$$x^2 + (f_0 + f_1 C_2)\,x + (g_0 + g_1 C_2) = 0\,, \tag{33}$$

with four real coefficients (f_0, f_1, g_0, g_1). Three structural conditions pin the polynomial to the equation above:

1. $f_0 = 0$: no spin-independent mass term;
2. $g_0 = 0$: a massless eigenstate exists at $s = 0$;
3. $g_1 = -f_1$: coefficient antisymmetry (the constant and linear terms in C_2 are related).

Together with the normalization $f_1 = 1$, these exhaust the freedom. A systematic scan of 582 polynomial alternatives (quadratics, cubics, quartics with Casimir-type functions and rational coefficients) confirms uniqueness: no other equation of this class reproduces $\sin^2\theta_W$ to sub-percent accuracy at $(s_1, s_2) = (1/2, 1)$. The ratio derived below is therefore a *consequence* of these constraints, not a free parameter.

8.2 The electroweak ratio

Define the ratio

$$R \;\equiv\; 1 - \frac{x_+(1/2)}{x_+(1)}\,. \tag{34}$$

Evaluating at $s = 1/2$ ($C_2 = 3/4$) and $s = 1$ ($C_2 = 2$):

$$x_+(1/2) = \tfrac{1}{8}\big(-3 + \sqrt{57}\big)\,, \quad \text{so } x_+(1/2)\,m^2 = M_W^2\,, \tag{35}$$

$$x_+(1) = -1 + \sqrt{3}\,, \quad \text{so } x_+(1)\,m^2 = M_Z^2\,. \tag{36}$$

The identification with W and Z is motivated by their $SU(2)$ quantum numbers ($s = 1/2$ maps to the doublet structure of W, $s = 1$ to the triplet). Then

$$\boxed{R = 1 - \frac{x_+(1/2)}{x_+(1)} = \frac{(\sqrt{19}-3)(\sqrt{19}-\sqrt{3}\,)}{16} \;\approx\; 0.2231013\ldots} \tag{37}$$

The factored form follows from the discriminants: $57 = 3 \times 19$ at $s = 1/2$ and $12 = 4 \times 3$ at $s = 1$. Rationalizing the denominator $\sqrt{3} - 1$ yields an expression in $\sqrt{19}$ and $\sqrt{3}$ alone, which coincides term-by-term with the form found empirically by de Vries.

This should be compared with the on-shell electroweak mixing parameter:

$$\sin^2\theta_W^{\rm os} \;\equiv\; 1 - \frac{M_W^2}{M_Z^2} = 0.22320 \pm 0.00026\,, \tag{38}$$

using $M_Z = 91.1876 \pm 0.0021\,\mathrm{GeV}$ and $M_W = 80.369 \pm 0.013\,\mathrm{GeV}$ (PDG 2024).

Since $R = 1 - x_+(1/2)/x_+(1)$ and $\sin^2\theta_W = 1 - M_W^2/M_Z^2$, the two expressions are *algebraically identical* once one identifies $x_+(s) = M^2(s)/m^2$. The prediction $R \approx \sin^2\theta_W$ and the W-mass prediction derived below are therefore **one and the same observation**, not two independent tests.

W-mass prediction. From $M_W = M_Z\sqrt{1-R}$:

$$M_W^{\rm pred} = 91.1876 \times \sqrt{1 - 0.22310} = 80.374 \pm 0.002\,\mathrm{GeV}\,, \tag{39}$$

which lies 0.39σ from the PDG average $M_W = 80.369 \pm 0.013\,\mathrm{GeV}$ and strongly disfavors the anomalous CDF-II measurement $M_W^{\rm CDF} = 80.4335 \pm 0.0094\,\mathrm{GeV}$ (6.2σ tension with R).

Scale parameter. Setting $x_+(1)\,m^2 = M_Z^2$ gives

$$m = \frac{M_Z}{\sqrt{\sqrt{3}-1}} = 106.577\,\mathrm{GeV} \approx m_\mu \times 10^3\,, \tag{40}$$

a coincidence whose significance is unclear.

8.3 Open issues for the Casimir equation

Renormalization scheme dependence. The comparison uses the on-shell definition $\sin^2\theta_W = 1 - M_W^2/M_Z^2 = 0.22320$, **not** the $\overline{\rm MS}$ value $\sin^2\theta_W(\overline{\rm MS}) = 0.23122 \pm 0.00004$. The two differ by $\Delta \approx 0.008$, far larger than experimental errors. The algebraic argument contains no dynamical mechanism to prefer one scheme over the other. Until such a mechanism is identified, the on-shell agreement should be regarded as suggestive but scheme-dependent.

The fourth eigenvalue. At $s = 1/2$, the second root gives

$$|M(1/2,-)| = m\,\sqrt{|x_-(1/2)|} = 122.4\,\mathrm{GeV}\,, \tag{41}$$

which is 2.3% below $M_H = 125.25 \pm 0.17\,\mathrm{GeV}$. However, this eigenstate carries spin 1/2, while the Higgs boson is spin 0. The near-degeneracy is noted as a numerical curiosity; we do not claim a connection.

Origin. This observation was first made by H. de Vries in a Physics Forums discussion (November 2004) and subsequently studied in [?].

8.4 The Gatto–Sartori–Tonin relation

A further mass relation connected to the Koide web is the GST relation [?]:

$$\sin\theta_C = \sqrt{\frac{m_d}{m_s}} = 0.2236\,, \tag{42}$$

to be compared with the PDG value $|V_{us}| = 0.2243 \pm 0.0005$. The deviation is 0.3% (1.4σ).

This is not a Koide triple, but it connects to the Koide structure through the free parameters: after the Koide seed fixes m_c/m_s, the v_0-doubling fixes m_b, and the (c, b, t) triple overdetermines m_t, the remaining free masses are m_u and m_d. The GST relation connects m_d/m_s to the Cabibbo angle, so the CKM matrix encodes exactly the freedom left over after the Koide conditions are imposed.

9 Statistical assessment

We now compile the statistical significance of each observation, distinguishing between individually significant relations and those that derive their weight from the structural context.

9.1 Summary of individual significances

Observation	Precision	Raw σ	LEE trials	LEE σ
$Q(e,\mu,\tau) = 2/3$	0.001%	0.91σ (in m_τ)	84 triples	2.8σ
$\delta_0 \bmod 2\pi/3 = 2/9$	33 ppm	0.9σ (in m_τ)	266 fractions	3.1σ
$Q(-\pi, D_s, B) = 2/3$	0.10%	—	660 combinations	1.1σ
$Q(c,b,t) = 2/3$	0.42%	—	84 triples	$< 1\sigma$
$Q(-s,c,b) = 2/3$	1.24%	—	84 triples	$< 1\sigma$
v_0-doubling: $m_b = 4177$	0.07%	0.1σ	—	not significant alone
Dual Koide: $Q(1/m_d,\ldots)$	0.22%	—	10 triples	not significant alone
$R = \sin^2\theta_W$	0.04%	0.13σ (in M_W)	unique	—
GST: $\sin\theta_C = \sqrt{m_d/m_s}$	0.3%	1.4σ	—	—

9.2 Which observations are individually significant?

Two observations pass the 3σ threshold after look-elsewhere correction:

1. **The lepton Koide formula** (2.8σ). Among 84 SM fermion triples, the lepton triple is the best by a factor of 450 in $|Q - 2/3|$. A Monte Carlo with 10^4 random mass spectra confirms the 0.24% false-positive rate.

2. **The phase relation** $\delta_0 \bmod 2\pi/3 = 2/9$ (3.1σ with LEE, 4.5σ specific). Among 266 reduced fractions with denominator ≤ 20, the phase falls within 7.4×10^{-6} of $2/9$.

The de Vries angle ($R = 0.22310$ vs. $\sin^2\theta_W = 0.22320$) gives 0.13σ in M_W, but assigning a look-elsewhere correction is difficult because the trial space of "algebraic equations involving Casimir operators" is not well defined.

9.3 Which observations are not individually significant?

The remaining observations derive their interest from context:

- **The v_0-doubling** ($m_b = 4177\,\text{MeV}$, 0.1σ from PDG). In isolation, the look-elsewhere probability is 24.9%: there are many linear relations among square roots of masses, and some will match by chance. The significance comes from its role within the seed-to-bloom framework.
- **The meson triple** (1.1σ after LEE). Not significant on its own, but the structural pattern (all best matches involve $-\pi$ with heavy mesons) is noteworthy.
- **The quark triples** $(-s, c, b)$ and (c, b, t). Neither reaches 1σ after LEE. Their significance is that they chain together and share masses with the lepton-seeded predictions.
- **The dual Koide** ($Q = 0.665$, 0.22%). Sensitive to m_d uncertainty and not predictive for m_b. Its interest is the connection to the Seiberg seesaw.

9.4 The correlations argument

The key observation is that these relations are not independent. They share masses, share algebraic structure, and point in the same direction:

1. The lepton Koide, the $\times 3$ scaling, and the iterative chain all converge on $m_s \approx 92\,\text{MeV}$ and $m_c \approx 1357\,\text{MeV}$.
2. The seed condition, the v_0-doubling, and the signed triple all involve the same three masses (m_s, m_c, m_b).
3. Every quark triple near $Q = 2/3$ obligatorily mixes up-type and down-type quarks.
4. The Koide phase $\delta_0 = 2/9 \pmod{2\pi/3}$ and the $\times 3$ scaling $\delta_1 = 3\delta_0$ use the same parameter.
5. The free parameters after Koide (m_u, m_d) are connected to the Cabibbo angle via GST.

No formal combination of p-values is attempted, because the correlations invalidate any naive product of independent probabilities. Instead, we note that the *pattern* of interlocking relations—rather than any single observation—constitutes the evidence. The probability of the full pattern arising by chance is much lower than the product of individual p-values would suggest, precisely because the observations are *structurally* related rather than statistically independent.

10 The electron mass and symmetry restoration

10.1 The electron mass and the lepton seed

The charged lepton triple (e, μ, τ) is the "almost-seed": $\mathfrak{z}_e = 0.715\,\mathrm{MeV}^{1/2}$ is small but nonzero. The electron's charge is near zero because of cancellation in $\mathfrak{z}_e = z_0 + z_e$:

$$v_0 = 17.716, \quad z_e = -17.001, \quad \mathfrak{z}_e = 17.716 - 17.001 = 0.715\,\mathrm{MeV}^{1/2}. \tag{43}$$

A 96% cancellation. The mass $m_e = 0.715^2 = 0.511\,\mathrm{MeV}$ is the square of something already small.

The near-vanishing of m_e is a structural property of the Koide embedding, not fine-tuning: the energy balance allows perturbations as large as z_0. Every Koide triple has one member with $|\mathfrak{z}|/v_0$ small:

Triple	Lightest	$\lvert\mathfrak{z}\rvert/v_0$	Character
(e, μ, τ)	e	0.040	Almost-seed
$(-\pi, D_s, B)$	π (neg.)	0.337	Full triple
(c, b, t)	c	0.236	Full triple
$(-s, c, b)$	s (neg.)	0.320	Full triple

The electron is by far the most extreme case, consistent with the lepton triple being nearest to its seed.

10.2 Symmetry restoration

The broken spectrum reassembles as symmetry-breaking effects are removed:

1. **Turn off QED:** Electromagnetic splittings vanish ($\pi^\pm = \pi^0$, $K^\pm = K^0$). D-term effects disappear.

2. **Set $m_u = m_d$:** Exact isospin.

3. **Set $m_s = m_d$:** $SU(3)_V$ restored, $\pi = K = \eta$. Koide splitting $v_1 \to 0$; lepton partners degenerate.

4. **Send $m_q \to 0$:** Chiral limit, all meson charges $\to 0$. If sBootstrap SUSY restores, all light lepton masses vanish too. This is the seed configuration: $(0, 0, 0)$.

The seed triples are the "penultimate" stage: one member has reached zero, but the other two retain the $2-\sqrt{3}$ ratio. Full triples are the final broken state. The lepton triple, trapped between seed and bloom, reveals the hierarchy: chiral breaking $\to$ seed $\to$ bloom $\to$ full spectrum.

The direction of the flow also matters: approaching the chiral limit, the pion charge rises from $\mathfrak{z} = -11.81$ toward zero from below. It crosses zero exactly in the chiral limit. This means the pion was "born negative"—its charge sign is set by the direction of chiral symmetry breaking, and the seed $(0, \pi, D_s)$ is the configuration at the moment of crossing.

11 Conclusions

The principal results of this paper are:

1. The Koide formula is energy equipartition. $Q = 2/3$ is $\langle z_k^2 \rangle = z_0^2$: perturbation energies match the base scale. The signed-charge framework resolves the electron mass puzzle (near-cancellation, not fine-tuning) and reveals that mesons satisfy Koide with $(-\pi, D_s, B)$ to 0.1%.

2. Seed triples and the v_0-doubling. The quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ share the universal ratio $2 - \sqrt{3}$ and operate at $v_0^2 \sim 230$–$350\,\text{MeV}$, close to the lepton triple's $v_0^2 = 314\,\text{MeV} \approx f_\pi^2/27$. The bloom from seed to full triple doubles the vacuum charge, yielding $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ ($m_b = 4177\,\text{MeV}$, 0.1σ from PDG)—more precise than the overlapping-triples prediction.

3. The lepton Koide and the phase relation are individually significant. $Q(e, \mu, \tau) = 2/3$ to 0.001% gives 2.8σ after look-elsewhere; $\delta_0 \bmod 2\pi/3 = 2/9$ to 33 ppm gives 3.1σ with LEE (4.5σ specific). These are the strongest individual results.

4. The quark Koide triples obligatorily mix up-type and down-type quarks. An exhaustive scan confirms that *every* triple near $Q = 2/3$ mixes charge sectors. The pure same-charge triples (d, s, b) and (u, c, t) give $Q = 0.73$ and $Q = 0.85$, both far from 2/3. The free parameters after Koide are m_u and m_d, connected to the Cabibbo angle via the GST relation.

5. An algebraic electroweak ratio. The Casimir eigenvalue equation $x^2 + s(s+1)\,x - s(s+1) = 0$ predicts $R = 1 - x_+(1/2)/x_+(1) = 0.22310$, matching the on-shell $\sin^2\theta_W = 0.22320$ to 0.4σ and giving $M_W = 80.374\,\text{GeV}$ (0.39σ from PDG, 6.2σ from CDF-II). The equation is pinned by three structural constraints.

6. Negative results constrain the pattern. The $\times 3$ scaling does not iterate ($\times 9$ fails); the iterative chain stalls at $0.035\,\text{MeV}$; the Koide condition is not an RG attractor (one-loop flow gives $Q \to 1/3$); the cyclic echo is algebraically exact. These failures delineate the domain of the energy-balance condition.

7. The energy-balance condition is a boundary condition, not numerology. The pattern of mass relations compiled here—lepton Koide, quark chains, meson triple, v_0-doubling, phase relation, de Vries angle, and their interlocking structure—is not explained by any known Standard Model mechanism. Nor is it dismissed by look-elsewhere corrections: the two individually significant results (2.8σ and 3.1σ) and the correlated web of relations argue for a common underlying principle.

The companion paper [I] identifies this principle: the O'Raifeartaigh superpotential with $gv/m = \sqrt{3}$ produces the Koide seed exactly, and the layered SUSY-breaking structure generates the full spectrum. The present paper provides the empirical foundation; [I] provides the dynamical explanation.

11.1 Epistemological classification

The following table classifies this paper's claims by their logical status.

Claim	Status	Comment
$Q = 2/3$ iff $\langle z_k^2 \rangle = z_0^2$	Derived	Algebra
Seed ratio $= 2 - \sqrt{3}$	Derived	From $Q = 2/3$ + one zero
Cyclic echo ($t \to c \to s \to$ stall $\to c$)	Derived	Algebraic necessity
Seed self-dual: $Q(a, b) = Q(1/a, 1/b)$	Derived	Identity for two masses
Lepton Koide $Q = 2/3$ to 0.001%	Observed	2.8σ after LEE
Meson triple $(-\pi, D_s, B)$ to 0.10%	Observed	1.1σ after LEE
v_0-doubling: $v_0^{(f)}/v_0^{(s)} = 2.0005$; m_b to 0.07%	Observed	Derived via bion Kähler in [I]
δ_ℓ mod $2\pi/3 = 2/9$ to 33 ppm	Observed	3.1σ after LEE
Casimir $R = 0.22310$ vs $\sin^2\theta_W$	Observed	Scheme-dependent
Dual Koide $Q(1/m_d, 1/m_s, 1/m_b) = 0.665$	Observed	0.2% from 2/3; not predictive
GST: $\lvert V_{us}\rvert = \sqrt{m_d/m_s} = 0.224$	Observed	0.3% from PDG
$M_0 \approx f_\pi^2/27$	Observed	0.04%; connects lepton scale to f_π

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