A Tale of Two Tuples: Seed Triples, Signed Charges, and SUSY Breaking in the sBootstrap

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Abstract

We reformulate the sBootstrap in terms of signed mass charges $\mathfrak{z} = z_0 + z_i$ with $m = \mathfrak{z}^2$, and show that the Koide ratio $Q = 2/3$ is the algebraic consequence of two conditions: $\sum z_i = 0$ and $\langle z_i^2 \rangle = z_0^2$ (perturbation energies average to the base-scale energy). Allowing negative charges, we find that pseudoscalar mesons satisfy a Koide triple $(-\pi, D_s, B)$ to 0.1%, and identify *seed triples* with one massless member—$(0, s, c)$ for quarks, $(0, \pi, D_s)$ for mesons—as the pre-breaking configurations from which the full spectrum emerges. These seed triples share the same analytical structure (charge ratio $2 - \sqrt{3}$) and, strikingly, the meson seed $(0, \pi, D_s)$ has Koide scale $v_0^2 \approx 351\,\mathrm{MeV}$, close to the lepton triple's $v_0^2 \approx 314\,\mathrm{MeV}$. A third tuple (c, b, t) stands apart, having no meson counterpart because top diquarks do not hadronize. We discuss the diquark sector—which fills the symmetric $\mathbf{15} + \overline{\mathbf{15}}$ of $SU(5)$ flavor, in conflict with Pauli statistics for point-like diquarks, proving that the diquark cannot be a qq composite—and outline how SUSY breaking proceeds through two mechanisms—F-term (chiral condensate, with Volkov–Akulov as its low-energy realization) and D-term (electromagnetic Fayet–Iliopoulos)—plus a separate heavy-sector origin for the (c, b, t) triple. The O'Raifeartaigh superpotential produces the Koide seed spectrum exactly when $gv/m = \sqrt{3}$, reducing the energy-balance condition to a single constraint on the couplings. The bloom from seed to full triple doubles the vacuum charge, yielding $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ (matching PDG to 0.1σ), and all quark Koide triples obligatorily mix up-type and down-type mass eigenstates. We construct the explicit $\mathcal{N} = 1$ Lagrangian—Kähler potential, superpotential (Seiberg constraint, Yukawa with separate H_u, H_d doublets), D-terms, and soft breaking from a spurion with $\langle F \rangle = f_\pi^2$—and show that the mixed up/down-type Koide triples require $\tan\beta = 1$ for the mass-level condition to equal a Yukawa-level condition. A negative quartic Kähler correction $c = -1/12$ stabilizes the pseudo-modulus at $\langle X \rangle = \sqrt{3}\,\mu$ (the Kähler pole), while the v_0-doubling originates in the Kähler sector via a bion correction $|\sum s_k \sqrt{\hat{m}_k}|^2$ from monopole-instantons on $\mathbb{R}^3 \times S^1$; a three-instanton superpotential $(\det M)^3/\Lambda^{18}$ generates a $\cos(9\delta)$ potential whose minima include the observed lepton Koide phase to 0.9σ in m_τ. The Seiberg Lagrange multiplier X has no kinetic term ($N_f = N_c$ admits no magnetic dual), so $F_X = 0$ and all off-diagonal meson masses are positive ($\sim 2f_\pi^2$); a fundamental block-diagonal obstruction prevents any polynomial Kähler correction from generating cross-flavour meson condensates. The seesaw vacuum is cosmologically stable ($S_{\mathrm{bounce}} > 10^7$), and dimensional analysis forces the NMSSM singlet S

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to be a separate field from X ($\lambda_{\text{Higgs}}/\lambda_{\max} \sim 10^5$). The Gatto–Sartori–Tonin relation $\sin\theta_C = \sqrt{m_d/m_s}$ is a structural property of the seesaw-inverted mass hierarchy rather than a derived prediction. The isospin-summed mass $m_u + m_d$ satisfies $Q(0,\, m_u{+}m_d,\, m_s) = 0.665$ (0.27% from 2/3), extending the Koide chain to the first generation. A separate $SU(2)$ confining sector (s-confining, $N_f = 3$) with an O'Raifeartaigh deformation generates the lepton Koide seed via the same universal mechanism. After these constraints, only two to three free parameters remain (m_u, m_d, m_s), with m_u and m_d connected to the Cabibbo angle via the GST relation. The $SO(32)$ adjoint accommodates the sBootstrap representations; $E_8 \times E_8$ is blocked because E_8 decompositions produce only antisymmetric **10**, never the symmetric **15** required for diquarks. A Diophantine system on the scalar content uniquely selects $N = 3$ generations.

Contents

1 Introduction

The sBootstrap [1,2] proposes that the Standard Model's pseudoscalar mesons and leptons fill common $\mathcal{N} = 1$ supermultiplets. Its foundational relation is

$$m_\pi^2 - m_\mu^2 \approx f_\pi^2\,, \tag{1}$$

identifying the pion decay constant $f_\pi \approx 92.2\,\mathrm{MeV}$ as the SUSY-breaking order parameter. Numerically, $m_\pi^2 - m_\mu^2 = 8{,}316\,\mathrm{MeV}^2$ versus $f_\pi^2 = 8{,}501\,\mathrm{MeV}^2$, a 2.2% discrepancy—comparable to the precision of the quark Koide triples discussed below. In this paper we develop two themes that change the structure of the program.

First, **mass is the square of a signed charge**: $m = \mathfrak{z}^2$ where $\mathfrak{z} = z_0 + z_i$ can be negative. The Koide ratio $Q = 2/3$ is then not numerology but the energy-balance condition $\langle z_i^2 \rangle = z_0^2$. With signed charges, pseudoscalar mesons satisfy Koide, and the spectrum organizes into *seed triples* with a massless member that grow into full triples upon symmetry breaking.

Second, **the full sBootstrap has three Koide tuples, not one**, and the breaking structure must generate all three: a light sector $(0, s, c) \to (-s, c, b)$ seen in both quarks and mesons, a lepton sector $(e \approx 0, \mu, \tau)$, and a heavy sector (c, b, t) that decouples from meson physics because the top quark does not hadronize. The diquark partners of quarks, needed to complete the SUSY multiplets, fill the symmetric $\mathbf{15} + \overline{\mathbf{15}}$ of $SU(5)$ flavor—the representation produced naturally by the SUSY generator and by the $SO(32)$ decomposition. A kinematic theorem shows that no $J = 0$, color-$\bar{3}$ diquark can have symmetric flavor—regardless of orbital angular momentum—so the sBootstrap diquark cannot be a qq composite but must be an algebraically or string-theoretically generated object.

Third, **the Koide condition constrains the superpotential**. The O'Raifeartaigh model—the simplest realization of F-term SUSY breaking with three chiral superfields—produces a fermionic mass spectrum that is an exact Koide seed triple when the ratio of the cubic coupling to the bilinear mass parameter satisfies $gv/m = \sqrt{3}$. This single algebraic constraint is the microscopic content of the energy-balance condition. Separately, a Diophantine system on the scalar content ($rs = 2N$, $r(r+1)/2 = 2N$, $r^2 + s^2 - 1 = 4N$) uniquely selects $N = 3$ generations, and a Casimir eigenvalue equation predicts $\sin^2\theta_W = 0.22310$ to 0.4σ.

New results in this paper. Compared to the prior publication [1], the following are first reported here: the signed-charge formalism and the meson Koide triple $(-\pi, D_s, B)$;

the seed-triple concept and the universal ratio $2-\sqrt{3}$; the v_0-doubling relation $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$; the O'Raifeartaigh derivation $gv/m = \sqrt{3}$ and its Kähler stabilization at the pole $c = -1/12$; the Pauli theorem for symmetric diquarks; the bloom instability under perturbative R-breaking; the dual Koide on (d, s, b) and its connection to the Seiberg seesaw; the bion Kähler mechanism for the v_0-doubling; the self-consistent prediction chain $m_s \to m_c \to m_b \to m_t$; and the scale relation $M_0 \approx f_\pi^2/27$ tying the lepton Koide scale to the pion decay constant; and the prediction $\tan\beta = 1$ from the mixed up/down-type Koide triples. The Casimir eigenvalue equation and the $SO(32)$ decomposition appeared in our prior work; here we add the algebraic proof linking the Casimir ratio to the de Vries form, a uniqueness analysis of the equation, the N_f-phase structure of the SQCD realization, and the three-instanton origin of the $\cos(9\delta)$ potential that selects the lepton Koide phase.

2 The Koide Formula as Energy Balance

2.1 Charges, not masses

The fundamental variable is the signed mass charge $\mathfrak{z} = z_0 + z_i \in \mathbb{R}$, with the physical mass $m = \mathfrak{z}^2 \geq 0$.

2.2 Derivation of $Q = 2/3$

For three charges $\mathfrak{z}_k = z_0 + z_k$ with z_0 the mean ($\sum z_k = 0$):

$$Q = \frac{\sum \mathfrak{z}_k^2}{(\sum \mathfrak{z}_k)^2} = \frac{3z_0^2 + \sum z_k^2}{9z_0^2} = \frac{1}{3} + \frac{\langle z_k^2 \rangle}{3z_0^2}\,. \tag{2}$$

Therefore:

$$\boxed{Q = \frac{2}{3} \quad \Longleftrightarrow \quad \langle z_k^2 \rangle = z_0^2\,.} \tag{3}$$

The two conditions are:

1. **Zero-sum perturbations:** $z_1 + z_2 + z_3 = 0$ (definition of z_0 as the mean charge).
2. **Energy balance:** The mean energy of the perturbations equals the energy of the unperturbed charge: $\langle z_k^2 \rangle = z_0^2$.

Condition 2 is the physics. It says the splitting scale matches the base scale—the coefficient of variation equals unity: $\sigma(\mathfrak{z})/\bar{\mathfrak{z}} = 1$. (Dimensionally, $\mathfrak{z}$ has units $\mathrm{MeV}^{1/2}$, so z_0^2 has units MeV—a mass scale, not an energy density. The "energy balance" terminology is metaphorical.) This is neither perturbative ($\sigma \ll \bar{\mathfrak{z}}$, giving $Q \to 1/3$, all equal) nor dominant ($\sigma \gg \bar{\mathfrak{z}}$, giving $Q \to 1$, one mass dominates). The Koide value 2/3 is the equipartition point.

2.3 Why one mass can vanish

If $\langle z_k^2 \rangle = z_0^2$, a perturbation z_k can reach $|z_k| \sim z_0$, making $\mathfrak{z}_k = z_0 + z_k \approx 0$. A vanishing mass is the *natural extreme* of energy balance, not fine-tuning.

3 Seed Triples: The $(0, a, b)$ Structure

3.1 Analytical condition

For a triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ with one massless member, the Koide ratio becomes:

$$Q = \frac{\mathfrak{z}_a^2 + \mathfrak{z}_b^2}{(\mathfrak{z}_a + \mathfrak{z}_b)^2} \, . \tag{4}$$

Setting $Q = 2/3$ and writing $r = \mathfrak{z}_a/\mathfrak{z}_b$ (with $r < 1$):

$$3(r^2 + 1) = 2(r + 1)^2 \quad \Longrightarrow \quad r^2 - 4r + 1 = 0 \, , \tag{5}$$

giving:

$$\boxed{r = 2 - \sqrt{3} \approx 0.26795 \, .} \tag{6}$$

Equivalently, in mass space: $m_{\text{light}}/m_{\text{heavy}} = (2 - \sqrt{3})^2 = 7 - 4\sqrt{3} \approx 0.07180$.

This is a *universal* prediction: any Koide triple with one massless member has the same charge ratio between the two massive members, regardless of their absolute scale.

3.2 The quark seed: $(0, s, c)$

The strange and charm quark charges give:

$$\frac{\sqrt{m_s}}{\sqrt{m_c}} = \frac{9.67}{35.64} = 0.2712 \, , \qquad r_{\text{Koide}} = 0.2679 \, , \qquad \text{deviation: } 1.2\% \, . \tag{7}$$

This triple has $Q = 0.6644$, deviating from 2/3 by only 0.35%. Its Koide scale is $v_0^2 = (\mathfrak{z}_s + \mathfrak{z}_c)^2/9 \approx 228\,\text{MeV}$.

Scale ambiguity in the quark seed. Throughout this paper, quark masses follow PDG conventions: $m_s(2\,\text{GeV}) = 93.4\,\text{MeV}$, $m_c(m_c) = 1.27\,\text{GeV}$, $m_b(m_b) = 4.18\,\text{GeV}$, $m_t = 172.76\,\text{GeV}$ (pole mass). This is a mixed convention, with the light quark evaluated at 2 GeV and heavier quarks at their own scale. QCD mass ratios are RG invariants at leading order (the anomalous dimension γ_m is flavor-universal), so Q is scheme-independent. However, the seed ratio $\sqrt{m_s}/\sqrt{m_c}$ uses masses at *different* scales: at a common $\overline{\text{MS}}$ scale the ratio shifts to ~ 0.29, worsening the seed match from 1.2% to $\sim 8\%$. This is a significant limitation of the quark seed claim. The meson seed, which uses unambiguous physical (pole) masses, is free of this scheme issue and provides the cleaner test of the seed hypothesis.

A historical note: Harari, Haut, and Weyers [6] observed a quark mass ratio pattern that, in the language of the present paper, corresponds to an approximate seed triple—$(0, d, s)$—at a time when the up quark mass was still thought to be possibly zero. With current masses, $\sqrt{m_d}/\sqrt{m_s} = 0.224$, deviating from $2 - \sqrt{3}$ by 16%—too large for a good Koide triple. But if m_s were $\sim 150\,\text{MeV}$ and $m_d \sim 7\,\text{MeV}$ (values common in that era), the ratio improves considerably. The seed idea was already present, ahead of its time.

3.3 The meson seed: $(0, \pi, D_s)$

The pion and D_s meson charges give:

$$\frac{\sqrt{m_\pi}}{\sqrt{m_{D_s}}} = \frac{11.81}{44.37} = 0.2663\,, \qquad r_{\text{Koide}} = 0.2679\,, \qquad \text{deviation: } 0.6\%\,. \tag{8}$$

This gives $Q = 0.6679$, deviating by 0.18%. It is the **meson counterpart** of the quark seed $(0, s, c)$—both involve the strange-charm sector. Indeed, $D_s = c\bar{s}$ is the meson that directly contains the s and c quarks.

3.4 The lepton triple as an "almost-seed"

The charged lepton triple (e, μ, τ) has $\mathfrak{z}_e = 0.715\,\text{MeV}^{1/2}$, which is small but nonzero. If m_e were zero, we would need $\sqrt{m_\mu}/\sqrt{m_\tau} = 2 - \sqrt{3} = 0.2679$; the actual value is 0.2439, deviating by 9%. So the leptons are *not* an exact seed—the electron, while very light, is not massless. But the triple is clearly the same family: the near-critical Koide phase puts one member close to zero.

3.5 Scale coincidence

The v_0^2 values of the seed triples cluster remarkably:

Triple	v_0^2 (MeV)	v_0 (MeV$^{1/2}$)
(e, μ, τ)	314	17.72
$(0, \pi, D_s)$	351	18.73
$(0, s, c)$	228	15.10

The lepton and meson seed triples share a Koide scale around 310–350 MeV, squarely in the QCD scale range. This is the sBootstrap connection made quantitative: **the lepton triple and the meson seed triple operate at the same scale.**

For the lepton triple, the coincidence is numerically sharp:

$$M_0 \;\approx\; \frac{f_\pi^2}{27}\,, \tag{9}$$

to 0.3%: the right-hand side gives 314.9 MeV versus the fitted $M_0 = 313.8\,\text{MeV}$. Equivalently, $v_0 \approx f_\pi/(3\sqrt{3})$. If this identification is exact and $\delta_0 \bmod 2\pi/3 = 2/9$ (the three-instanton value), all three charged lepton masses follow from f_π alone, each predicted to $\sim$0.3%.

4 A Tale of Two Tuples

4.1 From seed to full triple

The seed triples grow into full triples upon symmetry breaking:

Sector	Seed triple	Full triple	v_0^2 (full)
Quarks	$(0, s, c)$	$(-s, c, b)$	913 MeV
Mesons	$(0, \pi, D_s)$	$(-\pi, D_s, B)$	1230 MeV
Leptons	$(0 \approx e, \mu, \tau)$	(e, μ, τ)	314 MeV

In each case:

- The seed has a massless (or near-massless) member, two massive members in the ratio $2-\sqrt{3}$, and $Q \approx 2/3$.
- The full triple has **no** massless member: the zero has been lifted (to m_e, or the sign-flipped strange/pion mass). The b quark and B meson are always present as flavor degrees of freedom; what the breaking does is give them their *distinctive masses*—the specific values that complete the Koide triple. Before breaking, the b-direction sits at the flat direction ($\mathfrak{z} = 0$); after, it acquires $\mathfrak{z}_b = 64.65\,\mathrm{MeV}^{1/2}$, the value selected by the energy balance with c and the sign-flipped s. The bloom is not particle creation but *mass generation*.

The "tale of two tuples" is this: **each sector presents first as a seed with a massless member, then "blooms" into a full triple when SUSY breaks.** The massless member is the necessary initial condition; the breaking lifts it and generates the full spectrum.

The vacuum-charge doubling (new). The vacuum charge $v_0 = \frac{1}{3}\sum \mathfrak{z}_k$ of the quark seed is $v_0^{(s)} = (\sqrt{m_s}+\sqrt{m_c})/3 = 15.10\,\mathrm{MeV}^{1/2}$. For the full triple, $v_0^{(f)} = (-\sqrt{m_s}+\sqrt{m_c}+\sqrt{m_b})/3 = 30.21\,\mathrm{MeV}^{1/2}$. Their ratio is

$$v_0^{(f)}/v_0^{(s)} = 2.0005 \approx 2\,.$$

If the bloom exactly doubles the vacuum charge, then $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$, giving $m_b = 4{,}177\,\mathrm{MeV}$—matching the PDG value $4{,}180 \pm 30\,\mathrm{MeV}$ to 0.1σ (0.06σ including propagated input uncertainties; see §11.3). This relation is independent of the Koide condition: it constrains m_b from (m_s, m_c) alone, and is more precise (0.07%) than the overlapping-triples prediction (0.5%). A tension exists, however: the exact seed Koide gives $m_c = 1{,}301\,\mathrm{MeV}$ (not the PDG 1,270), and applying the v_0-doubling to this exact seed value gives $m_b = 4{,}233\,\mathrm{MeV}$ (1.8σ from PDG). The v_0-doubling works *with the physical* m_c, not the exact seed value—so these two constraints are mutually inconsistent at the stated precision. The resolution may be that the seed Koide fixes the ratio m_c/m_s only approximately (the 2.4% charm deviation being a higher-order correction to the $gv/m = \sqrt{3}$ condition), while the v_0-doubling constrains m_b directly from the physical masses. In this reading, the seed condition is the approximate one and the v_0-doubling the more fundamental. This hierarchy is not derived; it is imposed by the data.

4.2 Precision summary

Triple	Q	Dev. from 2/3	v_0^2 (MeV)	δ (rad)
		Seed triples		
$(0, \pi, D_s)$	0.6679	0.18%	351	$\approx 3\pi/4$
$(0, s, c)$	0.6644	0.35%	228	$\approx 3\pi/4$
$(0 \approx e, \mu, \tau)$	0.6667	0.001%	314	2.317
		Full triples		
(e, μ, τ)	0.66666	0.001%	314	2.317
$(-\pi, D_s, B)$	0.6674	0.10%	1230	2.806
$(-s, c, b)$	0.6750	1.24%	913	2.744
(c, b, t)	0.6695	0.42%	29 580	2.163

The Koide phases δ cluster in the range 2.2–2.8 rad (126°–161°), all displaced from the seed value $3\pi/4 \approx 2.356$. The lepton triple barely moved from its seed; the quark and meson triples bloomed further. All phases lie within one $\mathbb{Z}_3$ fundamental domain ($[0, 2\pi/3]$ modulo $2\pi/3$).

The lepton triple is unique: seed and full triple are nearly identical, because the electron mass is small enough to serve as the "zero" while being nonzero enough to make Q extremely precise. The $(-s, c, b)$ triple has the worst deviation (1.24%); this is expected because the bloom is a nonperturbative process that does not exactly preserve Q. The v_0-doubling constrains m_b independently (to 0.07% via $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$), so the 1.24% Koide deviation is the residual after the more fundamental v_0-constraint is satisfied.

4.3 Orthogonality of seed and full triples

Each Koide triple defines a perturbation vector $\vec{z} = (z_1, z_2, z_3)$ constrained to the plane $\sum z_k = 0$ with length $|\vec{z}|^2 = 3z_0^2$. The angle between seed and full triples measures how much the breaking rotates the spectrum in charge space:

Pair of triples	Angle
$(0, s, c)$ vs. $(-s, c, b)$	22°
$(0, \pi, D_s)$ vs. $(-\pi, D_s, B)$	26°
(e, μ, τ) vs. $(0, \mu, \tau)$	0.8°
$(0, s, c)$ vs. $(0, \pi, D_s)$	0.3°

Two results are notable. First, the quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ are **nearly parallel** (0.3°): the meson seed is the composite image of the quark seed, with the GOR map preserving direction while changing scale. Second, the lepton triple is essentially its own seed (0.8°)—the bloom has barely begun.

5 The Meson Koide Triple: $(-\pi, D_s, B)$

With the pion charge negative, the triple $(-\pi, D_s, B)$ satisfies:

$$Q(-\pi, D_s, B) = \frac{m_\pi + m_{D_s} + m_B}{(-\sqrt{m_\pi} + \sqrt{m_{D_s}} + \sqrt{m_B})^2} = 0.6674\,, \tag{10}$$

deviating from 2/3 by 0.10%—the second most precise Koide triple known. The energy balance:

$$\frac{\langle z_k^2 \rangle}{z_0^2} = 1.002\,. \tag{11}$$

For comparison, the "standard" all-positive quark triples (d, s, b) and (u, c, t) deviate from $Q = 2/3$ by 9.7% and 27% respectively at $\overline{\text{MS}}$ masses. **The meson triple is more precise than any quark triple**, including (c, b, t) (0.42%).

The pion's negative charge has physical meaning: as a pseudo-Goldstone boson, the pion would be massless ($\mathfrak{z} = 0$) if chiral symmetry were exact. It sits just below zero ($\mathfrak{z}_\pi = -11.81$), approaching zero from the negative side as $m_q \to 0$. The sign of $\mathfrak{z}$ encodes Goldstone nature: pseudo-Goldstone bosons sit on the negative side of the critical point, having crossed through zero from positive territory during chiral symmetry breaking.

Look-elsewhere correction. Eleven pseudoscalar mesons (π, K, η, η', D, D_s, B, B_s, B_c, η_c, η_b) yield $\binom{11}{3} = 165$ triples, times four independent sign patterns, giving 660 combinations. Scanning all 660, only $(-\pi, D_s, B)$ and the closely related $(-\pi, D_s, B_s)$ (deviation 0.14%) lie within 0.2% of $Q = 2/3$. A Monte Carlo with 10^7 random triples drawn from $[100, 10{,}000]$ MeV gives $p_{\text{single}} = 2.1 \times 10^{-4}$ for a match at 0.10%; correcting for 660 trials yields $p_{\text{LEE}} = 0.13$ (1.1σ). The meson Koide triple is therefore *not* statistically significant on its own, unlike the lepton triple (2.8σ after LEE). Its interest lies in the structural pattern: all six closest hits share the sign assignment $(-\pi, D_{(s)}, B_{(s,c)})$, i.e. a negative pion paired with charm-containing and bottom-containing mesons—the combination predicted by the sBootstrap's $SU(5)$ flavor structure.

6 The Third Tuple: (c, b, t)

6.1 A tuple without mesons

The triple (c, b, t) with all positive signs gives $Q = 0.6695$, deviating by 0.42%. Its Koide scale $v_0^2 \approx 29.6$ GeV is far above the QCD scale.

This triple has **no meson counterpart**: the top quark decays weakly ($t \to Wb$, lifetime $\sim 5 \times 10^{-25}$ s) before the strong interaction can bind it ($\tau_{\text{QCD}} \sim 1/\Lambda_{\text{QCD}} \sim 10^{-24}$ s). No T-meson ($t\bar{q}$) exists. The meson matrix $M^i{}_j$ extends only to b-flavored states; the top sector lives purely at the quark level.

6.2 Diquarks and the top

In the sBootstrap, diquarks $[q_i q_j]$ are the scalar partners of antiquarks $\bar{q}_k$, with the baryon acting as the SUSY generator [4,5]. For five active flavors (u, d, s, c, b), the diquarks form a representation of $SU(5)_f$.

The top diquarks $[tq]$ are kinematic objects only—they do not hadronize. Yet their masses should follow the same pattern as other diquarks: **there is no dynamical reason for top diquark masses to differ from the others**. The (c, b, t) Koide triple governs the quark masses that enter the diquark sector, and its energy balance $\langle z_k^2 \rangle = z_0^2$ constrains the possible diquark spectrum even though the top diquarks are never observed as bound states.

6.3 The (c, b, t) triple has no seed

Unlike the $(0, s, c)$ and $(0, \pi, D_s)$ seeds, there is no obvious $(0, x, y)$ seed for the (c, b, t) triple. No pair of quarks in this sector satisfies $\sqrt{m_{\text{light}}}/\sqrt{m_{\text{heavy}}} \approx 2 - \sqrt{3}$: one has $\sqrt{m_c}/\sqrt{m_t} = 0.086$ (too small) and $\sqrt{m_c}/\sqrt{m_b} = 0.551$ (too large).

This means (c, b, t) is *not* a "bloomed seed" in the same sense as $(-s, c, b)$. It may represent a fundamentally different structure—a triple that was never seeded by a massless member but instead emerged fully formed at a higher breaking scale, perhaps connected to electroweak symmetry breaking rather than to chiral symmetry breaking.

7 Mass Relations and Iterative Chains

The preceding sections established the Koide energy-balance condition $Q = 2/3$ and identified the triples that satisfy it. In this section we quantify how special the lepton triple really is, introduce the angular parametrization that reveals a hidden rational structure in the Koide phase, and trace an iterative chain that connects the heavy quarks to the strange sector. We report both successes and failures: the scaling rule that generates the quark triple from the lepton triple works once and then stops, and the iterative chain stalls at a mass too light for any known fermion. These negative results constrain the scope of the pattern.

7.1 The lepton Koide formula: how special?

Using PDG 2024 values $m_e = 0.51100\,\text{MeV}$, $m_\mu = 105.658\,\text{MeV}$, $m_\tau = 1776.86 \pm 0.12\,\text{MeV}$:

$$Q(e,\mu,\tau) = \frac{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2}{m_e + m_\mu + m_\tau} = 1.500014 \pm 0.000015\,, \tag{12}$$

where $Q = (\sum \sqrt{m})^2 / \sum m$ is the reciprocal of the convention in §2: the Koide condition is $Q = 3/2$ here, equivalently $\sum m / (\sum \sqrt{m})^2 = 2/3$ there. We use the 3/2 convention in this section to match the most common form in the Koide literature. The deviation from 3/2 is $|Q - 3/2| = 1.38 \times 10^{-5}$, corresponding to 0.91σ given the uncertainty dominated by δm_τ.

Look-elsewhere effect. The Standard Model contains 9 charged fermions (3 leptons, 6 quarks). An exhaustive scan of all $\binom{9}{3} = 84$ mass triplets yields the ranking in Table 1.

Table 1: Top-ranked SM triplets by proximity to $Q = 3/2$. The gap between rank 1 and rank 2 is a factor of 450.

Rank	Triplet	$\lvert Q - 3/2\rvert$
1	(e,μ,τ)	1.38×10^{-5}
2	(c,b,t)	6.2×10^{-3}

To assess the statistical weight of the lepton result, we performed a Monte Carlo study: 10,000 random mass sets of 9 fermions drawn log-uniform on $[0.1, 200{,}000]\,\text{MeV}$ were scanned for the best triplet. The fraction with $|Q - 3/2|_{\min} \leq 1.38 \times 10^{-5}$ gives a look-elsewhere-corrected significance of $\mathbf{2.8}\boldsymbol{\sigma}$. This is suggestive but not conclusive: the lepton Koide formula is unlikely to be a pure accident, but the evidence does not reach the discovery threshold.

Extended scan with mesons. Enlarging the pool to 30 particles (9 fundamental fermions, W, Z, H, f_π, v_{EW}, and 16 pseudoscalar and vector mesons) with all 2^3 sign combinations per triple yields $\binom{30}{3} = 4060$ unique triples. The lepton triple remains rank 1 ($|Q - 3/2| = 1.4 \times 10^{-5}$, 0.001%), but (c,b,t) drops to rank 113 (0.42%) and $(-s,c,b)$ to rank 324 (1.24%): 112 meson-containing triples are closer to $Q = 3/2$ than (c,b,t). Twenty-one triples fall within 0.1% of 3/2, versus ~ 12 expected from a uniform distribution—a ratio of 1.7, not statistically remarkable. The second-ranked triple $(\tau, v_{\text{EW}}, B_c)$ reaches $Q = 1.50001$ (0.004%), only $3\times$ worse than the leptons, but involves a composite meson and a scale rather than three fundamental masses. *Only (e,μ,τ) involves three independent fundamental parameters; the significance of quark Koide triples is diluted by the composite-meson pool.*

7.2 The Koide angle parametrization

Any triple satisfying $Q = 3/2$ exactly can be written [3]

$$\sqrt{m_k} = \sqrt{M_0}\left(1 + \sqrt{2}\cos\left(\frac{2\pi k}{3} + \delta_0\right)\right), \qquad k = 0, 1, 2\,, \tag{13}$$

where $M_0 > 0$ is the scale parameter and $\delta_0 \in [0, 2\pi)$ is the Koide phase. This is not an approximation: any three non-negative numbers whose Q-ratio equals 3/2 are exactly parametrized by some (M_0, δ_0).

Fitting to the charged lepton masses:

$$M_0 = 313.84\,\mathrm{MeV}\,, \qquad \delta_0 = 2.3166\ \mathrm{rad}\,. \tag{14}$$

The scale $M_0 \approx 314\,\mathrm{MeV}$ is the Koide scale v_0^2 of §3, now identified as the mass parameter of the angular decomposition.

The phase is close to 2/9. Unlike $Q = 2/3$, which is the algebraic consequence of the energy-balance condition (§2), the phase δ_0 is a free parameter that the energy balance does not fix. Its value is an empirical datum, not a derived quantity. The parametrization (13) has a $2\pi/3$ periodicity in δ_0 (permuting the three masses). The physically meaningful quantity is the reduced phase

$$\delta_0 \bmod \frac{2\pi}{3} = 0.22222\ \mathrm{rad}\,. \tag{15}$$

This is strikingly close to the fraction $2/9 = 0.22222\ldots$:

$$\delta_0 \bmod \frac{2\pi}{3} - \frac{2}{9} = +7.4 \times 10^{-6}\,, \tag{16}$$

a residual of 33 ppm relative to 2/9 (using PDG 2024 central masses; within the m_τ uncertainty of 0.12 MeV, the residual vanishes at $m_\tau \approx 1{,}776.97\,\mathrm{MeV}$, i.e. 0.9σ above central).

Statistical significance of the rational approximation. There are 266 reduced fractions p/q with $q \leq 20$ and $0 < p/q < 2\pi/3$. The probability that a random phase (uniform on $[0, 2\pi/3)$) falls within 7.4×10^{-6} of *any* of these 266 fractions gives a look-elsewhere-corrected significance of $\mathbf{3.1}\boldsymbol{\sigma}$. Evaluated specifically against the fraction 2/9 alone (i.e., without look-elsewhere correction), the significance is $\mathbf{4.5}\boldsymbol{\sigma}$. The deviation from 2/9 is 0.9σ in the m_τ uncertainty, consistent with an exact match.

7.3 The scaling rule: $M_1 = 3M_0$, $\delta_1 = 3\delta_0$

A natural question is whether other Koide triples can be obtained from the lepton parameters by a simple scaling. The ansatz

$$M_1 = 3\,M_0 = 941.5\,\mathrm{MeV}\,, \qquad \delta_1 = 3\,\delta_0 = 6.9497\ \mathrm{rad}\,, \tag{17}$$

inserted into (13), predicts a quark triple. Table 2 compares predictions to PDG values.

The agreement for s and b is at the percent level; charm shows a larger (7.1%) discrepancy. For the physical masses, $M_0(-s, c, b)/M_0(e, \mu, \tau) = 2.91$, not exactly 3—the $\times 3$ rule

Table 2: Masses predicted by the $\times 3$ scaling rule vs. PDG central values. The quark Koide triple uses a negative sign for $\sqrt{m_s}$, as in §4.

Quark	Predicted (MeV)	PDG (MeV)	Deviation
s	92.3	93.4	1.2%
c	1360	1270	7.1%
b	4197	4180	0.4%

is approximate. By contrast, the vacuum-charge doubling (§4) predicts m_b to 0.1σ. The predicted triple satisfies the Koide condition with a negative square root for the strange quark:

$$Q(-\sqrt{m_s},\ \sqrt{m_c},\ \sqrt{m_b}) = \frac{(-\sqrt{m_s}+\sqrt{m_c}+\sqrt{m_b})^2}{m_s+m_c+m_b} = \frac{3}{2}\,, \tag{18}$$

holding to machine precision (residual 2.2×10^{-16}), since the masses are generated from the parametrization (13) which enforces $Q = 3/2$ identically.

Renormalization caveat. The PDG quark masses used in the comparison are quoted at different scales: $m_s(2\,\mathrm{GeV})$, $m_c(m_c)$, $m_b(m_b)$, and m_t as the pole mass. The predicted masses emerge at a single internal scale set by M_1. Running all masses to a common $\overline{\mathrm{MS}}$ scale would change the individual deviations in Table 2 but not the structural point: the $\times 3$ scaling produces a recognizable quark triple. As noted in §11.10, QCD mass *ratios* are RG invariants at leading order, so the Koide condition itself is scheme-independent, but the comparison of absolute masses to PDG values carries an $O(\alpha_s)$ ambiguity.

7.4 The second scaling fails

Iterating the scaling rule:

$$M_2 = 9\,M_0 = 2825\,\mathrm{MeV}\,, \qquad \delta_2 = 9\,\delta_0 = 20.849\ \mathrm{rad}\,, \tag{19}$$

gives predicted masses (478, 92.3, 16,377) MeV. Only the middle value echoes m_s. The lightest (478 MeV) does not match any known quark mass, and the heaviest (16.4 GeV) falls between m_b and m_t with no candidate. No SM triplet accommodates these values.

The scaling stops at one step. This is a negative result that constrains any attempt to build a recursive tower of Koide triples: the multiplicative rule $M \to 3M$, $\delta \to 3\delta$ is not a general principle but a one-off relation between the lepton and quark sectors. Whether this single success is accidental or reflects a deeper connection (perhaps related to the three colors of QCD, or to the three generations) remains open.

7.5 Iterative chain: descending from t and b

An alternative approach to connecting the quark masses is purely algebraic: given two known masses, solve the Koide equation $Q = 3/2$ for the third. Starting from the top and bottom quarks:

Step 1: $Q(m_b, m_?, m_t) = 3/2$**.** The Koide equation with (m_b, m_t) known is quadratic in $\sqrt{m_?}$, yielding four branches (two signs for $\sqrt{m_?}$, two roots of each quadratic). The solutions are:

$$m_? = \{1357,\ 60{,}280,\ 1.34 \times 10^6,\ 3.55 \times 10^6\}\,\text{MeV}\,. \tag{20}$$

Only the first branch gives a physically viable mass: $m_? = 1357\,\text{MeV}$, which we identify with the charm quark (PDG: $m_c(m_c) = 1270\,\text{MeV}$, deviation 6.8%).

Step 2: $Q(m_c, m_?, m_b) = 3/2$**.** With $m_c = 1357\,\text{MeV}$ (the chain value, not the PDG value) and $m_b = 4180\,\text{MeV}$:

$$m_? = 92.17\,\text{MeV}\,, \tag{21}$$

which we identify with the strange quark. The negative square root emerges naturally from the algebra—the solution selects the branch with $\sqrt{m_?} < 0$ in the signed-charge framework, consistent with the $(-s, c, b)$ structure of §4.

Step 3: the chain stalls. Continuing from (m_s, m_c) with $m_s = 92.17\,\text{MeV}$:

$$m_? = 0.035\,\text{MeV}\,. \tag{22}$$

This is far too light for either u (2.16 MeV) or d (4.67 MeV). The chain does not reach the first-generation quarks; it stalls at a mass two orders of magnitude below m_u.

Step 4: cyclic echo. The stalled mass provides an algebraic consistency check. Taking $m_{\text{stall}} = 0.035\,\text{MeV}$ and $m_s = 92.17\,\text{MeV}$, and solving $Q(m_{\text{stall}}, m_?, m_s) = 3/2$:

$$m_? = 1357\,\text{MeV} \quad (\text{to machine epsilon})\,. \tag{23}$$

The chain returns to its starting point: $t \to c \to s \to \text{stall} \to c \to \cdots$. This is not a numerical coincidence but an algebraic necessity: the Koide condition, combined with the specific masses of b and t, defines a closed orbit in (m_1, m_2, m_3) space. The cycle is:

$$\begin{aligned} \cdots \to\ (b,t) \xrightarrow{Q=3/2} c \ \to\ (c,b) \xrightarrow{Q=3/2} s \\ \to\ (s,c) \xrightarrow{Q=3/2} \text{stall} \ \to\ (\text{stall}, s) \xrightarrow{Q=3/2} c \ \to\ \cdots \end{aligned}$$

The chain captures c, b, s but misses u, d, and the leptons entirely. The first generation is outside the algebraic orbit seeded by the third generation.

7.6 Cross-checks and master comparison

Two independent routes lead to the quark masses: the lepton-seeded scaling rule (§7.3) and the top-bottom iterative chain (§7.5). Table 3 compares all predictions.

The convergence of the two chains on $m_s \approx 92\,\text{MeV}$ and $m_c \approx 1357$–$1360\,\text{MeV}$ is the central positive result. Both routes, starting from completely different inputs (lepton masses vs. heavy quark masses), produce the same strange and charm masses to within 0.2% of each other. Against PDG values, the strange quark prediction is excellent ($\sim 1\%$), and the bottom quark prediction is even better (0.4%); the charm prediction shows a persistent $\sim 7\%$ overshoot.

The negative results are equally informative:

Table 3: Master comparison of predicted quark masses (MeV). Chain A descends from (t, b) via the Koide equation. Chain B applies the $\times 3$ scaling to the lepton triple. The small discrepancy between the two (1357 vs. 1360 MeV for charm) arises from different inputs: Chain A uses the PDG m_t and m_b directly, while Chain B uses the fitted (M_0, δ_0) from leptons.

Quark	Chain A (from t, b)	Chain B (from leptons)	PDG	Status
s	92.2	92.3	93.4	1–1.2% deviation
c	1357	1360	1270	7–7.1% deviation
b	(input)	4197	4180	0.4% deviation
t	(input)	—	172 760	—
stall	0.035	—	—	no SM match
M_2 set	—	(478, 92, 16377)	—	no SM match

- The $\times 3$ scaling does not iterate: the $\times 9$ step produces no viable triple.
- The iterative chain does not reach the first generation: it stalls at 0.035 MeV, two orders of magnitude below m_u.
- The chain is algebraically cyclic: it loops through (c, s, stall) indefinitely, never escaping to new masses.

These failures delineate the boundary of the Koide pattern: it connects the second and third quark generations (and, via the scaling rule, the leptons), but does not extend to the first generation or beyond the Standard Model spectrum. Whatever dynamics underlies the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ operates within a limited domain.

8 Composite Scalar Counting and Generation Uniqueness

The sBootstrap rests on a single postulate: every scalar of the Supersymmetric Standard Model is a composite of two quarks (diquark, color triplet) or of a quark–antiquark pair (meson, color singlet), with the electric charge given by the sum of the constituents' charges. We now show that matching the number of such composites to the fermionic degrees of freedom of N generations uniquely forces $N = 3$.

8.1 Setup and Diophantine system

Let there be N generations, each containing quarks of two types: *down* (charge $-1/3$) and *up* (charge $+2/3$). Not every quark need participate as a string endpoint; we allow r light down-type quarks and s light up-type quarks that form composites. The Standard Model with right-handed neutrinos has $4N$ real (Weyl) fermionic degrees of freedom per charge sector in each color channel. Matching composites to fermions, charge sector by charge sector, yields the following system.

1. **Charge $+1/3$ composites** (one up $\times$ one down, analogous to charged sleptons): rs complex scalars must match $2N$ real fermionic d.o.f., giving

$$rs = 2N\,. \tag{24}$$

2. **Charge $-2/3$ composites** (symmetric down–down pairs, color sextet): $\binom{r+1}{2} = r(r+1)/2$ complex scalars must match $2N$ real fermionic d.o.f., giving

$$\frac{r(r+1)}{2} = 2N\,. \tag{25}$$

3. **Neutral composites** (classified by $SU(r+s)$, minus the overall singlet trace; includes the partner of the right-handed neutrino): $(r+s)^2 - 1 = r^2 + s^2 + 2rs - 1$ states, of which $2rs$ are charged and already counted. The neutral count is therefore $r^2 + s^2 - 1$, to be matched to $4N$ real d.o.f., giving

$$r^2 + s^2 - 1 = 4N\,. \tag{26}$$

The alternative neutral equation $r^2 + s^2 - 1 = 2N$ (left-handed neutrinos only) produces no positive-integer solution when combined with (24)–(25); thus the sBootstrap *requires* right-handed neutrinos.

8.2 The uniqueness theorem

Theorem 1 (Generation uniqueness). *The system* (24)–(26) *has a unique solution over the positive integers:*

$$\boxed{(N,\, r,\, s) = (3,\, 3,\, 2)\,.} \tag{27}$$

Proof. Equating the right-hand sides of (24) and (25) gives $rs = r(r+1)/2$, hence

$$s = \frac{r+1}{2}\,, \tag{28}$$

which requires r odd. Substituting (24) and (28) into (26):

$$r^2 + \Big(\frac{r+1}{2}\Big)^2 - 1 \;=\; 4 \cdot \frac{r\,(r+1)/2}{2} \;=\; r(r+1)\,.$$

Expanding the left side,

$$r^2 + \frac{r^2+2r+1}{4} - 1 = r^2 + r\,,$$

and clearing the denominator:

$$4r^2 + r^2 + 2r + 1 - 4 = 4r^2 + 4r \quad\Longrightarrow\quad r^2 - 2r - 3 = 0\,,$$

i.e. $(r-3)(r+1) = 0$. Since $r > 0$, we obtain $r = 3$, whence $s = 2$ and $N = rs/2 = 3$. □

The solution has $r+s = 5$ light quarks acting as string endpoints, furnishing the fundamental representation of an $SU(5)$ flavor group. The top quark is the unique "elephant"—too heavy to serve as a constituent of the scalar composites—in precise agreement with the Standard Model, where $m_t \gg \Lambda_{\rm QCD}$ prevents top-flavored hadrons from forming.

8.3 Partial solutions and the hexagonal-number family

Equations (24) and (25) alone, without the neutral constraint, require $2N$ to be a hexagonal number $h_k = k(2k-1)$:

$$2N = 1,\ 6,\ 15,\ 28,\ 45,\ 66,\ \ldots$$

Of these, only $2N = 1$ and $2N = 6$ have $r \le 16$ (the asymptotic-freedom bound $2n_f < 33$ limits $r + s \le 16$). The first is inadmissible ($N = 1/2$); the second gives $N = 3$.

Remark (The next hexagonal solution). The case $(N, r, s) = (14, 7, 4)$ with $r + s = 11$ light flavors and $2N = 28$ total satisfies (24)–(25) but fails (26): $r^2 + s^2 - 1 = 64 \ne 4 \times 14 = 56$. Asymptotic freedom ($b_0 > 0$ in the QCD beta function) would also require $2n_f < 33$, i.e. $n_f \le 16$; with $N = 14$ generations one would need $2N = 28$ flavors, which violates the bound decisively. The neutral equation and the asymptotic-freedom bound independently single out $N = 3$.

8.4 Identification with $SU(5)$ representations

With $r = 3$ and $s = 2$, the $r + s = 5$ light quarks form the fundamental $\mathbf{5}$ of $SU(5)_f$. The composites organize into three $SU(5)$ representations:

Mesons (quark–antiquark, color singlet): The $\mathbf{5} \otimes \bar{\mathbf{5}}$ decomposes as

$$\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} \oplus \mathbf{1}\,. \tag{29}$$

The $\mathbf{24}$ carries the sleptons (charged and neutral) and the meson matrix. Under $SU(5) \supset SU(3)_d \times SU(2)_u$:

$$\mathbf{24} = (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{2}) + (\mathbf{8}, \mathbf{1})\,. \tag{30}$$

Diquarks (quark–quark, color triplet): The symmetric product

$$\mathbf{5} \otimes_S \mathbf{5} = \mathbf{15} \tag{31}$$

decomposes as

$$\mathbf{15} = (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{2}) + (\mathbf{6}, \mathbf{1})\,, \tag{32}$$

and its conjugate $\overline{\mathbf{15}}$ provides the antidiquarks.

Proposition 1 (Symmetric product is forced). *The counting equation* (25) *selects the symmetric product* $\mathbf{15}$*, not the antisymmetric* $\overline{\mathbf{10}}$.

Proof. The charge $-2/3$ composites are down–down pairs. Under $SU(3)_d$, the symmetric product $\mathbf{3} \otimes_S \mathbf{3} = \mathbf{6}$ has dimension $r(r+1)/2 = 6$, matching equation (25). The antisymmetric product $\mathbf{3} \otimes_A \mathbf{3} = \bar{\mathbf{3}}$ has dimension $r(r-1)/2 = 3 \ne 2N = 6$. The symmetric color sextet, and hence the $\mathbf{15}$, is the unique choice consistent with the sBootstrap counting. □

	Number of states at $\lvert Q_{\text{em}}\rvert$						
	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	2	Dim
$\mathbf{24}$	10	2×3	—	2×1	2×3	—	24
$\mathbf{15}$	1	3	9	1	—	1	15
$\overline{\mathbf{15}}$	1	3	9	1	—	1	15
Total	12	2×6	2×9	2×2	2×3	2×1	54

Table 4: Charge census of $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$ of $SU(5)$. Notation "$2{\times}k$" means k states at $+\lvert Q_{\text{em}}\rvert$ and k at $-\lvert Q_{\text{em}}\rvert$; the $\mathbf{24}$ is self-conjugate. In the $\mathbf{15}$, the 9 states at $\lvert Q_{\text{em}}\rvert = 2/3$ split as 3 from the $(\mathbf{3},\mathbf{2})_{1/6}$ component at $Q_{\text{em}} = +2/3$ and 6 from the $(\mathbf{6},\mathbf{1})_{-2/3}$ color sextet at $Q_{\text{em}} = -2/3$; the $\overline{\mathbf{15}}$ is the conjugate. Charges $\pm 4/3$ in the $\mathbf{24}$ arise from the leptoquark components $(\mathbf{3},\mathbf{2})_{-5/6}$ and $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$. The 12 neutral states comprise $8+1+1 = 10$ from the $\mathbf{24}$ plus one each from $\mathbf{15}$ and $\overline{\mathbf{15}}$.

8.5 Charge census of $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$

Treating the $SU(5)$ simultaneously as a Georgi–Glashow unification group [1], one assigns standard electric charges via $Q_{\text{em}} = T_3 - Y/6$ to every component. Table 4 displays the resulting charge census for $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$.
Three cross-checks tie the census to the Diophantine system:

- **Neutral composites:** The 12 states at $Q_{\text{em}} = 0$ match $r^2 + s^2 - 1 = 9 + 4 - 1 = 12 = 4N$.
- **Charge $+1/3$ composites** (one up × one down): $rs = 3 \times 2 = 6$ composites, matching equation (24) with $rs = 6 = 2N$. These states distribute as 3 from $(\bar{\mathbf{3}},\mathbf{2})_{5/6} \subset \mathbf{24}$ and 3 from $\overline{\mathbf{15}}$.
- **Charge $-2/3$ composites** (symmetric down–down): $r(r+1)/2 = 6$ symmetric pairs fill the color sextet $(\mathbf{6},\mathbf{1})_{-2/3} \subset \mathbf{15}$, matching equation (25) with $r(r+1)/2 = 6 = 2N$.

8.6 Anomaly cancellation and asymptotic freedom

The representation $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$ must be anomaly-free if it is to appear in a consistent gauge theory.

Proposition 2 (Anomaly cancellation). *The cubic $SU(5)$ anomaly of $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$ vanishes.*

Proof. The anomaly coefficient $A(\mathbf{R})$ for $SU(n)$ satisfies $A(\text{adjoint}) = 0$ (the adjoint is real), $A(\mathbf{15}) = n + 4 = 9$, and $A(\overline{\mathbf{15}}) = -(n+4) = -9$. Therefore

$$A(\mathbf{24}) + A(\mathbf{15}) + A(\overline{\mathbf{15}}) = 0 + 9 + (-9) = 0\,.$$

□

Proposition 3 (Asymptotic freedom). *An $SU(5)$ gauge theory with matter in the $\mathbf{24}\oplus\mathbf{15}\oplus\overline{\mathbf{15}}$ is asymptotically free.*

Proof. The one-loop beta-function coefficient is

$$b_0 = \frac{11}{3}\,C_2(G) - \frac{2}{3}\sum_{\text{Weyl}} T(\mathbf{R})\,,$$

where $C_2(G) = n = 5$ for $SU(5)$, $T(\mathbf{adj}) = n = 5$, and $T(\mathbf{15}) = (n+2)/2 = 7/2$. The Weyl fermion content consists of one adjoint and one $\mathbf{15} + \overline{\mathbf{15}}$ (treating each as a Weyl fermion), giving $\sum T = 5 + 7/2 + 7/2 = 12$. Therefore

$$b_0 = \frac{11}{3} \times 5 - \frac{2}{3} \times 12 = \frac{55-24}{3} = \frac{31}{3} > 0\,.$$

□

Both properties are nontrivial: anomaly cancellation ensures the gauge symmetry is quantum-mechanically consistent, and $b_0 > 0$ ensures the theory is well-defined in the ultraviolet. Together they confirm that the representation forced by the sBootstrap counting is physically viable.

8.7 Summary of the counting argument

The logical chain is:

1. The sBootstrap postulate (composites = scalars of the SSM) yields three Diophantine equations in (N, r, s).
2. The unique positive-integer solution is $(3, 3, 2)$: three generations, with the symmetric $\mathbf{15}$ forced over the antisymmetric $\overline{\mathbf{10}}$.
3. The solution organizes into $SU(5)$ representations $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$, which are anomaly-free and asymptotically free.
4. Right-handed neutrinos are required: without them, no integer solution exists.
5. The top quark is excluded from the composite sector by the counting itself ($s = 2 < N = 3$), matching its kinematic decoupling ($m_t \gg \Lambda_{\mathrm{QCD}}$).

9 The Diquark Sector and SU(5) Flavor

9.1 Counting partners

In the full sBootstrap [1], the scalar partners of the Standard Model fermions include both mesons ($q\bar{q}$) and diquarks (qq). For five active flavors, the meson matrix gives $5 \times 5 = 25$ scalars. The diquarks $[q_i q_j]$ form a representation of $SU(5)_f$:

$$\mathbf{5} \otimes \mathbf{5} = \overline{\mathbf{10}}_A \oplus \mathbf{15}_S\,. \tag{33}$$

The symmetric $\mathbf{15}_S$ gives 15 diquarks including same-flavor states $[uu]$, $[dd]$, $\ldots$, while the antisymmetric $\overline{\mathbf{10}}_A$ gives 10 diquarks with $i \neq j$ only.

9.2 The symmetric representation

The sBootstrap SUSY generator acts asymmetrically on the two fermion sectors. When it acts on a *lepton*, it produces a meson: $\mathcal{Q} \cdot \ell \sim q\bar{q}$. Since the quark and antiquark are distinguishable, the meson lies in the full $\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} + \mathbf{1}$. There is no Pauli constraint, and the meson spectrum is perfectly reproduced.

When it acts on a *quark*, it produces a diquark: $\mathcal{Q} \cdot q_i \sim q_i q_j$. The bosonic SUSY generator applied to a single flavor index i naturally produces the **symmetric** product—the $\mathbf{15}_S$ representation, not the antisymmetric $\overline{\mathbf{10}}$. This is the representation identified in the $SO(32)$ decomposition [1]: the adjoint of $SO(32)$ under $SU(5)_f$ decomposes to include the $\mathbf{15} + \overline{\mathbf{15}}$ (diquarks) alongside the $\mathbf{1} + \mathbf{24}$ (mesons).

The 15 includes 5 same-flavor states ($[uu]$, $[dd]$, $[ss]$, $[cc]$, $[bb]$) absent from the $\overline{\mathbf{10}}$. Among these are exotic $+4/3$-charged diquarks such as $[uu]$. The cost: an additional chiral $+4/3$ quark per generation, as identified in [1]. Whether these exotic states appear in nature as resonances, or whether they decouple at some high scale, is an open question.

9.3 The Pauli theorem for scalar diquarks

The requirement of the symmetric $\mathbf{15}$ conflicts with Fermi statistics—not merely for $L = 0$ diquarks, but for *all* scalar diquarks in the color $\bar{3}$ channel, regardless of internal orbital structure.

Theorem. *For a spin-0 ($J = 0$) diquark in the color $\bar{3}$ (antisymmetric) representation, the flavor wave function is antisymmetric for* ***all*** *values of the orbital angular momentum L and spin S.*

Proof. $J = 0$ requires $L = S$ (since $|L - S| \leq J \leq L + S$). The exchange symmetry of the spin-orbital wave function is $(-1)^{L+S+1} = (-1)^{2L+1} = -1$ for all L. Combined with the color $\bar{3}$ factor (-1), the non-flavor product is $(-1)(-1) = +1$ (symmetric). Total antisymmetry then requires flavor to be antisymmetric: $\overline{\mathbf{10}}$. □

This result is purely kinematic—independent of dynamics, binding mechanisms, or spatial extent. No wave function, however extended, can produce a symmetric-flavor scalar diquark in color $\bar{3}$ while respecting Fermi statistics. The key cases:

L	S	Spin-orbital $(-1)^{2L+1}$	× Color	Flavor
0	0	-1	$\times(-1) = +1$	must be A
1	1	-1	$\times(-1) = +1$	must be A
2	2	-1	$\times(-1) = +1$	must be A

The $L = 1$, $S = 1$ row closes the "P-wave diquark" escape route that might otherwise seem available.

The theorem has a sharp consequence: **the sBootstrap diquark in the 15 cannot be a qq composite.** It must be either:

1. An algebraically generated object—the image of the Miyazawa supercharge [4, 5] acting on an antiquark, whose symmetry is determined by the superalgebra rather than by the statistics of constituent quarks; or

2. A fundamental string-theoretic object, as in the $SO(32)$ decomposition [1], where the $\mathbf{15}$ arises from the adjoint representation of the gauge group and the open-string diquark is not built from point-like quarks.

The contrast with the meson sector is instructive. When the SUSY generator acts on a lepton, it produces a meson ($q\bar{q}$): the quark and antiquark are distinguishable, no Pauli constraint applies, and the meson spectrum is reproduced without tension. When it acts on a quark, it produces a diquark in the "wrong" representation—and the theorem shows this cannot be fixed by any internal structure of a qq pair. The asymmetry between the two SUSY directions is absolute: mesons work as composites; diquarks do not. This is an unresolved obstruction: the sBootstrap requires the symmetric **15**, but QCD composite diquarks cannot fill it. The proposed resolutions—Miyazawa supercharges and SO(32) string embeddings—are directions for future work, not established mechanisms. The diquark sector remains the weakest point of the program.

10 Two Algebraic Structures

The sBootstrap particle content and mass spectrum are constrained by two *independent* algebraic structures. Part A (§10.1–10.3) derives the representation content from an $SO(32)$ embedding. Part B (§10.4–10.6) derives an electroweak mass ratio from a Casimir eigenvalue equation. No algebraic connection between the two has been found (see §10.7).

10.1 Part A: The SO(32) embedding chain

The $SO(32)$ gauge group of Type I / heterotic string theory admits the maximal-subgroup chain

$$SO(32) \supset SU(16) \times U(1)_A \supset SU(5) \times SU(3) \times U(1)_A \times U(1)_B \,. \tag{34}$$

The first step is the standard embedding $SO(2n) \supset SU(n) \times U(1)$; the second decomposes $SU(16)$ under the regular subalgebra $SU(5) \times SU(3) \times U(1)_B$. Here $SU(5)$ is the Georgi–Glashow unification group and $SU(3)$ is a *horizontal* (flavor) symmetry acting on the three light generations—it is **not** $SU(3)_c$.

The fundamental **32** of $SO(32)$ decomposes as

$$\mathbf{32} = \underbrace{(\mathbf{5},\mathbf{3})}_{R_1} \oplus \underbrace{(\bar{\mathbf{5}},\bar{\mathbf{3}})}_{R_2} \oplus \underbrace{(\mathbf{1},\mathbf{1})}_{R_3} \oplus \underbrace{(\mathbf{1},\mathbf{1})}_{R_4}, \tag{35}$$

where $\dim = 15 + 15 + 1 + 1 = 32$ and the four summands carry distinct $U(1)_A \times U(1)_B$ charges.

10.2 Decomposition of the adjoint

The adjoint of $SO(32)$ is the antisymmetric square of the fundamental: $\mathbf{496} = \wedge^2(\mathbf{32})$. Distributing over the four summands in (35) and using the identity

$$\wedge^2(\mathbf{5},\mathbf{3}) = \big[\wedge^2(\mathbf{5}) \otimes S^2(\mathbf{3})\big] \oplus \big[S^2(\mathbf{5}) \otimes \wedge^2(\mathbf{3})\big] = (\mathbf{10},\mathbf{6}) \oplus (\mathbf{15},\bar{\mathbf{3}}) \,, \tag{36}$$

one obtains the full decomposition listed in Table 5.

10.3 Physical content and the projection problem

Restricting to the adjoint sector $Q_A = Q_B = 0$ and dropping the horizontal $SU(3)$ factor, one reads off the $SU(5)$ content:

$$\mathbf{24} \oplus \mathbf{1} \,, \tag{37}$$

Table 5: Decomposition of the $SO(32)$ adjoint **496** under $SU(5)\times SU(3)\times U(1)_A\times U(1)_B$.

$SU(5)\times SU(3)$ rep	Dim	Q_A	Q_B	Origin
Adjoint sector ($Q_A = Q_B = 0$, 226 states):				
$(\mathbf{24}, \mathbf{8})$	192	0	0	$R_1 \otimes R_2$
$(\mathbf{24}, \mathbf{1})$	24	0	0	$R_1 \otimes R_2$
$(\mathbf{1}, \mathbf{8})$	8	0	0	$R_1 \otimes R_2$
$(\mathbf{1}, \mathbf{1})$	1	0	0	$R_1 \otimes R_2$
$(\mathbf{1}, \mathbf{1})$	1	0	0	$R_3 \otimes R_4$
Matter sector (Q_A, $Q_B \neq 0$, 270 states):				
$(\mathbf{10}, \mathbf{6})$	60	+2	+2	$\wedge^2(R_1)$
$(\overline{\mathbf{10}}, \bar{\mathbf{6}})$	60	−2	−2	$\wedge^2(R_2)$
$(\mathbf{15}, \bar{\mathbf{3}})$	45	+2	+2	$\wedge^2(R_1)$
$(\overline{\mathbf{15}}, \mathbf{3})$	45	−2	−2	$\wedge^2(R_2)$
$(\mathbf{5}, \mathbf{3})_+$	15	+2	−14	$R_1 \otimes R_3$
$(\mathbf{5}, \mathbf{3})_-$	15	0	+16	$R_1 \otimes R_4$
$(\bar{\mathbf{5}}, \bar{\mathbf{3}})_+$	15	−2	+14	$R_2 \otimes R_3$
$(\bar{\mathbf{5}}, \bar{\mathbf{3}})_-$	15	0	−16	$R_2 \otimes R_4$
Total	**496**			

which is precisely the meson/slepton content of the sBootstrap. The matter sector at $|Q_A| = 2$ contains

$$\mathbf{15} \oplus \overline{\mathbf{15}} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{5} \oplus \bar{\mathbf{5}}\,, \tag{38}$$

among which the $\mathbf{15}+\overline{\mathbf{15}}$ are the symmetric diquarks demanded by the sBootstrap counting (§9.2).

Thus the sBootstrap representations appear in the $SO(32)$ adjoint: the **24** in the neutral sector ($Q_A = Q_B = 0$) and the $\mathbf{15} \oplus \overline{\mathbf{15}}$ in the matter sector ($|Q_A| = 2$). However, the remaining $496 - 54 = 442$ states (the $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{5}$, $\bar{\mathbf{5}}$, the SU(3)-horizontal copies, and the neutral singlets) must be projected out or made massive. The mechanism for this projection—whether by an orbifold, a GSO-type condition, or a flux background—is an open problem.

This embedding was first noted in [1]. Its role here is to provide a UV origin for the symmetric **15** that the Pauli theorem (§9.3) forbids at the level of point-particle composites: in the string picture, diquarks are open-string states whose symmetry is determined by the gauge group, not by Fermi statistics of constituents.

Why $SO(32)$ and not $E_8 \times E_8$. Anomaly cancellation in ten dimensions selects two gauge groups: $SO(32)$ and $E_8 \times E_8$. The latter cannot accommodate the sBootstrap. Every decomposition of E_8 under $SU(5)$ produces the antisymmetric **10**, never the symmetric **15**: the maximal-subgroup chain $E_8 \supset SU(5) \times SU(5)$ gives $\mathbf{248} = (\mathbf{24},\mathbf{1}) + (\mathbf{1},\mathbf{24}) + (\mathbf{10},\bar{\mathbf{5}}) + (\overline{\mathbf{10}},\mathbf{5}) + (\mathbf{5},\mathbf{10}) + (\bar{\mathbf{5}},\overline{\mathbf{10}})$, with no **15** appearing anywhere. The obstruction is irreducible: it reflects the fact that E_8's simply-laced root lattice admits only antisymmetric tensor representations of $SU(5)$. By contrast, $SO(32)$'s adjoint $\mathbf{496} = \wedge^2(\mathbf{32})$ naturally contains the symmetric **15** through the identity $\wedge^2(\mathbf{5} \otimes \mathbf{3}) \supset S^2(\mathbf{5}) \otimes \wedge^2(\mathbf{3}) = (\mathbf{15}, \bar{\mathbf{3}})$. The Type I realization, where quark Chan-Paton factors directly generate $SO(32)$ from $5\times 3 + 1 = 16$ endpoint labels, is the most natural home.

The $E_8 \times E_8$ heterotic string is connected to Type I $SO(32)$ by a chain of dualities (S-duality to heterotic $SO(32)$, then T-duality on a non-simply-connected torus). These dualities may allow the sBootstrap content to appear in the $E_8 \times E_8$ frame through a non-standard embedding, but the direct decomposition route is blocked by the **15** vs. **10** obstruction.

10.4 Part B: The Casimir eigenvalue equation

Consider the eigenvalue equation

$$x^2 \,+\, C_2\, x \,-\, C_2 = 0\,, \qquad C_2 = s(s+1)\,, \tag{39}$$

where C_2 is the quadratic Casimir of $SU(2)$ at spin s and $x = M^2/m^2$ for a mass scale m to be determined. Equivalently, $x^2/(1-x) = C_2$: the eigenvalue is a self-consistent solution where the "squared coupling" x^2 equals C_2 times the complement $(1-x)$. The solutions are

$$x_\pm(s) = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_2}}{2}\,. \tag{40}$$

Structural constraints (codimension analysis). The most general two-parameter family of such equations is

$$x^2 + (f_0 + f_1 C_2)\, x + (g_0 + g_1 C_2) = 0\,, \tag{41}$$

with four real coefficients (f_0, f_1, g_0, g_1). Three structural conditions pin the polynomial to (39):

1. $f_0 = 0$: no spin-independent mass term;
2. $g_0 = 0$: a massless eigenstate exists at $s = 0$;
3. $g_1 = -f_1$: coefficient antisymmetry (the constant and linear terms in C_2 are related).

Together with the normalization $f_1 = 1$, these exhaust the freedom. A systematic scan of 582 polynomial alternatives (quadratics, cubics, quartics with Casimir-type functions and rational coefficients) confirms uniqueness: no other equation of this class reproduces $\sin^2\theta_W$ to sub-percent accuracy at $(s_1, s_2) = (1/2, 1)$. The ratio derived below is therefore a *consequence* of these constraints, not a free parameter.

10.5 The electroweak ratio

Define the ratio

$$R \;\equiv\; 1 - \frac{x_+(1/2)}{x_+(1)}\,. \tag{42}$$

Evaluating (40) at $s = 1/2$ ($C_2 = 3/4$) and $s = 1$ ($C_2 = 2$):

$$x_+(1/2) = \tfrac{1}{8}\left(-3 + \sqrt{57}\right), \quad \text{so } x_+(1/2)\, m^2 = M_W^2\,, \tag{43}$$

$$x_+(1) = -1 + \sqrt{3}\,, \quad \text{so } x_+(1)\, m^2 = M_Z^2\,. \tag{44}$$

The identification with W and Z is motivated by their $SU(2)$ quantum numbers ($s = 1/2$ maps to the doublet structure of W, $s = 1$ to the triplet). Then

$$R = 1 - \frac{x_+(1/2)}{x_+(1)} = \frac{(\sqrt{19}-3)(\sqrt{19}-\sqrt{3}\,)}{16} \approx 0.2231013\ldots \tag{45}$$

The factored form follows from the discriminants of (39): $57 = 3 \times 19$ at $s = 1/2$ and $12 = 4 \times 3$ at $s = 1$. Rationalizing the denominator $\sqrt{3} - 1$ yields an expression in $\sqrt{19}$ and $\sqrt{3}$ alone, which coincides term-by-term with the form found empirically by de Vries.

This should be compared with the on-shell electroweak mixing parameter:

$$\sin^2\theta_W^{\text{os}} \;\equiv\; 1 - \frac{M_W^2}{M_Z^2} = 0.22320 \pm 0.00026\,, \tag{46}$$

using $M_Z = 91.1876 \pm 0.0021\,\text{GeV}$ and $M_W = 80.369 \pm 0.013\,\text{GeV}$ (PDG 2024).

Since $R = 1 - x_+(1/2)/x_+(1)$ and $\sin^2\theta_W = 1 - M_W^2/M_Z^2$, the two expressions are *algebraically identical* once one identifies $x_+(s) = M^2(s)/m^2$. The prediction $R \approx \sin^2\theta_W$ and the W-mass prediction derived below are therefore **one and the same observation**, not two independent tests.

W-mass prediction. From $M_W = M_Z\sqrt{1-R}$:

$$M_W^{\text{pred}} = 91.1876 \times \sqrt{1 - 0.22310} = 80.374 \pm 0.002\,\text{GeV}\,, \tag{47}$$

which lies 0.39σ from the PDG average $M_W = 80.369 \pm 0.013\,\text{GeV}$ and strongly disfavors the anomalous CDF-II measurement $M_W^{\text{CDF}} = 80.4335 \pm 0.0094\,\text{GeV}$ (6.2σ tension with R).

Scale parameter. Setting $x_+(1)\,m^2 = M_Z^2$ gives

$$m = \frac{M_Z}{\sqrt{\sqrt{3}-1}} = 106.577\,\text{GeV} \approx m_\mu \times 10^3\,, \tag{48}$$

a coincidence whose significance is unclear.

10.6 Open issues for the Casimir equation

Renormalization scheme dependence. The comparison (46) uses the on-shell definition $\sin^2\theta_W = 1 - M_W^2/M_Z^2 = 0.22320$, **not** the $\overline{\text{MS}}$ value $\sin^2\theta_W(\overline{\text{MS}}) = 0.23122 \pm 0.00004$. The two differ by $\Delta \approx 0.008$, far larger than experimental errors. The algebraic argument (39) contains no dynamical mechanism to prefer one scheme over the other. Until such a mechanism is identified, the on-shell agreement should be regarded as suggestive but scheme-dependent.

The fourth eigenvalue. At $s = 1/2$, the second root of (39) gives

$$|M(1/2,-)| = m\,\sqrt{|x_-(1/2)|} = 122.4\,\text{GeV}\,, \tag{49}$$

which is 2.3% below $M_H = 125.25 \pm 0.17\,\text{GeV}$. However, this eigenstate carries spin 1/2, while the Higgs boson is spin 0. The near-degeneracy is noted as a numerical curiosity; we do not claim a connection.

Origin. This observation was first made by H. de Vries in a Physics Forums discussion (November 2004) and subsequently studied in [1,19].

10.7 Independence of the two structures

Parts A and B describe independent algebraic structures that share the same physical particles:

- The $SO(32)$ embedding (§10.1) governs the *representation content*—which $SU(5)$ multiplets appear.
- The Casimir equation (§10.4) governs the *mass/coupling pattern*—what numerical ratios emerge among the gauge-boson masses.
- The two structures live over different number fields: the adjoint decomposition involves only rational dimensions, while the eigenvalue ratio R lies in $\mathbb{Q}(\sqrt{3}, \sqrt{19}\,)$—an extension not generated by any invariant of $SO(32)$.

No algebraic homomorphism linking the embedding chain (34) to the eigenvalue equation (39) has been found. The gap between *content* (which states exist) and *dynamics* (what masses they acquire) remains the central open problem of the sBootstrap program.

11 SUSY Breaking: From Seeds to Spectrum

The SUSY-breaking story begins with the foundational observation (1): $m_\pi^2 - m_\mu^2 \approx f_\pi^2$. This is the Volkov–Akulov mass formula for a pseudo-Goldstino—the muon—whose superpartner—the pion—is a pseudo-Goldstone boson. The SUSY-breaking scale is $f_\pi \approx 92\,\text{MeV}$, and the breaking mechanism is chiral: $\langle \bar{q}q \rangle \neq 0$ breaks both chiral symmetry and the sBootstrap SUSY simultaneously. The Volkov–Akulov realization is the *physical starting point*; what follows in this section is the SQCD machinery that makes it precise.

The dynamics must accomplish three things: generate the seed-to-bloom transition for each tuple, preserve the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ through the breaking, and produce the correct splitting between meson and lepton masses at scale f_π.

11.1 The Seiberg framework

The natural arena is $\mathcal{N} = 1$ SQCD with gauge group $SU(3)_c$ and N_f flavors of chiral superfields Q^i, $\bar{Q}_j$ in the fundamental and antifundamental representations. For $N_f = N_c = 3$ (the case relevant to light QCD), Seiberg's exact results [8] establish that the theory confines, with the low-energy degrees of freedom being the composite meson superfield

$$M^i{}_j = Q^i \cdot \bar{Q}_j \,, \tag{50}$$

a 3×3 matrix of chiral superfields, together with baryons $B = \epsilon_{ijk} Q^i Q^j Q^k$ and $\bar{B}$. These satisfy the quantum-modified constraint

$$\det M - B\bar{B} = \Lambda^6 \,, \tag{51}$$

where Λ is the dynamical scale ($2N_c = 6$ for $N_c = 3$).

Each entry $M^i{}_j$ is a chiral superfield containing a complex scalar (the meson) and a Weyl fermion (the lepton, in the sBootstrap identification). The scalar and fermion are degenerate in the SUSY limit; SUSY breaking splits them.

11.2 The three chiral superfields of each seed

The seed triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ maps naturally onto three chiral superfields with the following mass structure before breaking:

Superfield	Mass	Role
Φ_0	$m_0 = 0$	Massless: flat direction
Φ_a	$m_a = \mathfrak{z}_a^2$	Light massive member
Φ_b	$m_b = \mathfrak{z}_b^2$	Heavy massive member

The ratio $\mathfrak{z}_a/\mathfrak{z}_b = 2 - \sqrt{3}$ is enforced by the Koide energy-balance condition applied to the seed.

In the Seiberg description, these three superfields are entries of the meson matrix $M^i{}_j$. For the $(0, s, c)$ seed, the relevant entries involve the strange and charm quarks; Φ_0 is the entry whose quark-mass combination vanishes in the seed limit (i.e., the direction in flavor space where $m_i + m_j \to 0$ through GOR).

11.3 F-term breaking: the O'Raifeartaigh mechanism

The VA relation (1) tells us *that* SUSY is broken at scale f_π. To understand *how*, we need the microscopic mechanism. The O'Raifeartaigh model [9] is the simplest realization of spontaneous F-term SUSY breaking. With three chiral superfields, the superpotential

$$W = f\,\Phi_0 + m\,\Phi_1\Phi_2 + g\,\Phi_0\Phi_1^2 \tag{52}$$

gives F-terms:

$$F^*_{\Phi_0} = f + g\,\phi_1^2\,, \tag{53}$$
$$F^*_{\Phi_1} = m\,\phi_2 + 2g\,\phi_0\phi_1\,, \tag{54}$$
$$F^*_{\Phi_2} = m\,\phi_1\,. \tag{55}$$

If $f \neq 0$ and $m \neq 0$, then $F^*_{\Phi_2} = 0$ requires $\phi_1 = 0$, but then $F^*_{\Phi_0} = f \neq 0$: SUSY is **necessarily broken**. The key features are:

1. A massless field (Φ_0) exists at tree level—it is the pseudo-modulus.
2. The breaking is triggered because F_{Φ_0} cannot vanish.
3. After including loop corrections, Φ_0 acquires a mass (the Coleman–Weinberg potential lifts the flat direction).

The structural parallel with the seed triples is:

O'Raifeartaigh	Flavor	Seed	After breaking
Φ_0 (massless)	b / B	$\mathfrak{z} = 0$ (flat direction)	$\mathfrak{z}_b = 64.65$ (mass generated)
Φ_1 (light)	s / π	$\mathfrak{z}_s = 9.67$	$\mathfrak{z} \to -9.67$ (sign-flipped)
Φ_2 (heavy)	c / D_s	$\mathfrak{z}_c = 35.64$	$\mathfrak{z}_c \approx 35.64$ (shifted)
		$f = 0$ (SUSY)	$f = f_\pi$ (broken)

This parallel is more than structural: the minimal O'Raifeartaigh model gives Φ_1 and Φ_2 a single mass parameter m, but the *fermionic* mass matrix at the vacuum ($\phi_1 = 0$, $\phi_0 = v$) is

$$W_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2gv & m \\ 0 & m & 0 \end{pmatrix},$$

with eigenvalues 0 (the goldstino) and $m_\pm = \sqrt{g^2v^2 + m^2} \pm gv$. The mass ratio $m_-/m_+ = (\sqrt{t^2+1} - t)^2$, where $t = gv/m$, is *not* unity. Setting $\sqrt{m_-}/\sqrt{m_+} = 2 - \sqrt{3}$ (the Koide seed ratio) requires $\sqrt{t^2+1} - t = 2 - \sqrt{3}$, which gives

$$t = \frac{gv}{m} = \sqrt{3},$$

the unique value at which $t^2 + 1 = 4$, so $m_\pm = (2 \pm \sqrt{3})\, m$. The Koide ratio for the triple $(0, m_-, m_+) = (0,\ (2-\sqrt{3})\, m,\ (2+\sqrt{3})\, m)$ evaluates to $Q = 4m/(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 4/6 = 2/3$ exactly, since $\sqrt{2-\sqrt{3}} \cdot \sqrt{2+\sqrt{3}} = 1$ and the squared sum telescopes to 6.

The Koide condition on the seed is therefore *one algebraic constraint on the superpotential parameters*: $gv = \sqrt{3}\, m$. The coupling g, the pseudo-modulus VEV v, and the mass parameter m are individually undetermined, but their ratio is fixed. This is the microscopic content of the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ applied to the seed.

The b quark and B meson are always present as flavor degrees of freedom; the seed configuration $(0, s, c)$ means the b-direction sits at the flat direction before breaking. The bloom is not particle creation but mass generation: breaking lifts Φ_0 from zero to $\mathfrak{z}_b \neq 0$, simultaneously flipping the sign of $\mathfrak{z}_s$, so that the same three flavors rearrange from $(0, s, c)$ into $(-s, c, b)$.

The bloom as a phase rotation. In the Koide parametrization (13), the seed sits at $\delta = 3\pi/4$, where $\cos(3\pi/4) = -1/\sqrt{2}$ forces one mass to vanish: $\sqrt{m_0} = \sqrt{M_0}(1 + \sqrt{2}\cos\delta) = 0$. A Koide-preserving bloom ($Q = 2/3$ maintained) is a pure rotation $\delta : 3\pi/4 \to \delta_{\text{phys}}$ at fixed scale v_0 and fixed amplitude $r = \sqrt{2}\, v_0$. The total mass $M = 6v_0^2$ is conserved—mass merely redistributes among the three eigenvalues. The sign flip is instantaneous: the seed sits exactly at the boundary $\mathfrak{z}_0 = 0$, and any infinitesimal rotation generates a nonzero charge with definite sign.

Explicit R-symmetry breaking in the O'Raifeartaigh model (adding a term $\delta W \sim \varepsilon\, \Phi_0 \Phi_2$) lifts the goldstino mass as $m_0 \sim \varepsilon^2/m$, but simultaneously pushes Q away from $2/3$—no nonzero ε preserves the Koide condition. The Koide seed is an *unstable fixed point* of the R-breaking perturbation. In the dynamical (SQCD) realization, the bloom must therefore be a nonperturbative rearrangement—not a smooth deformation of the O'Raifeartaigh spectrum.

A striking difference from standard O'Raifeartaigh: the pseudo-modulus Φ_0 normally acquires the *smallest* mass in the spectrum (a loop-suppressed Coleman–Weinberg mass, parametrically $\sim g^2 f/16\pi^2 m$). In the sBootstrap bloom, the b-direction acquires the *largest* mass ($m_b \gg m_s$ and $m_b > m_c$). The bloom is far more dramatic than a perturbative CW lifting—it is a nonperturbative, strong-dynamics effect in which confinement generates a mass of order $\Lambda_{\text{QCD}}^2/m_q$, not $g^2/(16\pi^2)$. Numerically, the one-loop CW mass for the pseudo-modulus at the Seiberg vacuum is $m_{\text{CW}} \approx 13\,\text{MeV}$, more than two orders of magnitude below the physical $m_b = 4{,}180\,\text{MeV}$. This inversion is a signature of the dynamical (SQCD) realization versus the perturbative (superpotential) one.

A candidate mechanism: fractional instantons. On $\mathbb{R}^3 \times S^1$, $\mathrm{SU}(N_c)$ instantons fractionalize into N_c monopole-instantons [17]. Each monopole-instanton carries a single fermionic zero mode per flavor; when the zero mode is lifted by the quark mass m_i, the amplitude acquires a factor $\sqrt{m_i}$ (not m_i). Since the $N_c = 3$ monopole contributions enter *additively* in the effective superpotential,

$$W_{\mathrm{mon}} \sim \zeta \left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right),$$

where $\zeta \propto e^{-S_0/3}$ is the monopole fugacity. This is *exactly* the sum-of-square-roots structure of the vacuum charge $v_0 = (\sum \sqrt{m_k})/3$. The v_0-doubling then maps to the counting of active monopole-instanton contributions: the seed (two active flavors) involves two monopole amplitudes, while the bloom (three active flavors, one with a sign flip) involves three, doubling v_0.

This mechanism is non-holomorphic in the sense required: the Kähler potential receives monopole–anti-monopole (bion) cross terms of the form $|\sum s_k \sqrt{m_k}|^2$, where m_k are the current quark masses. Expanding for $N_c = 3$:

$$K_{\mathrm{bion}} = \frac{\zeta^2}{\Lambda^2}\left(m_1 + m_2 + m_3 + 2s_1 s_2\sqrt{m_1 m_2} + 2s_1 s_3\sqrt{m_1 m_3} + 2s_2 s_3\sqrt{m_2 m_3}\right),$$

where the diagonal terms $s_k^2 m_k = m_k$ are self-bions and the cross terms $\sqrt{m_i m_j}$ are mixed bion amplitudes—their geometric mean structure is what produces $\sum \sqrt{m_k}$. Note that m_k here are the quark masses (superpotential couplings), *not* the meson VEVs $M^k{}_k \propto 1/m_k$ of the Seiberg seesaw; the bion amplitude involves quark zero modes, not meson fields. Organizing the bion amplitudes into seed-sector ($S_{\mathrm{seed}} = \sqrt{m_s} + \sqrt{m_c}$) and bloom-sector ($S_{\mathrm{bloom}} = -\sqrt{m_s} + \sqrt{m_c} + \sqrt{m_b}$) channels, the Kähler correction takes the form

$$\delta K \sim \lambda_1 \left|S_{\mathrm{seed}}\right|^2 + \lambda_2 \left|S_{\mathrm{bloom}} - 2\,S_{\mathrm{seed}}\right|^2.$$

The second term is a perfect square that vanishes *at the minimum* when $S_{\mathrm{bloom}} = 2\,S_{\mathrm{seed}}$, i.e., $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$—the v_0-doubling condition. The relation between bion couplings ($\lambda_3 = -4\lambda_2$ for cross-channel bions) is what enforces the doubling. Numerically, $S_{\mathrm{bloom}} - 2\,S_{\mathrm{seed}} = 0.023\,\mathrm{MeV}^{1/2}$ (effectively zero: $m_b = 4{,}177$ vs. PDG 4,180). The Hessian of $\lambda_2\left|S_{\mathrm{bloom}} - 2\,S_{\mathrm{seed}}\right|^2$ has rank 1: its single nonzero eigenvector is $\partial f/\partial m_i$ (the gradient of $f = \sqrt{m_b} - 3\sqrt{m_s} - \sqrt{m_c}$), while the two flat directions correspond to independent variation of m_s and m_c at fixed f. Thus the bion Kähler constrains exactly *one* mass combination; the remaining two masses are fixed by the Koide seed in the superpotential. Including propagated uncertainties ($\delta m_s = 0.8\,\mathrm{MeV}$, $\delta m_c = 20\,\mathrm{MeV}$), the prediction is $m_b = 4{,}177 \pm 40\,\mathrm{MeV}$, consistent with the PDG value $4{,}180 \pm 30\,\mathrm{MeV}$ at 0.06σ.

The principal caveat is that this analysis applies on $\mathbb{R}^3 \times S^1$; connecting to $\mathbb{R}^4$ physics requires Ünsal's continuity conjecture [18], which posits that center-symmetric confinement at small S^1 is continuously connected to the large-S^1 confining vacuum. The conjecture is supported by lattice evidence for $SU(N)$ pure Yang–Mills and by the absence of phase transitions in the center-stabilized theory, but remains unproven for theories with matter.

The identification $\langle F_{\Phi_0}\rangle = f \sim f_\pi^2$ is the sBootstrap's central claim: the SUSY-breaking F-term is the chiral condensate parameter.

In SQCD, this is realized dynamically rather than through a superpotential: the chiral condensate $\langle \bar{q}q\rangle \neq 0$ acts as the nonzero F-term, the impossibility of restoring chiral symmetry in the QCD vacuum is the dynamical analogue of $f \neq 0$, and the pion—the would-be Goldstone boson that acquires mass from explicit chiral breaking—plays the role of Φ_0 whose mass is lifted by loop corrections (here, by quark masses).

11.4 The Goldstino direction and the Volkov–Akulov realization

When SUSY is spontaneously broken, a massless fermion appears: the Goldstino, the fermionic analogue of the Goldstone boson. Its decay constant $f_{\rm VA}$ sets the SUSY-breaking scale through the Volkov–Akulov Lagrangian [12]:

$$\mathcal{L}_{\rm VA} = -f_{\rm VA}^2 \det\left(\delta_\mu^\nu + \frac{i}{f_{\rm VA}^2}\,\bar{\lambda}\bar{\sigma}^\nu\partial_\mu\lambda\right), \tag{56}$$

where λ is the Goldstino field. At low energies, the Goldstino couples derivatively with strength $\sim 1/f_{\rm VA}^2$, exactly as the pion couples with strength $\sim 1/f_\pi^2$ in the chiral Lagrangian.

The sBootstrap identification is:

$$f_{\rm VA} = f_\pi\,, \qquad \text{Goldstino} = \text{muon (partner of } \pi)\,. \tag{57}$$

More precisely, the muon is a *pseudo-Goldstino*: in the limit $m_q \to 0$ where the pion becomes exactly massless (a true Goldstone boson), its fermionic partner also becomes massless (a true Goldstino). The finite quark masses explicitly break both chiral symmetry (giving $m_\pi \neq 0$) and the sBootstrap SUSY (giving $m_\mu \neq m_\pi$), with the splitting controlled by f_π:

$$m_\pi^2 - m_\mu^2 = f_\pi^2\,. \tag{58}$$

This is the Volkov–Akulov relation for the pseudo-Goldstino: the mass splitting between the scalar (pion) and its fermionic partner (muon) is set by the SUSY-breaking scale squared.

The VA mechanism is not a separate breaking from the F-term: it is the *low-energy, composite-level description* of the same breaking. At the microscopic level, $\langle F\rangle \neq 0$ in the meson superfield; at the composite level, this manifests as the Goldstino coupling with $f_{\rm VA} = f_\pi$. These two quantities—the Goldstino decay constant and the pion decay constant—are *identified* in the sBootstrap; this is the central claim, not a derivation.

The identification should not be confused with the standard relation between the F-term VEV and the chiral condensate. In standard SQCD,

$$\langle F\rangle \sim \frac{|\langle\bar{q}q\rangle|}{f_\pi} \sim \frac{(250\,\text{MeV})^3}{92\,\text{MeV}} \approx 1.7\times10^5\,\text{MeV}^2\,,$$

while $f_\pi^2 \approx 8{,}500\,\text{MeV}^2$—a factor of ~20 discrepancy. The GOR relation [13], $m_\pi^2 f_\pi^2 = -(m_u+m_d)\langle\bar{q}q\rangle$, connects these through the small quark masses: the ratio $|\langle\bar{q}q\rangle|/f_\pi \sim 20$ is exactly $(m_u+m_d)^{-1}m_\pi^2 f_\pi \approx 20$, so the discrepancy is the chiral suppression factor, not a tuning. The sBootstrap identification $f_{\rm VA} = f_\pi$ is therefore a *low-energy effective* statement: the breaking scale appearing in the meson–lepton mass relation is f_π (the Goldstone decay constant), not the microscopic condensate $|\langle\bar{q}q\rangle|^{1/3}$.

The VA realization gives more than a mass formula—it organizes the entire breaking narrative. In the chiral limit ($m_q \to 0$), both pion and muon are exactly massless: the pion as the Goldstone boson of broken chiral symmetry, the muon as the Goldstino of broken sBootstrap SUSY. This is the *exact seed*: the B-meson direction sits at the flat direction ($\mathfrak{z} = 0$), and the π–μ pair is massless. Turning on quark masses ($m_q \neq 0$) does three things simultaneously:

1. Gives the pion its GOR mass (lifting the flat direction for π).

2. Gives the muon its mass (the pseudo-Goldstino acquires mass when the explicit breaking parameter m_q is nonzero).

3. Generates the B-meson mass (the flat direction for Φ_0 is lifted by the Coleman–Weinberg potential, i.e., by the same quark masses that feed the GOR formula).

The splitting between (1) and (2) is f_π^2—the VA relation. The seed-to-bloom transition of §4 is the same physics seen from the charge-space perspective: $m_q \neq 0$ lifts the zero, generates the b/B mass, flips the sign of $\mathfrak{z}_s$ and $\mathfrak{z}_\pi$, and produces the full triple. The VA equation (1) is not one fact among many; it is the generating relation from which the bloom follows.

A note on quantum numbers: the Goldstino of spontaneously broken spacetime SUSY is a neutral fermion (the fermionic component of the superfield whose F-term breaks SUSY). The muon carries electric charge and lepton number. The sBootstrap SUSY, however, is not a spacetime symmetry but an emergent hadronic symmetry—a Miyazawa-type superalgebra [5] acting on composite states. In this context, the "Goldstino" need not carry the quantum numbers of a spacetime supercharge; it carries the quantum numbers appropriate to the broken hadronic generator. The muon's charge is inherited from the pion multiplet it partners.

11.5 D-term breaking: electromagnetic splittings

The unbroken $U(1)_{\rm em}$ gauge symmetry allows a Fayet–Iliopoulos (FI) term [10]:

$$\mathcal{L}_{\rm FI} = \xi\, D_{\rm em}\,, \tag{59}$$

which shifts scalar masses by $\delta m^2 = Q_{\rm em}\,\xi$, where $Q_{\rm em}$ is the electric charge. This is the supersymmetric version of the electromagnetic mass difference.

In QCD without SUSY, the electromagnetic mass splitting of the pion is given by the Das–Guralnik–Mathur–Low–Young (DGMLY) formula [15]:

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha_{\rm em}}{4\pi}\, m_\rho^2 \approx (35.5\,{\rm MeV})^2\,, \tag{60}$$

which in the SUSY framework becomes a D-term contribution:

$$\xi \sim \frac{3\alpha_{\rm em}}{4\pi}\, m_\rho^2 \sim (35\,{\rm MeV})^2\,. \tag{61}$$

In charge space, the D-term shifts $\mathfrak{z}$ by an amount proportional to $Q_{\rm em}$:

$$\mathfrak{z} \to \mathfrak{z} + \delta\mathfrak{z}\,, \qquad \delta\mathfrak{z} \approx \frac{Q_{\rm em}\,\xi}{2\mathfrak{z}}\,. \tag{62}$$

For the pion ($Q_{\rm em} = \pm 1$, $\mathfrak{z} \approx 11.8$): $\delta\mathfrak{z} \approx 0.2\,{\rm MeV}^{1/2}$, matching the observed $\mathfrak{z}_{\pi^\pm} - \mathfrak{z}_{\pi^0} = 11.82 - 11.62 = 0.20\,{\rm MeV}^{1/2}$.

The D-term effect is subleading to the F-term: $\xi \sim (35\,{\rm MeV})^2$ versus $f_\pi^2 \sim (92\,{\rm MeV})^2$. It operates *within* each supermultiplet entry, splitting charged from neutral mesons, while the F-term splits bosons from fermions across the supermultiplet.

Witten's inequality [11] guarantees $m_{\pi^\pm} > m_{\pi^0}$ in QCD with electromagnetism—the charged pion is always heavier. In the SUSY framework this becomes the statement that the D-term contribution is positive for $Q_{\rm em} \neq 0$, consistent with the FI sign.

11.6 Mapping the breaking to the tuples

The tuple structure maps onto a layered breaking with two independent mechanisms (F-term and D-term) plus a separate heavy-sector origin. The Volkov–Akulov realization of §11.4 is not a third mechanism: it is the low-energy, composite-level description of the same F-term breaking. The VA relation (1) *sets the scale* ($f_{\rm VA} = f_\pi$); the O'Raifeartaigh/SQCD dynamics of §11.3 *provides the mechanism* ($\langle F\rangle \neq 0$).

Layer 1: F-term/VA (chiral condensate, scale $\sim f_\pi$). The chiral condensate $\langle \bar{q}q\rangle \neq 0$ breaks both chiral symmetry and the sBootstrap SUSY. This is the primary breaking, affecting the light sector. The seed triples $(0, s, c)$ and $(0, \pi, D_s)$ are the pre-breaking configurations; the massless member is the flat direction whose F-term is forced nonzero by confinement. After breaking:

- The zero member is lifted and acquires negative charge ($\mathfrak{z} < 0$), becoming the sign-flipped member of the full triple ($-s$, $-\pi$).
- The full triples $(-s, c, b)$ and $(-\pi, D_s, B)$ emerge, incorporating the b-quark and B-meson sectors.
- The lepton triple (e, μ, τ) is the fermionic shadow: the leptons acquire masses through the same condensate, with the muon as pseudo-Goldstino.

Layer 2: D-term (electromagnetism, scale $\sim \alpha_{\bf em} m_\rho$). The $U(1)_{\rm em}$ FI term generates electromagnetic mass splittings within each multiplet. This is a perturbative correction on top of the F-term breaking, affecting only charged particles. It does not create new Koide triples but shifts existing ones by $O(\alpha_{\rm em})$.

Layer 3: The heavy sector (scale $\sim v_0^{(cbt)}$). The (c, b, t) triple at $v_0^2 \approx 29.6\,\text{GeV}$ operates far above the chiral scale. It has no seed (no massless member), no meson shadow (top does not hadronize), and its breaking may be connected to a different sector of the theory—possibly the electroweak Higgs mechanism.

This layered structure involves three distinct scales, each with different physical meaning:

$$\text{F-term (boson–fermion splitting):} \quad f_\pi \approx 92\,\text{MeV}\,, \quad f_\pi^2 \approx 8{,}500\,\text{MeV}^2\,; \tag{63}$$

$$\text{D-term (EM fine structure):} \quad \xi^{1/2} \approx 35\,\text{MeV}\,, \quad \xi \approx 1{,}200\,\text{MeV}^2\,; \tag{64}$$

$$\text{Heavy sector (Koide scale):} \quad v_0^{(cbt)} \approx 172\,\text{MeV}^{1/2}\,, \quad (v_0^{(cbt)})^2 \approx 29{,}600\,\text{MeV}\,. \tag{65}$$

Comparing the squared scales: $f_\pi^2 \gg \xi$ (F-term dominates over D-term by a factor ~ 7, consistent with the D-term being a perturbative $O(\alpha_{\rm em})$ correction). The heavy-sector scale $(v_0^{(cbt)})^2$ carries dimensions of MeV (not MeV^2) because v_0 is a charge (MeV$^{1/2}$), not an energy; direct comparison with f_π requires specifying which physical quantity is being compared.

11.7 The O'Raifeartaigh structure in SQCD

In Seiberg's SQCD with $N_f = N_c = 3$, the quantum constraint (51) provides a natural setting for the O'Raifeartaigh mechanism. The constraint $\det M - B\bar{B} = \Lambda^6$ means the moduli space has a specific geometry: not all field directions are flat. When quark masses are added via a tree-level superpotential $W_{\text{tree}} = m_i M^i{}_i$ (sum over flavors), the F-terms become:

$$F_{M^i{}_j} = m_i\,\delta^i_j + \Lambda^6\,(\text{constraint terms})\,. \tag{66}$$

For the seed triple $(0, s, c)$, the "0" member corresponds to a direction in the meson matrix where the tree-level mass vanishes—the entry involving the lightest quark combination. In the exact chiral limit ($m_u = m_d = 0$), this entry is truly massless, and the constraint (51) forces $\langle F\rangle \neq 0$ for that entry. This is the dynamical O'Raifeartaigh mechanism: confinement (encoded in Λ^6) plays the role of the parameter f in the superpotential, and the impossibility of setting all F-terms to zero is a consequence of the strong dynamics rather than a choice of superpotential.

The pseudo-modulus and the Koide seed. In the ISS magnetic dual, the tree-level superpotential contains a Yukawa coupling $h\,\mathrm{Tr}(q\,M\,\tilde{q})$ between the magnetic quarks q, $\tilde{q}$ and the meson matrix M. The O'Raifeartaigh parameters map as $g \to h$, $m \to h\mu$, $v \to \langle X\rangle$ (the pseudo-modulus VEV), so the seed Koide condition $gv/m = \sqrt{3}$ becomes $\langle X\rangle/\mu = \sqrt{3}$—the magnetic Yukawa h cancels. The Koide seed fixes the pseudo-modulus VEV, not the coupling h. At one-loop Coleman–Weinberg level $\langle X\rangle = 0$; a non-canonical Kähler correction provides the mechanism.

Kähler stabilization of the pseudo-modulus. In the minimal O'Raifeartaigh model ($W = fX + m\phi\tilde{\phi} + gX\phi^2$) with canonical Kähler potential, the CW potential is monotonically increasing and selects $\langle X\rangle = 0$. Polynomial superpotential deformations (including cubic, quartic, and dimension-five operators) either vanish at the vacuum, reinforce $v = 0$, or produce minima at $gv/m \ll \sqrt{3}$.

A negative quartic Kähler correction resolves this:

$$K = |X|^2 + \frac{c}{\Lambda_K^2}\,|X|^4\,, \qquad c < 0\,.$$

The tree-level potential becomes $V_{\text{tree}} = f^2/(1 + 4c\,v^2/\Lambda_K^2)$, which has a pole at $v_{\text{pole}} = \Lambda_K/(2\sqrt{|c|})$. Near the pole the CW contribution is negligible ($V_{\text{CW}} \sim 10^{-4}\,m^4 \ll V_{\text{tree}}$), and the effective potential is minimized at v_{pole}. Demanding $v_{\text{pole}} = \sqrt{3}\,m/g$ gives the exact condition

$$c = -\,\frac{\Lambda_K^2}{12\,m^2}\,.$$

With $\Lambda_K = m$ this is $c = -1/12$, an algebraic result: $\Lambda_K/(2\sqrt{1/12}) = \sqrt{12}/2 = \sqrt{3}$. No numerical tuning is required. At this VEV the fermion spectrum is $(0,\ (2-\sqrt{3})\,m,\ (2+\sqrt{3})\,m)$ with $Q = 4/6 = 2/3$ exactly (since $(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 4 + 2 = 6$), confirming the Koide seed. The Kähler metric $K_{X\bar{X}} = 1 - v^2/(3m^2)$ vanishes at $v = \sqrt{3}\,m$, signaling the boundary of the effective field theory.

The ISS mechanism (Intriligator–Seiberg–Shih [14]) establishes metastable SUSY-breaking vacua for $N_c < N_f < 3N_c/2$, where the rank condition in the magnetic dual prevents all F-terms from vanishing. The sBootstrap case $N_f = N_c = 3$ is strictly *below* the ISS window—at $N_f = N_c$ the low-energy description is qualitatively different, governed by the quantum-modified constraint (51) on the moduli space rather than by a free magnetic dual. However, physical QCD has $N_f = 3$ light flavors and $N_f = 4$–5 if charm and bottom are included at their respective thresholds. With charm included, $N_f = 4$

falls within the ISS window ($3 < 4 < 4.5$); with both charm and bottom, $N_f = 5$ falls in the conformal window ($4.5 < 5 < 9$), where the Seiberg seesaw constraint does not apply directly. The sBootstrap's multi-scale structure—with the seed triples involving s, c, and b quarks at different scales—requires treating the effective N_f as scale-dependent: $N_f = 3$ below m_c (confining, Seiberg constraint), $N_f = 4$ above m_c (ISS metastable SUSY breaking), and $N_f = 5$ above m_b (conformal window). Note that $N_f = 4$ is the *unique* integer satisfying $3 < N_f < 4.5$ for $N_c = 3$, providing a dynamical rationale for exactly four light flavors (u, d, s, c) in the SUSY-breaking sector.

ISS vacuum lifetime. The metastable ISS vacuum decays to the SUSY-preserving ground state by tunneling with bounce action $S_{\text{bounce}} \sim (8\pi^2/3)\, N_c\, (\Lambda_{\text{mag}}/h\mu)^4$, where μ is the lightest quark mass controlling the tunneling. For $\mu = m_s = 93\,\text{MeV}$, $h \sim O(1)$, and $\Lambda_{\text{mag}} \sim 300\,\text{MeV}$: $S_{\text{bounce}} \sim 10^2$–$10^3$ (depending on h), giving a lifetime exceeding the age of the universe by hundreds of orders of magnitude for any h of order unity. The metastable vacuum is cosmologically acceptable.

The Seiberg seesaw and dual Koide. At the Seiberg vacuum with $B = \bar{B} = 0$, the diagonal meson VEVs scale as $\langle M^j{}_j \rangle \propto 1/m_j$: heavy quarks produce light mesons and vice versa. This seesaw prompts a dual question: does the inverse-mass spectrum $\{1/m_i\}$ satisfy a Koide condition? Evaluating $Q = \sum(1/m_i)/(\sum \sqrt{1/m_i})^2$ for the pure down-type quarks (d, s, b):

$$Q(1/m_d,\, 1/m_s,\, 1/m_b) = 0.6652\,,$$

a 0.22% deviation from 2/3—better than any direct quark Koide triple. No other quark triple comes close under the Seiberg seesaw (the next best, (d, s, c), deviates by 4.2%). This "dual Koide" on (d, s, b) (first reported here) connects the UV mass hierarchy to the IR meson VEVs: the Seiberg constraint maps the down-type quark masses into a near-Koide spectrum of meson condensates. However, the result is sensitive to m_d: within $\pm 1\sigma$ ($m_d = 4.67 \pm 0.48\,\text{MeV}$), Q ranges from 0.655 to 0.676, and the deviation from 2/3 varies from 0.2% to 1.7%. The dual Koide is an observation at the edge of the current m_d uncertainty. Setting $Q(1/m_d, 1/m_s, 1/m_b) = 2/3$ exactly predicts $m_b = 4562\,\text{MeV}$ (9% from PDG, 12.7σ)—much worse than v_0-doubling ($m_b = 4177\,\text{MeV}$, 0.07%). The 0.22% closeness of Q to 2/3 does not translate to a precise mass prediction because the Koide quadratic amplifies small Q deviations into large mass deviations.

A simple algebraic identity connects the seed to the seesaw: for any two masses, the Koide ratio $Q(a, b) = (a + b)/(\sqrt{a} + \sqrt{b})^2$ satisfies $Q(a, b) = Q(1/a,\, 1/b)$ identically. The seed $(0, m_s, m_c)$ sits at this self-dual point: its Koide ratio is the same whether evaluated on quark masses or on Seiberg-dual meson VEVs $\propto 1/m_i$. Thus the seed Koide in the electric (UV) theory is *automatically* a seed Koide in the magnetic (IR) theory—the Seiberg duality preserves the seed condition without fine-tuning. For the full bloomed triple $(-s, c, b)$, self-duality breaks: $Q(-s, c, b) \neq Q(1/m_s, 1/m_c, 1/m_b)$ in general. The dual Koide $Q(1/m_d, 1/m_s, 1/m_b) = 0.665$ is therefore a *nontrivial* constraint on the down-type sector, not a tautological restatement of the seed.

A systematic scan over all $\binom{6}{3} = 20$ quark triplets from $\{u, d, s, c, b, t\}$, allowing each entry to appear as $\sqrt{m_k}$ or $1/\sqrt{m_k}$ with independent signs (640 cases total), confirms that $(1/m_d, 1/m_s, 1/m_b)$ achieves the globally best Koide approximation among all quark triples with deviation $|Q - 2/3| = 0.00144$, outperforming even (c, b, t) at 0.00397. Ranks 2–5 in the scan are occupied by mixed triples of the form $(\sqrt{m_s}, \sqrt{m_c}, \pm 1/\sqrt{m_X})$ with

$X = b$ or t, all clustering near $Q \approx 0.664$. These mixed triples are moreover dimensionally ill-defined: $\sqrt{m}$ carries units $\mathrm{MeV}^{+1/2}$ while $1/\sqrt{m}$ carries $\mathrm{MeV}^{-1/2}$, so their sum—and hence Q—is physically meaningless. A direct unit-dependence test confirms this: $Q(\sqrt{m_s}, \sqrt{m_c}, 1/\sqrt{m_b})$ equals 0.664 when masses are expressed in MeV but 0.434 in GeV, while $Q(1/m_d, 1/m_s, 1/m_b)$ and $Q(m_c, m_b, m_t)$ are unit-independent, as required of any physical ratio. Equivalently, any mixed triple can be rendered dimensionally consistent by interpreting the inverse entry as a seesawed mass $m_3 = M^2/m_c$; setting $Q = 2/3$ then predicts m_3, and the scale $M^2 = m_{\exp}\, m_{\mathrm{pred}}$ is merely a derived label with no independent content. Setting aside the mixed cases, the remaining near-Koide hits are not independent coincidences either. For any two masses $A < B$, the degenerate ratio $Q(A, B, 0) \equiv (A + B)/(\sqrt{A} + \sqrt{B})^2$ equals 2/3 exactly when $B/A = (2 + \sqrt{3})^2 = 7 + 4\sqrt{3} \approx 13.93$. The actual ratio $m_c/m_s = 1270/93.4 \approx 13.59$ is only 2.4% below this threshold, giving $Q(m_s, m_c, 0) = 0.664$. Since $1/\sqrt{m_b} \approx 0.0155\ \mathrm{MeV}^{-1/2}$ and $1/\sqrt{m_t} \approx 0.0025\ \mathrm{MeV}^{-1/2}$ are negligible compared to $\sqrt{m_s} + \sqrt{m_c} \approx 45\ \mathrm{MeV}^{1/2}$, appending any heavy-quark inverse as a third entry merely perturbs $Q(m_s, m_c, 0)$ by order $(1/\sqrt{m_X})/(\sqrt{m_s} + \sqrt{m_c})^2 \lesssim 10^{-4}$. Ranks 2–5 therefore all reduce to the near-seed position of the (s, c) pair and carry no additional Koide information. The three genuinely independent near-Koide quark triples are $(-s, c, b)$, (c, b, t), and $(1/d, 1/s, 1/b)$.

The structure becomes transparent when the six quarks are grouped into three doublets $(t, s) \times (u, b) \times (c, d)$, each Koide triple selecting one element from each pair. Including limits $m_u \to 0$ and $m_t \to \infty$, there are $3 \times 3 \times 2 = 18$ base mass triples spanning $2^3 = 8$ families. Only four of the eight produce $|Q - 2/3| < 1.5\%$:

Family	Best triple	Type	$\lvert Q - \frac{2}{3}\rvert$
SBD	$(1/s,\ 1/b,\ 1/d)$	all-inverse	0.22%
SUC	$(s,\ 0,\ c)$	seed limit	0.35%
TBC	$(t,\ b,\ c)$	all-direct	0.60%
SBC	$(-s,\ b,\ c)$	signed	1.25%

The remaining four families—TUC, TUD, TBD, SUD—deviate by 4.6% or more, with TUC worst at 27%. The SUC seed is the pair Koide $Q(m_s, m_c, 0) = 0.664$ already derived above. Note that the doublets (t, s), (u, b), (c, d) pair each up-type quark with a down-type from a *different* generation, related by a cyclic permutation; this is the same structure that appears in the CKM off-diagonal entries.

Charge decomposition and the seesaw. The Koide condition $Q = 2/3$ is equivalent to $\langle z_k^2 \rangle = z_0^2$ (§2), an energy balance between the mean charge z_0 and the perturbations $z_k = \mathfrak{z}_k - z_0$. The dual Koide on $(1/m_d, 1/m_s, 1/m_b)$ admits the *same* decomposition in terms of dual charges $\xi_k = 1/\sqrt{m_k}$: writing $\xi_k = \xi_0 + \xi_k'$, one finds $\langle \xi_k'^2 \rangle / \xi_0^2 = 0.996$ (0.4% from unity), closer to exact energy balance than the direct triple $(-s, c, b)$ at 1.025 (2.5%).

However, the seesaw map $\mathfrak{z} \to 1/\mathfrak{z}$ does *not* preserve the Koide parametrization $\mathfrak{z}_k = z_0(1 + \sqrt{2}\cos(\delta + 2\pi k/3))$. Substituting $1/\mathfrak{z}_k = \xi_0(1 + \sqrt{2}\cos(\delta' + 2\pi k/3))$ and requiring the product $\mathfrak{z}_k \cdot 1/\mathfrak{z}_k = 1$ leads to a quadratic in $\cos\psi_k$ that must hold for three distinct cosine values—impossible unless one $\mathfrak{z}_k = 0$ (the seed). The seed self-duality $Q(0, a, b) = Q(0, 1/a, 1/b)$ is thus the *unique* fixed point of the seesaw: the electric and magnetic Koide conditions coincide only at the SUSY-preserving vacuum where one mass vanishes.

This means the three independent Koide triples decompose into two sectors:

- **Electric** (charges $\mathfrak{z} = \pm\sqrt{m}$): $(-s, c, b)$ and (c, b, t), constraining $\{s, c, b, t\}$.

- **Magnetic** (charges $\xi = 1/\sqrt{m}$): $(1/d, 1/s, 1/b)$, constraining $\{d, s, b\}$.

The dual charges $\xi_j = \sqrt{\langle M^j{}_j \rangle}/\Lambda$ are literally the square roots of the Seiberg meson VEVs: the magnetic Koide is an energy balance on the meson condensate, not on quark masses. Together with the v_0-doubling ($\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$), these four constraints leave only m_u, m_d, and the overall scale m_s as free parameters—consistent with the parameter budget of §11.12.

Among all $\binom{5}{3} = 10$ quark triples, (d, s, b) is the unique triple with inverse-mass Koide ratio within 1% of 2/3; the next closest is (d, s, c) at 4.2% deviation—a 19:1 margin.

11.8 How the three supermultiplets join

The seed-to-bloom transition generates three separate Koide triples. But the full sBootstrap requires these triples to be *embedded in a single meson matrix.* The 3×3 light-flavor matrix is:

$$M^{(3)} = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} \longleftrightarrow \begin{pmatrix} \pi^0/\eta & \pi^+ & K^+ \\ \pi^- & \pi^0/\eta & K^0 \\ K^- & \bar{K}^0 & \eta' \end{pmatrix}. \tag{67}$$

When charm and bottom are included, the matrix extends to 5×5. We display it schematically, with the members of the meson Koide triple $(-\pi, D_s, B)$ marked by boxes:

$$M^{(5)} = \begin{pmatrix} \pi^0/\eta & \boxed{\pi^+} & K^+ & D^0 & \boxed{B^+} \\ \pi^- & \pi^0/\eta & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & \eta' & \boxed{D_s^-} & B_s \\ \bar{D}^0 & D^+ & D_s^+ & \eta_c & B_c \\ B^- & \bar{B}^0 & \bar{B}_s & \bar{B}_c & \eta_b \end{pmatrix}, \tag{68}$$

where rows are indexed by (u, d, s, c, b) and columns by $(\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b})$.

The three members of the meson Koide triple occupy different off-diagonal blocks of this matrix: $\pi^+ = u\bar{d}$ is in the light (u, d) block, $D_s^- = s\bar{c}$ bridges the light-charm sector, and $B^+ = u\bar{b}$ bridges the light-bottom sector. They do not form a "slice" in any simple linear-algebraic sense—they are scattered entries connected by the Koide condition on their *masses*, not by their positions in the matrix. The meson seed $(0, \pi, D_s)$ drops the B^+ and replaces it with zero (the entry that would involve the top quark, absent from the matrix).

This scattered structure is the central puzzle: the Koide triples are *mass relations* among matrix entries, not submatrices. The question is whether the Seiberg constraint $\det M - B\bar{B} = \Lambda^6$ (appropriately generalized to 5×5), combined with the SUSY-breaking dynamics, produces correlations among these scattered entries that enforce the energy-balance condition $\langle z_k^2 \rangle = z_0^2$.

11.9 The energy balance is not an RG attractor

One might hope that the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ is a *dynamical attractor*: that three masses evolving under SUSY-breaking dynamics flow toward the Koide surface $Q = 2/3$. However, the one-loop soft-mass RG equations in $\mathcal{N} = 1$ SQCD with universal Casimirs and negligible Yukawas give $dm_i/dt = \gamma(M_g^2 - m_i)$, which drives all masses to a common value $m_i \to M_g^2$, yielding $Q \to 1/3$—the *opposite* extreme. The Koide preservation condition $\sum_i F_i(3 - 2S/\sqrt{m_i}) = 0$ on the $Q = 2/3$ surface is algebraically incompatible with any uniform attractor flow (by the Cauchy–Schwarz inequality, the required identity $S \sum 1/\sqrt{m_i} = 9/2$ contradicts $S \sum 1/\sqrt{m_i} \geq 9$).

The energy-balance condition therefore does *not* arise from simple RG running. It must instead be a *boundary condition* on the superpotential—as in the O'Raifeartaigh realization of §11.3, where $gv = \sqrt{3}\, m$ produces the exact Koide seed. The condition is set at the scale of SUSY breaking and then preserved by the QCD-invariant mass ratios, rather than being an RG attractor.

11.10 The exact seed and what breaks it

The seed configuration $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ requires *two* independent conditions:

1. One member at $\mathfrak{z} = 0$ exactly (the flat direction).
2. The ratio of the massive members equals $2 - \sqrt{3}$:

$$\frac{\sqrt{m_a}}{\sqrt{m_b}} = 2 - \sqrt{3} \approx 0.26795\,. \tag{69}$$

These are logically distinct. The chiral limit ($m_u = m_d = 0$, $\langle \bar{q}q \rangle \neq 0$) enforces condition (1) for the meson sector: the pion is an exact Goldstone boson, $m_\pi = 0$, and in the picture of §11.3 the B-meson direction sits at the flat direction before acquiring mass. But the chiral limit says *nothing* about condition (2)—it does not constrain $\sqrt{m_{D_s}}/\sqrt{m_B}$ or $\sqrt{m_s}/\sqrt{m_c}$ to equal $2 - \sqrt{3}$.

The physical deviations from the exact ratio are small but measurable:

Seed	Exact ratio	Physical ratio	Deviation
$(0, \pi, D_s)$	0.2680	$\sqrt{m_\pi}/\sqrt{m_{D_s}} = 0.2663$	-0.6%
$(0, s, c)$	0.2680	$\sqrt{m_s}/\sqrt{m_c} = 0.2712$	$+1.2\%$
$(0 \approx e, \mu, \tau)$	0.2680	$\sqrt{m_\mu}/\sqrt{m_\tau} = 0.2439$	-9.0%

The meson seed is closest to the exact ratio (0.6% deviation). The quark seed deviates by 1.2%. A natural thought is that QCD running might shift the quark ratio: perhaps at some scale μ^* the ratio $\sqrt{m_s(\mu^*)}/\sqrt{m_c(\mu^*)}$ hits $2 - \sqrt{3}$ exactly. But this is *not possible within QCD alone.* At leading order, all quark masses run with the same anomalous dimension γ_m:

$$m_i(\mu) = m_i(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_m/\beta_0}, \tag{70}$$

so the ratio $m_s(\mu)/m_c(\mu)$—and therefore $\sqrt{m_s}/\sqrt{m_c}$—is an RG invariant of QCD. Higher-order corrections break this invariance, but only at the $O(\alpha_s^2)$ level, far too small to account for 1.2%.

The quark seed deviation is therefore *intrinsic*: the $\overline{\text{MS}}$ mass ratio m_s/m_c is a fixed number (at any QCD scale), and it misses the Koide target by 1.2%. Three possible explanations present themselves:

Electroweak running. Above the electroweak scale, quark masses are controlled by Yukawa couplings y_i to the Higgs, which *do* run differently for different flavors: the β-functions for y_s and y_c involve different electroweak quantum numbers ($SU(2)_L$ doublet structure, hypercharge). The ratio y_s/y_c evolves with scale, and there may exist a scale μ^* above the electroweak threshold where $\sqrt{y_s(\mu^*)}/\sqrt{y_c(\mu^*)} = 2 - \sqrt{3}$. This would identify the "seed scale" as an electroweak-sector phenomenon, not a QCD one—consistent with the sBootstrap's identification of the breaking with $SU(2)_L \times U(1)_Y$ rather than $SU(3)_c$.

Charge running. Even if mass ratios are QCD-invariant, the Koide decomposition $\mathfrak{z}_k = z_0 + z_k$ may have its own flow. The Koide scale $v_0 = z_0^2$ sets the common scale, and the fluctuations z_k encode the deviations. As shown in §11.9, simple RG running drives $Q \to 1/3$ rather than $2/3$, so the energy-balance condition cannot be an attractor in the usual sense. However, the O'Raifeartaigh realization (§11.3) shows that the condition is a boundary condition on the superpotential ($gv = \sqrt{3}\,m$). The 1.2% deviation of the quark seed ratio may then reflect the distance between the physical quark masses and the exact superpotential constraint.

Meson vs. quark deviations. The meson deviation (0.6%) is smaller than the quark deviation (1.2%), and for a different reason: meson masses are *not* simple functions of quark mass ratios. The pion mass is given by the GOR relation $m_\pi^2 f_\pi^2 = -(m_u + m_d)\langle\bar{q}q\rangle$; the D_s mass involves heavy-quark effective theory; and both receive chiral perturbation theory corrections. These nonperturbative QCD effects are the dominant source of the meson seed deviation, and are in principle calculable at NLO in ChPT.

A remarkable feature of the VA relation. The foundational equation $m_\pi^2 - m_\mu^2 = f_\pi^2$ contains no QCD coupling constant: no α_s, no N_c, no $\Lambda_{\rm QCD}$. It is a pure mass relation among three experimentally measured quantities—two masses and a decay constant—holding to 2.2%. In the sBootstrap, this is natural: the relation is a SUSY mass splitting, set by the breaking scale f_π, and SUSY mass relations do not involve the gauge coupling of the confining theory. Standard QCD *explains* f_π in terms of $\langle\bar{q}q\rangle$ and α_s; the sBootstrap *uses* f_π as the fundamental parameter, with QCD providing the dynamical origin but not appearing in the mass formula. The absence of QCD factors in (1) is a hint that the relevant symmetry is not $SU(3)_c$ but the hadronic superalgebra that connects pions to leptons.

The lepton "seed" has the worst ratio match (-9%), which is consistent with the observation that the lepton triple is *not* a true seed: $m_e \neq 0$, and the electron was never at the flat direction. The leptons are an "almost-seed" whose excellent $Q = 0.66666$ arises not from the seed structure but from the smallness of m_e, which makes the two conditions approximately degenerate. For the leptons, condition (1) is approximate rather than exact, and condition (2) is correspondingly relaxed.

11.11 What the (c, b, t) triple tells us

The (c, b, t) triple ($Q = 0.6695$, $v_0^2 = 29.6\,\text{GeV}$) has neither a seed nor a meson shadow. It operates at a scale far above $\Lambda_{\rm QCD}$, suggesting its origin is different:

1. It may be governed by the electroweak Higgs mechanism rather than the chiral condensate. In a two-Higgs-doublet model with $\tan\beta = 1$ (as predicted by the

mixed-triple Koide condition, §11.12), $y_t \approx \sqrt{2}$; the Koide condition on (c, b, t) would then constrain the Yukawa coupling matrix.

2. It may represent a Volkov–Akulov realization at the electroweak scale: if there is a second SUSY breaking at $f' \sim v_{\rm EW}$, the (c, b, t) triple would be its analog of the (π, μ) system, but with f' replacing f_π.

3. Or it may be a purely kinematic accident—the quark masses happen to satisfy the Koide condition without a dynamical reason. The 0.42% precision makes this unlikely but not impossible.

Interpretation (1) is most natural within the sBootstrap: the (c, b, t) triple involves quarks whose masses are dominated by Yukawa couplings to the Higgs, not by the chiral condensate. Since $m_i = y_i\, v_{\rm EW}/\sqrt{2}$, the Koide condition $Q = 2/3$ translates into a constraint on the Yukawa couplings (see §11.12): $\sum y_i/(\sum \sqrt{y_i})^2 = 2/3$. With $y_t \approx 1$ fixed by the top mass, this determines y_b/y_t and y_c/y_t through the Koide parametrization. The top Yukawa $y_t = 0.99$ is the only coupling of order unity; the charm and bottom Yukawas ($y_c = 0.010$, $y_b = 0.034$) are effective couplings generated by the Seiberg seesaw, not independent fundamental parameters. In this picture, the entire fermion mass hierarchy derives from a single Yukawa coupling ($y_t \approx 1$) plus confining dynamics at $\Lambda \sim 300\,\text{MeV}$. The chiral sector and the electroweak sector would each independently produce a Koide-type energy balance, with the charm and bottom quarks serving as the bridge between the two.

CKM mixing and the Koide triples. A striking feature of the quark Koide triples is that they *obligatorily mix up-type and down-type quarks*: $(-s, c, b)$ contains one up-type (c) and two down-type (s, b), while (c, b, t) contains two up-type (c, t) and one down-type (b). An exhaustive scan of all $\binom{6}{3} = 20$ quark triples with all sign choices confirms that *every* triple close to $Q = 2/3$ is mixed—neither the pure down-type (d, s, b) nor the pure up-type (u, c, t) comes close ($Q = 0.73$ and $Q = 0.85$ respectively). This mixing of mass eigenvalues from different Yukawa sectors is puzzling in the Standard Model, where up-type and down-type masses are eigenvalues of independent matrices Y_u and Y_d. It is natural, however, in an $SU(5)_f$ flavor-symmetric framework where all five quarks are treated democratically.

The Koide constraints on the two overlapping triples, together with the seed condition on $(0, s, c)$, determine (s, c, b) from a single input (m_t). The unconstrained masses are m_u and m_d—the first generation. The Gatto–Sartori–Tonin relation [20] $|V_{us}| \approx \sqrt{m_d/m_s}$ connects precisely these free parameters to the Cabibbo angle ($|V_{us}| = 0.224$ vs. PDG 0.2243, Table 6), suggesting that the CKM matrix encodes the residual freedom left undetermined by the Koide structure.

If interpretation (2) is also invoked, the sBootstrap has two breaking scales:

$$f_{\rm chiral} = f_\pi \approx 92\,\text{MeV}\,, \qquad f_{\rm EW} \sim v_{\rm EW} \approx 246\,\text{GeV}\,, \tag{71}$$

and the full spectrum requires both. The light sector (leptons, light mesons) is governed by f_π; the heavy sector (top, and by extension charm and bottom through the Koide network) is governed by $v_{\rm EW}$. The charm and bottom quarks, appearing in both the $(0, s, c)/(-s, c, b)$ light-sector triples and the (c, b, t) heavy-sector triple, are the "bridges" between the two scales.

11.12 Toward a Superpotential

Consider $\mathcal{N}=1$ SQCD with gauge group $SU(3)_c$ and $N_f=5$ flavors of chiral superfields Q^i, $\bar{Q}_j$ ($i,j=1\ldots5$, corresponding to u,d,s,c,b) in the fundamental and antifundamental representations. The tree-level superpotential is

$$W_{\text{tree}} = \text{Tr}(\hat{m}\, M)\,, \qquad \hat{m} = \text{diag}(m_u,\, m_d,\, m_s,\, m_c,\, m_b)\,,$$

where $M^i{}_j = Q^i \cdot \bar{Q}_j$ is the 5×5 composite meson superfield. Since $m_c, m_b \gg \Lambda_{\text{QCD}}$, the charm and bottom quarks are integrated out at tree level, reducing the effective theory to $N_f=N_c=3$ light flavors (u,d,s). In this regime, Seiberg's exact results give confinement with the composite meson $M^{(3)}$, baryons B, $\bar{B}$, and the quantum-modified constraint (51):

$$\det M^{(3)} - B\bar{B} = \Lambda_{\text{eff}}^6\,,$$

where Λ_{eff} is the dynamical scale of the effective three-flavor theory after threshold matching at m_c and m_b. Implementing this constraint via a Lagrange multiplier X gives the full effective superpotential

$$W_{\text{eff}} = \text{Tr}(\hat{m}^{(3)}\, M^{(3)}) + X\big(\det M^{(3)} - B\bar{B} - \Lambda_{\text{eff}}^6\big)\,.$$

The F-term equations for the 3×3 light-flavor block read

$$\begin{aligned}
\frac{\partial W_{\text{eff}}}{\partial M^i{}_j} &= \hat{m}_j\, \delta^i{}_j + X\,(\det M^{(3)})\,(M^{(3)\,-1})_i{}^j = 0\,,\\
\frac{\partial W_{\text{eff}}}{\partial B} &= -X\,\bar{B} = 0\,,\\
\frac{\partial W_{\text{eff}}}{\partial \bar{B}} &= -X\,B = 0\,.
\end{aligned}$$

The baryon equations force either $X=0$ or $B=\bar{B}=0$. If $X=0$ the meson equation reduces to $\hat{m}_j\,\delta^i{}_j = 0$, which has no solution for nonzero quark masses. Hence $B=\bar{B}=0$ and the constraint becomes $\det M^{(3)} = \Lambda_{\text{eff}}^6$. The diagonal meson equation then requires

$$\hat{m}_j + X\,\frac{\det M^{(3)}}{M^j{}_j} = 0 \qquad (\text{no sum on } j)\,,$$

which determines each diagonal entry of $M^{(3)}$ in terms of the quark masses and Λ_{eff}. For a seed configuration in which one quark mass vanishes—say $m_u \to 0$—the corresponding F-term $F_{M^1{}_1} = X\,(\det M^{(3)})(M^{(3)\,-1})_1{}^1$ remains nonzero so long as $\det M^{(3)} \neq 0$, which is guaranteed by the quantum constraint. Supersymmetry is therefore spontaneously broken, with order parameter

$$\langle F\rangle \;\sim\; f_\pi^2\,.$$

The scalar and fermionic components of $M^i{}_j$ acquire different masses through this breaking. The scalar component (pseudoscalar meson $q_i\bar{q}_j$) picks up an F-term contribution absent for the fermionic component (lepton, via the Miyazawa superalgebra):

$$\begin{aligned}
m^2_{M^i{}_j}\big|_{\text{scalar}} &= |m_i + m_j|^2 + f^2\,,\\
m^2_{M^i{}_j}\big|_{\text{fermion}} &= |m_i + m_j|^2\,,
\end{aligned}$$

where $f^2 \approx f_\pi^2$. The individual masses $|m_i + m_j|$ are the tree-level (perturbative) contributions; the physical masses are dominated by nonperturbative QCD effects that shift both meson and lepton by the same amount. The mass *splitting*, however, is universal across the multiplet:

$$m^2_{\text{meson}} - m^2_{\text{lepton}} = f^2 \approx f_\pi^2\,,$$

because f^2 is a SUSY-breaking contribution independent of the QCD dynamics. This is precisely the Volkov–Akulov relation (1). For heavier mesons the actual splitting is strongly flavor-dependent (§14, "kaon–muon puzzle").

The top quark, with $m_t \gg \Lambda_{\text{QCD}}$, does not hadronize and therefore does not participate in the confined meson matrix—the counting $N_f = 5$ is exact. Its mass, and corrections to the heavy-quark sector (c, b), originate instead from electroweak symmetry breaking. In $\mathcal{N} = 1$ SUSY, holomorphy of the superpotential requires separate Higgs doublets H_u and H_d for up-type and down-type quarks, giving

$$W_{\text{Yukawa}} = y^u_{ij}\, H_u\, Q^i\, \bar{Q}_j + y^d_{ij}\, H_d\, Q^i\, \bar{Q}_j\,,$$

with $\langle H_u \rangle = v \sin\beta/\sqrt{2}$ and $\langle H_d \rangle = v\cos\beta/\sqrt{2}$ ($v = 246\,\text{GeV}$, $\tan\beta = v_u/v_d$). This is relevant only for quarks whose masses are dominated by the electroweak condensate rather than the chiral condensate. For the light sector (u, d, s) the Yukawa contribution is negligible compared to $\text{Tr}(\hat{m}\, M)$. Note that $\hat{m}$ in the chiral mass term already *is* the current quark mass generated by the Higgs mechanism; the two terms are not additive. The Yukawa superpotential is written separately to make explicit the dependence on the Higgs doublets, which are needed for the heavy sector where $m_i \gg \Lambda_{\text{QCD}}$ and the mass cannot be attributed to chiral symmetry breaking.

The two-doublet structure has a direct consequence for the Koide triples. Since the quark Koide triples obligatorily mix up-type and down-type quarks (§**??**), the translation from masses to Yukawa couplings depends on $\tan\beta$. For the (c, b, t) triple, with c and t receiving mass from H_u and b from H_d:

$$Q(m_c, m_b, m_t) = Q(y_c \sin\beta,\ y_b\cos\beta,\ y_t\sin\beta)\,,$$

which equals $Q(y_c, y_b, y_t)$ only when $\sin\beta = \cos\beta$, i.e., $\tan\beta = 1$. The same conclusion holds for the $(-s, c, b)$ triple, where s and b come from H_d and c from H_u. Thus $\tan\beta = 1$ is the unique value at which the Koide condition on *masses* is equivalent to a Koide condition on *Yukawa couplings* for mixed up/down-type triples.

This prediction is independently motivated by the sBootstrap's $SU(5)_f$ flavor symmetry, which treats all five quarks democratically. A ratio $\tan\beta \neq 1$ would distinguish up-type from down-type quarks in the vacuum, breaking $SU(5)_f$ at the electroweak scale in a way not present in the chiral sector. Equal Higgs VEVs, $v_u = v_d = v/\sqrt{2} = 174\,\text{GeV}$, preserve the democratic structure. At $\tan\beta = 1$, the top Yukawa is $y_t = \sqrt{2}\, m_t/v_u = 2m_t/v \approx 1.40 \approx \sqrt{2}$.

The Koide condition on the (c, b, t) triple constrains the diagonal Yukawa matrix $Y = \text{diag}(y_c, y_b, y_t)$. At $\tan\beta = 1$, all Yukawas are $y_i = 2m_i/v$ regardless of type, and the Koide relation reads

$$\frac{y_c + y_b + y_t}{(\sqrt{y_c} + \sqrt{y_b} + \sqrt{y_t})^2} = \frac{2}{3}\,,$$

which is equivalent to writing the square roots of the Yukawa couplings as

$$\sqrt{y_k} = \sqrt{y_0}\,\big(1 + \sqrt{2}\,\cos(2\pi k/3 + \delta)\big)\,, \qquad k = 0, 1, 2\,,$$

where $y_0 = \left(\sum\sqrt{y_k}\right)^2/9$ and δ is the Koide phase. For the physical (c, b, t) masses, $\delta \approx 0.069$ and $Q = 0.6695$ (a 0.42% deviation from 2/3). The traceless part of $\sqrt{Y}$ lies in the Cartan subalgebra of $SU(3)_f$:

$$\sqrt{Y} = \sqrt{y_0}\left(I_3 + \tfrac{\sqrt{6}}{2}\,\hat{n}\cdot\boldsymbol{\lambda}_{\text{diag}}\right),$$

where $\boldsymbol{\lambda}_{\text{diag}} = (\lambda_3, \lambda_8)$ are the diagonal Gell-Mann matrices and $\hat{n} = (\cos\theta, \sin\theta)$ is a unit vector in the Cartan plane. An exact trigonometric identity gives $\theta = \pi/6 - \delta$, so the Koide phase is literally the angle of the Yukawa matrix in flavor space. The Koide condition fixes the amplitude at $\sqrt{6}/2$; only $\hat{n}$ (equivalently δ) is free. The bloom from seed to physical triple is a rigid rotation in the (λ_3, λ_8) plane by $-\Delta\delta$.

The total Lagrangian is

$$\mathcal{L} = \int d^4\theta\, K + \left[\int d^2\theta\, W + \text{h.c.}\right] + \mathcal{L}_D + \mathcal{L}_{\text{soft}}\,,$$

with Kähler potential (canonical plus stabilization and bion corrections),

$$K = \underbrace{\mathrm{Tr}(M^\dagger M) + |X|^2 + |B|^2 + |\bar{B}|^2 + |H_u|^2 + |H_d|^2 + |S|^2}_{K_{\text{can}}} \underbrace{-\ \frac{|X|^4}{12\,\mu^2}}_{K_{\text{stab}}} + \underbrace{\frac{\zeta^2}{\Lambda^2}\left|\sum_{k=1}^{N_c} s_k\sqrt{\hat{m}_k}\right|^2}_{K_{\text{bion}}},$$

The stabilization term $K_{\text{stab}} = -|X|^4/(12\mu^2)$ is the negative quartic Kähler correction from §11.12 (with $\Lambda_K = \mu$, $c = -1/12$); it pins the pseudo-modulus at $\langle X\rangle = \sqrt{3}\,\mu$ via the Kähler pole. The monopole fugacity is $\zeta \propto e^{-S_0/(2N_c)}$, $\hat{m}_k$ are the current quark masses (the couplings in $W \ni \mathrm{Tr}(\hat{m}\,M)$), and $s_k = \pm 1$ are monopole-instanton signs (encoding the seed-to-bloom phase). The bion term is non-holomorphic (containing $\sqrt{\hat{m}}$) and non-perturbatively suppressed by $e^{-2S_0/3} \sim 10^{-2}$ at the confinement scale. Its minimum enforces the v_0-doubling condition $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ (§11.3); superpotential combining mass, constraint, and Yukawa terms,

$$W = \underbrace{\mathrm{Tr}(\hat{m}\,M)}_{\text{chiral mass}} + \underbrace{X\left(\det M^{(3)} - B\bar{B} - \Lambda_{\text{eff}}^6\right)}_{\text{Seiberg constraint}} + \underbrace{c_3\,\frac{(\det M^{(3)})^3}{\Lambda_{\text{eff}}^{18}}}_{\text{3-instanton}} + \underbrace{y_c\,H_u\,{M^c}_c + y_b\,H_d\,{M^b}_b + y_t\,H_u\,Q^t\bar{Q}_t}_{\text{Yukawa}} + \lambda$$

where $M^{(3)}$ is the 3×3 light-flavor block (u, d, s) and Λ_{eff} is the dynamical scale of the confined $N_f = N_c = 3$ theory (including threshold matching at m_c and m_b). The Yukawa terms are written in the full five-flavor theory above the heavy-quark thresholds, where ${M^c}_c = Q^c\bar{Q}_c$ and ${M^b}_b = Q^b\bar{Q}_b$ are elementary bilinears; below threshold, they reduce to the matching conditions that generate Λ_{eff} and the Seiberg constraint. The top Yukawa $y_t\,H_u\,Q^t\bar{Q}_t$ stands apart because the top does not confine into a meson. At $\tan\beta = 1$, the tree-level lightest Higgs mass vanishes ($m_h = m_Z|\cos 2\beta| = 0$); the NMSSM coupling $\lambda_S\,S\,H_u{\cdot}H_d$ lifts the flat direction and gives $m_h = \lambda_S v/\sqrt{2} = 125\,\text{GeV}$ for $\lambda_S = 0.72$. The singlet S is a *separate* elementary field, distinct from the Seiberg Lagrange multiplier X (§**??**). The constraint $\tan\beta = 1$ ($v_u = v_d = v/\sqrt{2}$) is imposed by the Koide condition on mixed triples (§11.12). The D-term Lagrangian includes an optional Fayet–Iliopoulos parameter ξ for $U(1)_{\text{em}}$:

$$\mathcal{L}_D = -\frac{e^2}{2}\left(\sum_i Q_i\,|\phi_i|^2 + \xi\right)^2.$$

Finally, soft SUSY-breaking terms arise from a spurion $\langle F_S \rangle = f_\pi^2$:

$$\mathcal{L}_{\text{soft}} = -\tilde{m}^2\,\mathrm{Tr}(M^\dagger M) - \big(B_\mu\,\mathrm{Tr}(\hat{m}\,M) + A_y\big(y_c\,H_u\,M^c{}_c + y_b\,H_d\,M^b{}_b\big) + \text{h.c.}\big)\,, \qquad \tilde{m}^2 \sim f_\pi^2\,.$$

The soft scalar mass $\tilde{m}^2 \sim f_\pi^2 \approx (92\,\text{MeV})^2$ generates the universal meson–lepton splitting $m^2_{\text{meson}} - m^2_{\text{lepton}} = f_\pi^2$.

Scope of the Lagrangian. This Lagrangian produces: (i) the seed Koide condition ($gv/m = \sqrt{3}$ in the O'Raifeartaigh limit); (ii) the meson–lepton mass splitting (f_π^2); (iii) the v_0-doubling condition via the bion Kähler correction K_{bion}, whose minimum enforces $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$; (iv) the Yukawa couplings for the heavy sector; (v) the $\cos(9\delta)$ potential that selects the lepton Koide phase, via the three-instanton operator $(\det M^{(3)})^3/\Lambda_{\text{eff}}^{18}$. The bloom is non-holomorphic and lives in the Kähler sector: the bion term $|\sum s_k\sqrt{m_k}|^2$ is the leading monopole-instanton contribution on $\mathbb{R}^3 \times S^1$ (§11.3).

Flavour-universal Yukawa identity. At the seesaw vacuum $M_j = C/m_j$, the product of the Yukawa coupling times the meson VEV is flavour-independent:

$$y_j\,\langle M_j \rangle = \frac{m_j}{v_{\text{type}}}\,\frac{C}{m_j} = \frac{C}{v_{\text{type}}} = \frac{\sqrt{2}\,C}{v} = 5.07\ \text{MeV}\,,$$

where $C = \Lambda^2(m_u m_d m_s)^{1/3}$. The quark mass cancels exactly: heavy quarks couple weakly (small y_j) but have large meson VEVs (large M_j), and vice versa. This identity ensures that the Higgs F-terms from the meson sector are flavour-blind, automatically satisfying the Glashow–Weinberg condition for the absence of tree-level Higgs-mediated FCNCs.

Three Yukawa sectors. The six quarks divide into three sectors with distinct coupling structures:

Sector	Quarks	Mechanism	y range
Confined	u, d, s	$\mathrm{Tr}(\hat{m}\,M)$: mass term is Yukawa	10^{-5}–5×10^{-4}
Intermediate	c, b	Explicit $y_j H M^j{}_j$	7×10^{-3}–2.4×10^{-2}
Elementary	t	$y_t H_u Q^t \bar{Q}_t$	$0.99 \approx 1$

At $\tan\beta = 1$, both Yukawa matrices $Y^u = \mathrm{diag}(y_u, y_c, y_t)$ and $Y^d = \mathrm{diag}(y_d, y_s, y_b)$ are diagonal at tree level; CKM mixing must originate in the UV quark mass texture rather than in confined-phase meson dynamics (§**??**).

Particle spectrum and supertrace. The Lagrangian defines a 16×16 fermion mass matrix and a 32×32 scalar mass-squared matrix at the Seiberg vacuum. The fermion spectrum spans twenty orders of magnitude, from the ultra-heavy X–$M^s{}_s$ pair at $\sim \Lambda^6/(M_u M_d) \sim 8\times10^{10}\,\text{MeV}$ down to baryonic fermions at $\sim |X_0| \sim 10^{-9}\,\text{MeV}$. The off-diagonal meson fermion masses follow the inverted seesaw hierarchy $|X_0|\langle M_k\rangle \propto 1/m_k$, transmitting the quark mass ratios to the confined spectrum. The supertrace is $\mathrm{STr}[M^2] = 2\,\mathrm{Tr}(M^2_{\text{soft}}) = 18\,f_\pi^2 = 152{,}352\,\text{MeV}^2$: the W_{IJ}^2 piece cancels between scalar and fermion sectors, and the holomorphic third-derivative piece cancels between real and imaginary scalar components, leaving only the nine soft meson masses. Decomposed by electromagnetic charge, the supertrace becomes $\mathrm{STr}_Q[M^2] = 2\,n_Q\,f_\pi^2$ where n_Q counts meson fields with charge Q: $n_0 = 5$, $n_{\pm1} = 2$. The EM-weighted sum $\sum Q^2_{\text{em}}\,\mathrm{STr}_Q = 8\,f_\pi^2$ is nonzero because only mesons carry soft mass; this is the electromagnetic trace that connects to the FI parameter (§??). The Yukawa-induced F-terms ($F_{M^d{}_d} = y_c v/\sqrt{2}$, $F_{M^s{}_s} = y_b v/\sqrt{2}$) are nonzero at the seesaw vacuum but do not couple to the off-diagonal meson directions: $\partial F_{M^d{}_d}/\partial M^u{}_s\big|_{\text{diag}} = 0$ because the off-diagonal cofactors vanish at a diagonal matrix. All six off-diagonal meson masses are positive ($m^2 \approx 2f_\pi^2 = 16{,}928\ \text{MeV}^2$), and one-loop Coleman–Weinberg corrections are stabilising ($\delta m^2_{\text{CW}} \sim 10^{-9}\, m^2_{\text{tree}}$).

Mass spectrum from the Lagrangian. The mass of each particle is determined by a specific term:

Mass	Origin	Equation
$m^2_{\text{meson}} - m^2_{\text{lepton}} = f_\pi^2$	$\mathcal{L}_{\text{soft}}$: $\tilde{m}^2 \sim f_\pi^2$	Universal splitting
$m_c/m_s = (2+\sqrt{3})^2$	W: O'Raifeartaigh at $gv/m = \sqrt{3}$	Seed Koide
$\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$	K_{bion}: minimum of $\lvert S_{\text{bloom}} - 2S_{\text{seed}}\rvert^2$	v_0-doubling
$m_t = y_t v_u/\sqrt{2}$, $\tan\beta = 1$	W_{Yukawa}: $y_t H_u Q^t \bar{Q}_t$	External
$\delta m^2 = Q_{\text{em}}\,\xi$	$\mathcal{L}_D$: FI term	EM splitting
$\delta_\ell \bmod 2\pi/3 = 2/9$	W: $(\det M^{(3)})^3/\Lambda^{18}$	3-instanton

The first four rows account for six quark masses (m_u and m_d free, m_s sets the scale, m_c from seed, m_b from Kähler, m_t external). The meson–lepton splitting is universal across the multiplet, and the D-term provides the subdominant electromagnetic correction.

The parameters of this construction separate into those fixed by structure and those that remain free.

Fixed by the Koide seed. The O'Raifeartaigh condition $gv/m = \sqrt{3}$ translates, in the ISS magnetic dual, into $\langle X\rangle = \sqrt{3}\,\mu$ for the pseudo-modulus. This fixes the mass ratio $m_c/m_s = (2+\sqrt{3})^2$ within the seed triple, predicting $m_c = 1{,}301\,\text{MeV}$ from m_s alone (PDG: $1{,}270\,\text{MeV}$, 2.4% deviation).

Fixed by the vacuum-charge doubling. The bloom from seed to full triple doubles v_0, giving $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$ and predicting $m_b = 4{,}177\,\text{MeV}$ (0.1σ from PDG). This constraint is *non-holomorphic*: it is a linear relation among square roots of masses, while the superpotential is holomorphic in meson fields whose VEVs scale as $1/m_i$. The map $\mathfrak{z}_k = \sqrt{m_k} \sim 1/\sqrt{\langle M^k{}_k\rangle}$ is non-holomorphic. The v_0-doubling must therefore arise from the Kähler potential, nonperturbative effects, or emergent dynamics—it is not a tree-level superpotential term.

Fixed by SUSY breaking. The supertrace relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ fixes the splitting scale. At the Seiberg vacuum, $F_X = 0$ (SUSY is unbroken in the confined sector), but the Yukawa couplings produce $F_{H_u} = y_c M^c{}_c$ and $F_{H_d} = y_b M^b{}_b$. By the seesaw $y_j M_j = (m_j/v)(C/m_j) = C/v$, these F-terms are flavour-universal: $F_{H_u} = F_{H_d} = C/v_{\text{EW}} = 290\,\text{MeV}$. The identification $\langle F\rangle \sim f_\pi^2$ links Λ_{QCD} to m_s via the GOR relation.

***D*-term effects on the Koide condition.** In $\mathcal{N} = 1$ SUSY, D-terms shift scalar masses but leave fermion masses untouched. The lepton Koide triples, being conditions on *fermion* masses, are naturally protected against D-term perturbations. The meson Koide triple $(-\pi, D_s, B)$ involves scalar masses and *is* in principle affected by D-terms: a FI parameter ξ shifts $m^2 \to m^2 + \xi Q^2_{\text{em}}$. Since all three mesons carry unit charge, each

m^2 shifts by the same ξ, and the relative correction to Q is of order $\xi/(4m_\pi^2) \sim 0.1\%$ for $\xi \approx -80\,\mathrm{MeV}^2$. Numerically, the D-term shifts the meson Q by 0.03%, well below the 0.10% precision of the triple.

Remaining free parameters. After the seed Koide fixes m_c/m_s, the v_0-doubling fixes m_b, and $\langle X\rangle/\mu = \sqrt{3}$ is imposed, only two to three free parameters remain:

- m_u, m_d: unconstrained by any Koide triple; connected to the Cabibbo angle through the Gatto–Sartori–Tonin relation $|V_{us}| \approx \sqrt{m_d/m_s}$ (giving $V_{us} = 0.224$, PDG: 0.2243);
- m_s (or equivalently Λ_{QCD}): the overall scale of the light sector. The lepton Koide scale $M_0 \approx f_\pi^2/27$ (§3) ties the lepton scale to f_π and hence to Λ_{QCD}.

The top mass m_t is external—it does not participate in the confined meson matrix—but if $Q(c,b,t) = 2/3$ is imposed as a constraint rather than treated as a test, m_t is predicted from m_s alone. The Yukawa couplings $y_c = \sqrt{2}\,m_c/v_{\mathrm{EW}}$, $y_b = \sqrt{2}\,m_b/v_{\mathrm{EW}}$ are fixed by the predicted masses.

The (c,b,t) Koide triple provides an *overdetermination test.* Using the self-consistent predictions $m_c^{\mathrm{seed}} = 1{,}301\,\mathrm{MeV}$ and $m_b^{v_0} = 4{,}233\,\mathrm{MeV}$ (from the seed and v_0-doubling with input m_s only), together with the external m_t: $Q(c,b,t) = 0.668$, a 0.14% deviation from 2/3—closer than the 0.42% obtained with PDG masses. The self-consistent predictions satisfy the Koide conditions better than the measured values. If instead both overlapping triples are required to satisfy $Q = 2/3$ exactly, the overlap system determines $m_b^{\mathrm{pred}} = 4{,}159\,\mathrm{MeV}$ (0.5% from PDG) and $m_c^{\mathrm{pred}} = 1{,}369\,\mathrm{MeV}$ (7.8% from PDG). The bion Kähler prediction $m_b = 4{,}177\,\mathrm{MeV}$ (0.07%, 0.06σ) is more precise, suggesting that v_0-doubling rather than exact $Q = 2/3$ on both triples is the tighter constraint.

Conversely, imposing $Q(c,b,t) = 2/3$ exactly on the self-consistent m_c^{seed} and $m_b^{v_0}$ predicts $m_t = 171.3 \pm 1.5\,\mathrm{GeV}$ (where the uncertainty comes from $\delta m_s = 0.8\,\mathrm{MeV}$), within 0.9% of the PDG pole mass $m_t = 172.76 \pm 0.30\,\mathrm{GeV}$. The entire third-generation mass spectrum then follows from a single input m_s; using $m_s = 94.2\,\mathrm{MeV}$ (within 1σ of PDG) gives m_t exactly.

Summary of predictions. Table 6 collects the quantitative predictions.

Quantity	Predicted	PDG 2024	Source
m_c	$1{,}301\,\mathrm{MeV}$	$1{,}270 \pm 20$ (1.6σ)	Seed: $gv/m = \sqrt{3}$
m_b	$4{,}177\,\mathrm{MeV}$	$4{,}180 \pm 30$ (0.1σ)	Bion Kähler
m_t	$171.3 \pm 1.5\,\mathrm{GeV}$	172.76 ± 0.30	$Q(c,b,t) = 2/3$
$\sin^2\theta_W$	0.22310	0.22320 ± 0.00026 (0.4σ)	Casimir equation
V_{us}	0.224	0.2243 ± 0.0008 (0.9σ)	GST relation
M_0 (lepton)	$f_\pi^2/27 = 314.0\,\mathrm{MeV}$	$313.8\,\mathrm{MeV}$ (fitted)	Scale relation
m_h	$125.4\,\mathrm{GeV}$	125.25 ± 0.17 (0.6σ)	NMSSM: $\lambda v/\sqrt{2}$
$\tan\beta$	1	— (not directly measured)	Mixed-triple Koide
$\mathrm{STr}[M^2]$	$18\,f_\pi^2$	— (not directly measured)	Soft mass only

Table 6: Predictions from the sBootstrap Lagrangian. The quark masses use $m_s = 93.4\,\mathrm{MeV}$ as input; the bion m_b uses PDG m_c. The Casimir and GST predictions are independent of m_s.

12 The Electron Mass and the Lepton Seed

The charged lepton triple (e, μ, τ) is the "almost-seed": $\mathfrak{z}_e = 0.715\,\mathrm{MeV}^{1/2}$ is small but nonzero. The electron's charge is near zero because of cancellation in $\mathfrak{z}_e = z_0 + z_e$:

$$v_0 = 17.716, \quad z_e = -17.001, \quad \mathfrak{z}_e = 17.716 - 17.001 = 0.715\,\mathrm{MeV}^{1/2}\,. \tag{72}$$

A 96% cancellation. The mass $m_e = 0.715^2 = 0.511\,\mathrm{MeV}$ is the square of something already small.

The near-vanishing of m_e is a structural property of the Koide embedding, not fine-tuning: the energy balance allows perturbations as large as z_0. Every Koide triple has one member with $|\mathfrak{z}|/v_0$ small:

Triple	Lightest	$\lvert\mathfrak{z}\rvert/v_0$	Character
(e, μ, τ)	e	0.040	Almost-seed
$(-\pi, D_s, B)$	π (neg.)	0.337	Full triple
(c, b, t)	c	0.236	Full triple
$(-s, c, b)$	s (neg.)	0.320	Full triple

The electron is by far the most extreme case, consistent with the lepton triple being nearest to its seed.

13 Symmetry Restoration

The broken spectrum reassembles as symmetry-breaking effects are removed:

1. **Turn off QED:** Electromagnetic splittings vanish ($\pi^\pm = \pi^0$, $K^\pm = K^0$). D-term effects disappear.

2. **Set $m_u = m_d$:** Exact isospin.

3. **Set $m_s = m_d$:** $SU(3)_V$ restored, $\pi = K = \eta$. Koide splitting $v_1 \to 0$; lepton partners degenerate.

4. **Send $m_q \to 0$:** Chiral limit, all meson charges $\to 0$. If sBootstrap SUSY restores, all light lepton masses vanish too. This is the seed configuration: $(0, 0, 0)$.

The seed triples are the "penultimate" stage: one member has reached zero, but the other two retain the $2-\sqrt{3}$ ratio. Full triples are the final broken state. The lepton triple, trapped between seed and bloom, reveals the hierarchy: chiral breaking $\to$ seed $\to$ bloom $\to$ full spectrum.

The direction of the flow also matters: approaching the chiral limit, the pion charge rises from $\mathfrak{z} = -11.81$ toward zero from below. It crosses zero exactly in the chiral limit. This means the pion was "born negative"—its charge sign is set by the direction of chiral symmetry breaking, and the seed $(0, \pi, D_s)$ is the configuration at the moment of crossing.

14 Open Problems

14.1 The diquark sector

The Pauli theorem (§9.3) proves that no $J = 0$, color-$\bar{3}$ diquark can have symmetric flavor. The argument extends to all partial waves: $J = 0$ forces $L = S$, so the spatial–spin exchange sign is $(-1)^{L+S} = (-1)^{2S} = +1$, which combined with the antisymmetric color $\bar{3}$ gives overall $+1$; Pauli then requires antisymmetric flavor ($\overline{\mathbf{10}}$). The $\mathbf{15}$ is accessible only for $J \geq 1$. In SQCD, the same conclusion holds algebraically: the ε-tensor on color indices in $B^{abc} = \varepsilon_{ijk} Q^{ia} Q^{jb} Q^{kc}$ forces antisymmetric flavor regardless of statistics. The sBootstrap's $\mathbf{15}$ diquark must therefore be non-composite: either an algebraic image of the Miyazawa supercharge (§14.5) or a fundamental $SO(32)$ string state. However, the mixed anticommutator $\{Q_\alpha, \mathcal{Q}_{[ij],\beta}\} = \epsilon_{\alpha\beta}\, \mathcal{S}_{[ij]}$ produces a diquark generator in $\overline{\mathbf{10}}$ (antisymmetric), not the symmetric $\mathbf{15}$; thus the Miyazawa route alone does not resolve the $\mathbf{15}$ vs. $\overline{\mathbf{10}}$ conflict. The exotic $+4/3$-charged diquarks of the $\mathbf{15}$ would be a testable consequence: if their mass is set by the SUSY-breaking scale ($\sim f_\pi$), they would be sub-GeV and accessible at fixed-target experiments; if by Λ_{QCD} or higher, LHC pair-production searches for exotic charges apply. No mass prediction can be made without a concrete realization of the $\mathbf{15}$.

14.2 What determines the Koide phase?

The energy balance constrains but does not uniquely fix δ. The O'Raifeartaigh realization (§11.3) fixes the seed ratio $gv/m = \sqrt{3}$ but leaves δ undetermined—the phase controls how the seed blooms into a full triple. The bloom is a phase rotation $\delta : 3\pi/4 \to \delta_{\text{phys}}$ at fixed v_0, but explicit R-breaking perturbations push Q away from $2/3$ at any nonzero strength, making the seed an unstable fixed point. The bloom must therefore be nonperturbative.

The lepton phase satisfies $\delta_\ell \bmod 2\pi/3 = 2/9$ to 33 ppm (0.9σ in m_τ; 3.1σ after look-elsewhere over fractions with denominator ≤ 20). Since $2/9$ is not a rational multiple of π, this is not a geometric angle. It is, however, $2/9 = 2/(3 \times 3)$, which points to a $\mathbb{Z}_9$ discrete symmetry in the Koide modulus space. The $\mathbb{Z}_3$ mass-permutation symmetry ($\delta \to \delta + 2\pi/3$) is the unique subgroup; the quotient $\mathbb{Z}_9/\mathbb{Z}_3$ has three cosets, producing three inequivalent mass spectra from the same M_0. The simplest potential with $\mathbb{Z}_9$ symmetry, $V \supset -\cos\big(9(\delta - 2/9)\big)$, has the lepton value as one of its nine minima, with the $\mathbb{Z}_3$-equivalent minima at $\delta = 2/9 + 2k\pi/3$.

A dynamical origin for $\mathbb{Z}_9$ exists in SQCD: for $N_f = N_c = 3$, the diagonal $U(1)_{R+2A}$ (R-symmetry plus twice the axial symmetry) has anomaly coefficient $6N_c = 18 = 2 \times 9$. Instantons break $U(1)_{R+2A} \to \mathbb{Z}_{18}$, which contains $\mathbb{Z}_9$ as a subgroup. The meson field M carries $\mathbb{Z}_{18}$ charge 4 (from two quark zero modes at charge 2 each), so $\det M$ carries charge 12. The lowest-order holomorphic $\mathbb{Z}_{18}$-singlet beyond the Seiberg constraint is the three-instanton operator

$$\delta W_{\text{3-inst}} = c_3\, \frac{(\det M)^3}{\Lambda^{18}}\,, \tag{73}$$

since $(\det M)^3$ has $\mathbb{Z}_{18}$ charge $36 = 2 \times 18$. Near the SUSY vacuum $\det M = \Lambda^6$, parametrizing $\det M = \Lambda^6\, e^{3i\delta_M}$ yields $\delta W \propto e^{9i\delta}$, and the cross-term with the tree-level superpotential gives $V_{\text{eff}} \supset -V_0 \cos\big(9(\delta - \delta_0)\big)$ with $\delta_0 = \arg(c_3)/9$. This is exactly the $\cos(9\delta)$ potential identified above. The one-instanton vertex ($\det M$, charge 12) is already captured by the Seiberg constraint; the two-instanton vertex ($(\det M)^2$, charge 24, not a $\mathbb{Z}_{18}$ singlet) is forbidden. The three-instanton correction does not affect the meson mass spectrum: the F-term equations define an effective Lagrange multiplier $X_{\text{eff}} = X + 3c_3\Lambda^{12}/\Lambda^{18}$, but the constraint $\det M = \Lambda^6$ and the meson VEVs $M_j = C/m_j$ are unmodified. The correction acts only on the phase structure (through $\cos(9\delta)$), not on the magnitudes.

At strong coupling ($\Lambda \sim 300\,\text{MeV}$), the coefficient c_3 is $O(1)$ and the potential strength is $V_0^{1/4} \sim 65$–$85\,\text{MeV}$, comparable to $f_\pi = 92\,\text{MeV}$—this is not a parametrically suppressed effect. The same $\cos(9\delta)$ structure also arises from magnetic bion contributions on $\mathbb{R}^3 \times S^1$, where the $\mathbb{Z}_3$-symmetric combination of the SU(3) affine roots maps to $\cos(9\delta)$ in the meson phase variable.

The embedding is specific to $N_c = 3$: the condition $N_c^2 \mid 6N_c$ requires $N_c \mid 6$, which holds only for $N_c = 1, 2, 3, 6$.

The quark Koide phases, by contrast, do *not* sit at $\mathbb{Z}_9$ minima: the $(-s, c, b)$ and (c, b, t) phases are displaced by 39% and 22% of the $\mathbb{Z}_9$ spacing, respectively. This suggests that the lepton and quark phases are selected by different dynamical mechanisms—the instanton potential for leptons, and possibly the bloom mechanism or ISS dynamics for quarks.

The Coleman–Weinberg potential of the minimal O'Raifeartaigh model is flat in δ at one loop—the pseudo-modulus potential depends only on the modulus $|\Phi_0|$, not on the Koide phase. The three-instanton potential above provides the leading δ-dependent term that lifts this flat direction. The *modulus* direction $|\Phi_0|$ is a separate question: the CW potential stabilizes $|\langle\Phi_0\rangle|$ at zero, but the Koide seed requires $\langle X\rangle/\mu = \sqrt{3}$ (§11.12). A negative quartic Kähler correction $c\,|X|^4/\Lambda_K^2$ with $c = -\Lambda_K^2/(12\mu^2)$ pins the pseudo-modulus at the Kähler pole $v_{\text{pole}} = \sqrt{3}\,\mu$ exactly (an algebraic result, not numerical tuning; see §11.12). This coefficient is *not* radiatively generated: the one-loop CW quartic in the O'Raifeartaigh model has $c_4 = -20/3$ at $gf/m^2 = 1$ (exact), which is orders of magnitude too large; the ISS one-loop potential produces a positive quartic ($c_{\text{eff}} = +0.014$), the wrong sign entirely. The Kähler pole mechanism is therefore a tree-level statement about the effective Kähler potential, not a consequence of one-loop dynamics. However, magnetic bions on $\mathbb{R}^3 \times S^1$ generate a quartic Kähler correction of the correct sign. The bion-induced coefficient $c_{\text{bion}} = -3\pi^2\, e^{-4\pi/(N_c^2\alpha_s)}\, M^2/\alpha_s^2$ (where $M^2 = [\prod_f (m_f/\Lambda)]^2$ is the zero-mode mass factor) equals $-1/12$ at $\alpha_s = 0.143$ ($S_{\text{bion}} = 9.76$) in the ISS model with heavy magnetic quarks ($M^2 = 1$). With physical light-quark masses, zero-mode suppression reduces $|c_{\text{bion}}|$ by five orders of magnitude; the ISS magnetic dual—where the pseudo-modulus lives—is the appropriate setting for the bion calculation. The three-instanton operator $(\det M)^3$ shifts the Seiberg Lagrange multiplier X (dimension $[\text{mass}]^{-3}$ in the confining phase) but this is a different object from the ISS pseudo-modulus (dimension $[\text{mass}]^{+1}$ in the magnetic dual).

More broadly, the ISS Coleman–Weinberg potential does *not* transmit the Koide condition from the UV quark masses to the IR meson spectrum. In the $N_f = 4$ ISS model, the one-loop CW masses for the broken mesons χ_a are nearly degenerate ($m_{\text{CW}} \approx 27\,\text{MeV}$ for all four flavors), because the CW mass depends on m_a through the affine map $F_a = h\mu^2 - m_a$, and all quark masses satisfy $m_a \ll h\mu^2$. The resulting spectrum gives $Q \approx 1/3$ (the degenerate limit), regardless of the UV value. This confirms that flavor-sensitive mass structure must arise from non-perturbative effects—the bion Kähler correction (§11.3), which directly involves $\sqrt{m_k}$, rather than the one-loop effective potential.

14.3 Connecting the three tuples

The unified superpotential (§11.12) connects the light and heavy sectors through a layered structure: the seed Koide fixes m_c/m_s (from $gv/m = \sqrt{3}$), the v_0-doubling fixes m_b (from the Kähler sector), and the (c, b, t) Koide serves as an overdetermination test. After these constraints, only two to three free parameters remain: m_u, m_d (connected to the Cabibbo angle), and the overall scale m_s. The Koide condition does *not* extend to triples containing m_u or m_d: $Q(m_u, m_d, m_s) = 0.567$ (-15%) and $Q(m_u, m_c, m_t) = 0.849$ ($+27\%$), confirming that the lightest quarks are free parameters of the framework. The v_0-doubling is non-holomorphic and lives in the Kähler sector: as shown in §11.3, the bion Kähler correction $K_{\text{bion}} \sim |\sum s_k \sqrt{\hat{m}_k}|^2$ generates an effective potential whose minimum enforces $\sqrt{m_b} = 3\sqrt{m_s} + \sqrt{m_c}$. The Hessian has rank 1, constraining exactly the v_0-doubling combination while leaving m_s and m_c to the Koide seed. The extension from $\mathbb{R}^3 \times S^1$ to $\mathbb{R}^4$ relies on Ünsal's continuity conjecture.

14.4 Lepton masses are not mesino masses

The fermion mass matrix W_{IJ} at the Seiberg vacuum has light eigenvalues ($\sim 10^{-3}$–10^{-2} MeV) from the seesaw between the meson and Lagrange-multiplier sectors. These are far too small and too degenerate ($Q_{\text{light}} = 0.48$, not 2/3) to reproduce lepton masses. The bion-induced Kähler correction $\delta K_{ij} \sim \zeta^2 \sqrt{m_i m_j}/\Lambda^2 \sim 10^{-7}$ is negligible at the Seiberg vacuum, and a systematic scan over flavour-dependent Kähler parameters finds $Q \in [0.35, 0.50]$, never reaching 2/3. The obstruction is structural: Kähler rescaling $K^{-1/2} W_{IJ} K^{-1/2}$ is a congruence transformation that preserves the rank of W_{IJ} and cannot generate new mass eigenvalues. A further dimensionless obstruction: the holomorphic mesino mass ratio $m_{\psi_{us}}/m_{\psi_{ud}} = M_d/M_s = m_s/m_d = 20.0$ is preserved by any universal Kähler rescaling, but the target ratio $m_\mu/m_e = 206.8$ differs by a factor 10.3. Matching both masses simultaneously would require flavour-dependent anomalous dimensions ($\gamma_{ud} \approx 0.25$ vs. $\gamma_{us} \approx 0.46$), which are plausible for a strongly coupled theory but not calculable with existing methods. Moreover, the Kähler expansion breaks down at the seesaw vacuum: $M_u/\Lambda \approx 1362$, $M_d/\Lambda \approx 630$, and $M_s/\Lambda \approx 31$, so the canonical Kähler $K = \mathrm{Tr}(M^\dagger M)$ is a poor approximation and large non-perturbative corrections are expected. Lepton masses in this framework therefore require a mechanism beyond the confined-phase Kähler potential. A natural candidate is a *separate* confining sector. $SU(2)$ SQCD with $N_f = 3$ (the s-confining case $N_f = N_c + 1$) produces three antisymmetric mesons $L^{ij} = \epsilon^{ab}\Psi^i_a\Psi^j_b$ with dynamical superpotential $W_{\text{dyn}} = L^{12}L^{13}L^{23}/\Lambda_L^5$ (the Pfaffian). An O'Raifeartaigh deformation

$$W = f X_L + m\, L^{12}L^{23} + g\, X_L (L^{12})^2 + \frac{\epsilon}{\Lambda_L^5}\, L^{12}L^{13}L^{23}$$

breaks SUSY at a metastable vacuum ($F_{X_L} = f$) while the Pfaffian provides a distant SUSY-restoring vacuum. At $t \equiv g\langle X_L\rangle/m = \sqrt{3}$ (selected by a Kähler pole, as in the quark sector), the fermion mass matrix has the spectrum $(0, (2-\sqrt{3})m, (2+\sqrt{3})m)$ with $Q = 2/3$ *exactly*. This is the same O'Raifeartaigh–Koide seed that generates the quark Koide triples; the mechanism is universal, independent of gauge group. The bloom rotation handles the mass hierarchy, and a confinement scale $\Lambda_L \sim 300$ MeV gives the observed $M_0 = 313.8$ MeV. Below Λ_L, QED running is nearly flavour-universal and preserves $Q = 2/3$ to $O(\alpha^2/\pi^2) \sim 10^{-5}$, consistent with the observed 9 ppm deviation. A complete specification of the lepton sector—in particular the mediation mechanism coupling L^{ij}

to the Higgs doublets and the derivation of the overall lepton mass scale—is deferred to future work.

14.5 The baryon sector

In Seiberg's SQCD with $N_f = N_c = 3$, baryons are part of the low-energy spectrum. Their SUSY partners are "baryonic fermions" with no known candidates among observed particles.

The meson–baryon connection requires a Miyazawa supercharge [4, 5], which cannot lie in the fundamental $\mathbf{5}$: the product $\mathbf{5} \otimes (\mathbf{1} \oplus \mathbf{24})$ does not contain $\overline{\mathbf{10}}$, so a single-index generator cannot map mesons to baryons. The minimal choice is

$$\mathcal{Q}_{[pq],\alpha} \in \overline{\mathbf{10}} \text{ of } SU(5)_f \,, \qquad \Delta B = +1 \,,$$

with transformation law $[\mathcal{Q}_{[pq]}, M^i{}_j] = \delta^i_{[p}\, \tilde{\psi}_{B,q]j} - \frac{1}{5}\delta^i_j\, \tilde{\psi}_{B,pq}$, where $\tilde{\psi}_{B,lm} = \frac{1}{6}\epsilon_{lmijk}\, \psi_B^{ijk}$ is the dual baryonino. The anticommutator decomposes as

$$\overline{\mathbf{10}} \otimes \mathbf{10} = \mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75} \,,$$

giving $\{\mathcal{Q}, \bar{\mathcal{Q}}\} \supset \sigma^\mu P_\mu + T^A + Z$, where T^A are the $SU(5)_f$ generators and Z are central charges encoding the meson–baryon mass splitting $m_B^2 - m_M^2 = \langle Z \rangle \sim \Lambda_{\mathrm{QCD}}^2$. This Miyazawa SUSY evades the Haag–Łopuszański–Sohnius theorem because it is an approximate symmetry broken at scale Λ_{QCD}, analogous to $SU(6)$ spin–flavor symmetry in the quark model. The $\mathbf{75}$ channel on the right-hand side signals that the full bosonic subalgebra extends beyond $SU(5)_f$; its interpretation remains open. The $\overline{\mathbf{10}}$ representation of the Miyazawa supercharge is the same $SU(5)_f$ multiplet that appears in the $SO(32)$ adjoint decomposition (Table 5), suggesting that the baryon–meson connection and the string embedding may share a common algebraic origin.

14.6 The strong CP problem

The principle "Nature abhors chirality" predicts $\bar{\theta} = 0$ but provides no mechanism.

14.7 The kaon–muon puzzle

The Lagrangian predicts a universal soft splitting $m^2_{\text{meson}} - m^2_{\text{lepton}} = f_\pi^2$ across the meson matrix. For the pion–muon pair, this works: $m_\pi^2 - m_\mu^2 \approx f_\pi^2$ to 2.2%. But the tree-level formula $m_M^2 = |m_i + m_j|^2 + f_\pi^2$ badly underpredicts heavier meson masses: it gives $m_K \approx 133\,\mathrm{MeV}$ (physical: $498\,\mathrm{MeV}$) and $m_{D_s} \approx 1{,}367\,\mathrm{MeV}$ (physical: $1{,}968\,\mathrm{MeV}$). QCD confinement generates the bulk of these masses independently of the soft term. The universal splitting applies to the *soft* contribution, not to the total mass; it is visible only for (π, μ), where the soft term dominates over $|m_u + m_d|^2$.

14.8 Flavour-changing neutral currents

The off-diagonal meson scalars $M^d{}_s$, $M^s{}_d$ are the *same particles* as the SM kaon and antikaon—they are not new degrees of freedom. Their tree-level contribution to K–$\bar{K}$ mixing is the standard QCD effect already included in SM predictions. The question is whether the additional SUSY couplings (the $\lambda_S\, S\, H_u \cdot H_d$ vertex, the Yukawa $y_c\, H_u\, M^d{}_d$

and $y_b\, H_d\, M^s{}_s$) generate *extra* flavour-violating contributions beyond QCD. Because the mesinos are identified with the leptons in the sBootstrap framework, no new particles are introduced; the principal source of additional FCNC is the Yukawa-meson cross-coupling through Higgs exchange. A quantitative estimate of these effects requires matching the effective theory to the measured K–$\bar{K}$ Wilson coefficients [21] and will be addressed in future work.

14.9 The Casimir eigenvalue equation

The eigenvalue equation $x^2+C_2x-C_2 = 0$ produces the on-shell weak mixing angle to 0.4σ, but three issues remain open: (i) the equation is an ansatz, not derived from a Lagrangian; (ii) the match uses the on-shell renormalization scheme, and no argument selects this scheme over $\overline{\mathrm{MS}}$; (iii) the negative spin-$\frac{1}{2}$ root gives $|M| = 122.4\,\mathrm{GeV}$, tantalizingly close to $M_H = 125.25\,\mathrm{GeV}$ but with a spin mismatch.

14.10 The Higgs mass at $\tan\beta = 1$

At $\tan\beta = 1$, the D-term quartic $(g^2 + g'^2)(|H_u|^2 - |H_d|^2)^2/8$ vanishes on the D-flat direction $H_u = H_d$. The tree-level lightest CP-even Higgs mass is $m_h = m_Z|\cos 2\beta| = 0$—a known MSSM result. The meson Yukawa F-terms contribute only mass terms ($y_j^2|H|^2$), not quartics. In the NMSSM, a singlet coupling $\lambda\, S\, H_u \cdot H_d$ generates an extra quartic $\lambda^2|H_u{\cdot}H_d|^2$; at $\sin 2\beta = 1$ (i.e., $\tan\beta = 1$), $m_h^2 = \lambda^2 v^2/2$, and $\lambda = 0.72$ gives $m_h = 125\,\mathrm{GeV}$ at tree level. The Seiberg Lagrange multiplier X cannot serve as the NMSSM singlet: $[X] = \mathrm{mass}^{-3}$, so the coupling $\lambda_0\, X\, H_u \cdot H_d$ requires $[\lambda_0] = \mathrm{mass}^4$; writing $\lambda_0 = \hat{\lambda}\,\Lambda^4$, canonical normalisation gives $\lambda_{\mathrm{NMSSM}} = \hat{\lambda}$. The Higgs mass demands $\hat{\lambda} \approx 0.51$, but positive meson VEVs require $\hat{\lambda} < 2\Lambda^2/v^2 \approx 3 \times 10^{-6}$: a five-order-of-magnitude conflict. Therefore S must be a *separate* elementary singlet with standard NMSSM couplings $\lambda_S\, S\, H_u{\cdot}H_d + \frac{\kappa}{3}\, S^3$. At $\tan\beta = 1$ and $\lambda_S = 0.72$, the tree-level Higgs mass $m_h = \lambda_S v/\sqrt{2} = 125\,\mathrm{GeV}$ is recovered. The μ-term $\mu = \lambda_S\langle S\rangle$ is set by the NMSSM soft potential, leaving $\langle S\rangle$ as a free parameter of order the electroweak scale.

14.11 Vacuum stability, CKM mixing, and the Oakes relation

For $N_f = N_c = 3$, the Seiberg description uses composites M, B, $\tilde{B}$ subject to the quantum-modified constraint $\det M - B\tilde{B} = \Lambda^{2N_c}$. The Lagrange multiplier X enforces this constraint and has *no kinetic term*: it is not a dynamical field. Three independent arguments establish this: (i) Seiberg's original construction [7] introduces X solely to impose the constraint; (ii) the magnetic dual has $N_c' = N_f - N_c = 0$—there is no magnetic gauge group and no ISS-type pseudo-modulus; (iii) the quantum moduli space $\det M - B\tilde{B} = \Lambda^6$ is smooth and requires no additional dynamical field.

Because X is non-dynamical, $F_X = 0$ at the constrained vacuum: the equation $\partial W/\partial X = 0$ is a constraint, not an F-term energy contribution. The diagonal seesaw vacuum $M^i{}_i = C/m_i$ satisfies the constraint exactly, and all off-diagonal meson masses are positive:

$$m^2(M^a{}_b) = 2f_\pi^2 + 2\,|X_0|^2|M_c|^2 \approx 2f_\pi^2 = 16{,}928\ \ \mathrm{MeV}^2\,,$$

where the F-term contribution $|X_0|^2|M_c|^2$ is at most 5×10^{-7} MeV^2 (10^{-11} of the soft term). *There are no tachyonic modes in the off-diagonal meson sector.*

One-loop Coleman–Weinberg corrections are positive (stabilising) and suppressed by a factor 10^{-9} relative to the tree-level mass. Moreover, a *fundamental block-diagonal obstruction* prevents cross-flavour meson mixing: the 6×6 off-diagonal mass matrix decomposes into three independent 2×2 blocks for conjugate pairs $(M^a{}_b,\, M^b{}_a)$, and this structure is preserved by all single-trace polynomial Kähler invariants. The physical reason is index contraction: at a diagonal vacuum, each $M^a{}_b$ can only contract with its conjugate $M^b{}_a$, never with a field from a different flavour pair. Cross-pair mixing would require non-perturbative (instanton-generated) Kähler corrections or physics beyond the confined-meson effective theory.

Although the off-diagonal meson vacuum is exactly diagonal, the seesaw structure $M_i = C/m_i$ naturally embeds the Gatto–Sartori–Tonin relation. If a perturbation (from any source) were to generate off-diagonal condensates ϵ_{ab}, the seesaw hierarchy would ensure

$$\frac{\epsilon_{us}}{\epsilon_{ud}} = \sqrt{\frac{m_d}{m_s}} = 0.2236 = \sin\theta_C\big|_{\mathrm{GST}}\,,$$

reproducing the Weinberg–Oakes relation *algebraically*. The GST variant gives $\theta_C = 12.9^\circ$ (PDG: 13.04°, -0.9%). This is a structural property of the seesaw-inverted mass hierarchy, not a dynamical prediction: the Cabibbo angle is encoded in m_d/m_s as a free parameter, with the seesaw providing the natural scaffolding for the Oakes relation. The actual origin of CKM mixing—whether from non-perturbative Kähler corrections, physics at the UV completion scale, or quark mass-matrix texture—remains an open problem.

A notable extension to the first generation: the Koide condition does *not* hold for the triple (m_u, m_d, m_s) ($Q = 0.567$, 15% off), but $Q(0,\, m_u{+}m_d,\, m_s) = 0.665$ is only 0.27% from 2/3—closer than the established seed $Q(0,\, m_s,\, m_c) = 0.664$ at 0.35%. The Koide chain extends to the first generation but sees the isospin-summed mass $m_u + m_d = 6.83\,\mathrm{MeV}$ as a single seed entry; the splitting into $m_u \neq m_d$ is the remaining freedom parametrised by θ_C.

Without a baryon mass term, the global minimum lies at $M = 0$, $B\tilde{B} = -\Lambda^6$, with $V = \sum m_i^2 \approx 8{,}750\ \mathrm{MeV}^2$. However, the seesaw vacuum is cosmologically stable: Coleman–de Luccia bounce-action estimates give $S_4 \sim 2 \times 10^7$ (thick-wall) to 10^9 (thin-wall), both far above the threshold $S_4 > 400$ required for a lifetime exceeding the age of the universe. Adding a baryon mass term $W_B = m_B\, B\tilde{B}$ to the superpotential lifts the baryonic flat direction without affecting the seesaw vacuum (since $B = \tilde{B} = 0$ there): the meson masses M_i, the off-diagonal meson spectrum, and the Higgs sector are all unchanged. With the Kähler pole confining X near X_{seesaw}, any $m_B \gtrsim 1$ MeV eliminates vacuum (B) entirely. The natural scale $m_B \sim \Lambda = 300$ MeV (arising from instanton corrections in the confining phase) lifts the baryino mass from $|X_0| \sim 10^{-9}$ MeV to ~ 300 MeV and makes $V_B \sim 10^{20}\ \mathrm{MeV}^2 \gg V_A$.

14.12 The bloom mechanism

The Koide parametrization

$$\sqrt{m_k} = v_0\big(1 + \sqrt{2}\cos(\delta + 2\pi k/3)\big), \quad k = 0, 1, 2,$$

satisfies $Q = 2/3$ identically for all δ and v_0. The seed sits at $\delta = 3\pi/4$, where $\cos(3\pi/4) = -1/\sqrt{2}$ forces $\sqrt{m_0} = 0$. A Koide-preserving bloom is a pure rotation $\delta\colon 3\pi/4 \to \delta_{\mathrm{phys}}$ at fixed v_0; the total mass $M = 6v_0^2$ is conserved while mass redistributes among the three eigenvalues.

Adiabatic bion potential. The monopole-instanton effective superpotential on $\mathbb{R}^3 \times S^1$ takes the form $W_{\text{mon}} \sim \zeta(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})$. The bion (monopole–anti-monopole) contribution to the Kähler potential is $V_{\text{bion}} = |\sum_k s_k \sqrt{m_k}|^2$, where $s_k = \text{sign}(z_k)$ are the signs of the Koide amplitudes $z_k = v_0(1+\sqrt{2}\cos(\delta+2\pi k/3))$. On the interior of the Koide manifold (away from the seed) all three z_k are nonzero, and the adiabatic signs follow δ:

$$\sum_k s_k |z_k| = \sum_k z_k = 3v_0 = \text{const.}$$

This is an exact algebraic identity: the adiabatic bion potential $V_{\text{bion}} = (3v_0)^2 = 9v_0^2$ is *flat* on the entire Koide manifold and cannot select δ.

Frozen-sign bion. At the seed ($\delta = 3\pi/4$), one amplitude vanishes and its sign is topologically frozen: for the quark triple $(-s, c, b)$, the bloom from seed to physical values takes $s_0 : 0 \to -1$ while $s_{1,2} = +1$ remain fixed. With frozen signs $s_k = +1$ for all k, the frozen-sign potential

$$V_{\text{frozen}}(\delta) = \left(\sum_k |z_k| \right)^2$$

is *not* constant on the Koide manifold. It achieves its minimum value $9v_0^2$ at the seed ($\delta = 3\pi/4$, where $\sum |z_k| = 3v_0$, the triangle inequality saturated by $|z_0| = 0$) and grows for $\delta > 3\pi/4$. The frozen bion thus provides a restoring force that opposes the bloom.

Three-instanton driving force. The three-instanton operator $\delta W_{\text{3-inst}} = c_3(\det M)^3/\Lambda^{18}$ arises from the $\mathbb{Z}_{18}$ center symmetry of $SU(3)$ SQCD with $N_f = N_c = 3$ (the anomaly coefficient $6N_c = 18$ breaks $U(1)_{R+2A} \to \mathbb{Z}_{18}$; see the preceding subsection). Near the Seiberg vacuum, parametrizing $\det M = \Lambda^6 e^{3i\delta}$ gives

$$V_{\text{3-inst}} \propto -\cos(9\delta).$$

For the quark triple $(-s, c, b)$, the observed phase is $\delta_{\text{phys}} = 157.2^\circ$. The nearest minimum of $-\cos(9\delta)$ is at $\delta = 160^\circ$, just 2.8° away. The combined potential

$$V(\delta) = A\, V_{\text{frozen}}(\delta) + B\,(-\cos 9\delta)$$

has a stable equilibrium at the physical phase for $B/A = -2.07$: the three-instanton term drives the bloom outward from the seed, while the frozen-sign bion provides the restoring force that stabilizes the physical triple.

Lepton triple. For the lepton triple (e, μ, τ), all three amplitudes $z_k > 0$ throughout the bloom ($\delta_\ell = 132.7^\circ$, never crossing zero), so the adiabatic and frozen-sign bion potentials coincide: $V_{\text{frozen}} = 9v_0^2$ exactly (flat). The bion potential provides no selection. The near-rational relation $\delta_\ell \bmod 2\pi/3 \approx 2/9$ (33 ppm) is therefore *not* explained by the bion mechanism; its origin is the three-instanton potential $-\cos(9\delta)$ with a minimum at $\delta \equiv 2/9 \pmod{2\pi/3}$, as discussed in the preceding subsection. The lepton and quark phases are selected by the same instanton potential but through different dynamical channels: for quarks the frozen-sign bion competes with the instanton term, while for leptons the bion is inert and the instanton acts unopposed.

14.13 The $SO(32)$ projection

The adjoint **496** contains the sBootstrap representations: **24** in the neutral sector and $\mathbf{15} \oplus \overline{\mathbf{15}}$ in the matter sector at $|Q_A| = 2$. The remaining 442 states must be projected out or made massive. No projection mechanism has been identified. The alternative anomaly-free gauge group $E_8 \times E_8$ is blocked by the **15** vs. **10** obstruction (§10.3), constraining the string embedding to the Type I / $SO(32)$ corner of the landscape.

14.14 Generation uniqueness

The Diophantine system uniquely selects $N = 3$ generations, but the third equation ($r^2 + s^2 - 1 = 4N$) has the weakest physical motivation. A dynamical derivation of the counting constraints would strengthen the argument considerably.

15 Epistemological classification

The following table classifies the paper's claims by their logical status. "Derived" means following from stated assumptions by algebra or field theory; "observed" means an empirical pattern; "assumed" means an input without derivation.

Claim	Status	Comment
$Q = 2/3$ iff $\langle z_k^2 \rangle = z_0^2$	Derived	Algebra (§2)
Seed ratio $= 2 - \sqrt{3}$	Derived	From $Q = 2/3$ + one zero
O'R seed at $gv/m = \sqrt{3}$	Derived	Fermion eigenvalues
Kähler stabilization: $c = -1/12$	Derived	Pole condition (§11.12)
$\langle X \rangle / \mu = \sqrt{3}$ in magnetic dual	Derived	ISS dictionary
Pauli theorem (no symmetric J=0 diquark)	Derived	Angular momentum
Diophantine: $(N, r, s) = (3, 3, 2)$ unique	Derived	Number theory
Cyclic echo ($t \to c \to s \to$ stall $\to c$)	Derived	Algebraic necessity
Seed self-dual: $Q(a, b) = Q(1/a, 1/b)$	Derived	Identity for two masses
Cartan angle $\theta = \pi/6 - \delta$	Derived	Exact trigonometric identity
F-terms flavour-universal: $F = C/v_{\rm EW}$	Derived	Seesaw cancellation
Lepton Koide $Q = 2/3$ to 0.001%	Observed	2.8σ after LEE
Meson triple $(-\pi, D_s, B)$ to 0.10%	Observed	1.1σ after LEE
v_0-doubling: $v_0^{(f)}/v_0^{(s)} = 2.0005$ (0.025%); m_b to 0.07%	Observed	Derived via bion Kähler
δ_ℓ mod $2\pi/3 = 2/9$ to 33 ppm	Observed	3.1σ after LEE
Casimir $R = 0.22310$ vs $\sin^2\theta_W$	Observed	Scheme-dependent
Dual Koide $Q(1/m_d, 1/m_s, 1/m_b) = 0.665$	Observed	0.2% from 2/3; not predictive
GST: $\lvert V_{us}\rvert = \sqrt{m_d/m_s} = 0.224$	Derived	Tachyon ratio; 0.9% from PDG [20]
$M_0 \approx f_\pi^2/27$	Observed	0.04%; connects lepton scale to f_π
$\tan\beta = 1$	Derived	Mixed triples + $SU(5)_f$
$m_\pi^2 - m_\mu^2 = f_\pi^2$ (VA relation)	Assumed	2.2% match
$f_{\rm VA} = f_\pi$ identification	Assumed	Central ansatz
SU(5) flavor symmetry	Assumed	5 light quarks
$m_b = 4{,}177 \pm 40\,$MeV from bion minimum	Derived	0.06σ; requires Ünsal conjecture
$N_f = 4$ unique in ISS window	Derived	Only integer in $(3, 4.5)$
ISS vacuum cosmologically stable	Derived	$S_{\rm bounce} \gg 1$ for $h \sim O(1)$
ISS CW does not transmit Koide	Derived	$Q_{\rm CW} \to 1/3$ (affine map)
$c = -1/12$ from bion Kähler	Derived	At $\alpha_s = 0.143$ (ISS)
${\rm STr}[M^2] = 18\, f_\pi^2$	Derived	Soft mass only (canonical Kähler)
$m_h = \lambda_S v/\sqrt{2} = 125\,$GeV	Input	NMSSM singlet $S \neq X$; $\lambda_S = 0.72$
$\cos(9\delta)$ from $(\det M)^3/\Lambda^{18}$	Derived	$\mathbb{Z}_{18}$ charge counting
$\mathcal{Q}_{[ij]} \in \overline{\mathbf{10}}$ of $SU(5)_f$	Derived	Miyazawa algebra
Casimir structural constraints	Assumed	Reverse-engineered
No tachyons: $F_X = 0$ (Lagrange multiplier)	Derived	Block-diagonal obstruction
GST relation $\sin\theta_C = \sqrt{m_d/m_s}$	Structural	Seesaw encodes Oakes; not derived
$\mu = \lambda_S \langle S \rangle \sim O(v)$	Free	Standard NMSSM ($S \neq X$)
Kähler cannot generate $Q_\ell = 2/3$	Derived	Rank invariance of W_{IJ}
FCNC: mesons = SM kaons, not extra d.o.f.	Noted	Extra SUSY FCNC TBD
$X \neq S$: $\hat{\lambda}_{\rm Higgs}/\hat{\lambda}_{\rm max} \sim 10^5$	Derived	Dimensional analysis
O'R–Koide universal (gauge-independent)	Derived	$SU(2)$, $Sp(2)$, $SU(3)$ all give $Q = 2/3$
$SU(2)$ lepton sector: metastable O'R + Pfaffian	Derived	s-confining, $N_f = 3$
Seesaw vacuum cosmologically stable	Derived	$S_4 \sim 2 \times 10^7 \gg 400$
$m_B\, B\tilde{B}$ lifts baryon vacuum, no meson changes	Derived	$m_B \gtrsim 1$ MeV suffices
Adiabatic bion flat on Koide manifold	Derived	$\sum s_k \lvert z_k \rvert = 3v_0$ (identity)
Bloom driven by $\cos(9\delta)$, stabilised by bion	Derived	Physical δ within 2.8° of $\cos(9\delta)$ min

16 Conclusions

Seven principal results:

1. The Koide formula is energy equipartition. $Q = 2/3$ is $\langle z_k^2 \rangle = z_0^2$: perturbation energies match the base scale. The signed-charge framework resolves the electron mass puzzle (near-cancellation, not fine-tuning) and reveals that mesons satisfy Koide with $(-\pi, D_s, B)$ to 0.1%.

2. Seed triples set the stage for SUSY breaking. The quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ share the universal ratio $2 - \sqrt{3}$ and operate at $v_0^2 \sim 230$–$350\,\mathrm{MeV}$, close to the lepton triple's $v_0^2 = 314\,\mathrm{MeV} \approx f_\pi^2/27$ (recall that v_0^2 carries dimensions of MeV, not MeV^2, because $v_0 = z_0$ is a charge with dimensions $\mathrm{MeV}^{1/2}$). The massless member is the necessary condition for F-term (O'Raifeartaigh) SUSY breaking. Upon breaking, seeds bloom into full triples $(-s, c, b)$ and $(-\pi, D_s, B)$. The lepton triple is the "almost-seed" that has barely bloomed.

3. The seed condition is one superpotential constraint, stabilized by a Kähler pole. The O'Raifeartaigh model with superpotential $W = f\Phi_0 + m\Phi_1\Phi_2 + g\Phi_0\Phi_1^2$ produces a fermionic spectrum $(0, m_-, m_+)$ that is an exact Koide seed triple when $gv/m = \sqrt{3}$. The one-loop CW potential alone stabilizes the pseudo-modulus at $v = 0$; a negative quartic Kähler correction $c\,|\Phi_0|^4/\Lambda_K^2$ with $c = -\Lambda_K^2/(12m^2)$ pins it at the Kähler pole $v = \sqrt{3}\,m/g$ exactly. The Koide seed emerges at $M_0 = 2m/3$. The coefficient $c = -1/12$ is not radiatively generated but arises from magnetic bions on $\mathbb{R}^3 \times S^1$ at $\alpha_s = 0.143$ in the ISS magnetic dual.

4. Two breaking mechanisms plus a heavy-sector origin. F-term breaking (via the massless seed member) generates the light sector; the Volkov–Akulov relation $f_{\mathrm{VA}} = f_\pi$ is its low-energy realization, not a separate mechanism. D-term breaking ($U(1)_{\mathrm{em}}$ FI parameter) provides electromagnetic splittings as a perturbative correction. The heavy triple (c, b, t) stands apart: no seed, no meson shadow, scale far above Λ_{QCD}, possibly connected to electroweak breaking.

5. The Diophantine counting uniquely selects $N = 3$. Three equations on the scalar content—$rs = 2N$, $r(r+1)/2 = 2N$, $r^2 + s^2 - 1 = 4N$—have the unique positive-integer solution $(N, r, s) = (3, 3, 2)$, which maps to the $SU(5)$ representations $\mathbf{24} \oplus \mathbf{15} \oplus \overline{\mathbf{15}}$. This combination is anomaly-free and asymptotically free.

6. An algebraic electroweak ratio. The eigenvalue equation $x^2 + s(s+1)\,x - s(s+1) = 0$, with s the spin Casimir, predicts $R = 1 - x_+(1/2)/x_+(1) = 0.22310$, matching the on-shell $\sin^2\theta_W = 0.22320$ to 0.4σ and giving $M_W = 80.374\,\mathrm{GeV}$ (0.39σ from PDG). The equation is pinned by three structural constraints; R follows as a consequence, not a free parameter. The dynamical origin of the equation and the choice of renormalization scheme remain open.

7. An explicit Lagrangian with two to three free parameters. The full $\mathcal{N}=1$ Lagrangian (§11.12) comprises a Kähler potential with a negative quartic correction $c=-1/12$ (stabilizing the pseudo-modulus at $gv/m=\sqrt{3}$) and a bion correction $K_{\text{bion}} \sim |\sum s_k\sqrt{\hat{m}_k}|^2$, a superpotential combining the Seiberg constraint with Yukawa couplings (using separate H_u, H_d doublets) and an NMSSM coupling $\lambda_S\, S\, H_u{\cdot}H_d$ ($\lambda_S = 0.72$ for $m_h = 125\,\text{GeV}$, with S a separate elementary singlet, distinct from the Lagrange multiplier X), standard $SU(2)_L \times U(1)_Y$ D-terms (vanishing at $\tan\beta = 1$), and soft breaking $V_{\text{soft}} = f_\pi^2\,\text{Tr}(M^\dagger M)$. The supertrace $\text{STr}[M^2] = 18\,f_\pi^2$ receives contributions only from the nine soft meson masses. The mixed up/down-type Koide triples require $\tan\beta = 1$ for the mass-level condition to equal a Yukawa-level condition—a prediction independently motivated by $SU(5)_f$. The seed Koide fixes the pseudo-modulus VEV $\langle X\rangle = \sqrt{3}\,\mu$ (predicting m_c/m_s); the bion Kähler correction fixes m_b through the v_0-doubling condition, with Hessian rank 1 constraining exactly one mass combination. The self-consistent chain from m_s alone predicts $m_c = 1{,}301\,\text{MeV}$, $m_b = 4{,}177\,\text{MeV}$, and—if $Q(c,b,t) = 2/3$ is imposed—$m_t = 171.3 \pm 1.5\,\text{GeV}$ (Table 6). The lepton Koide scale satisfies $M_0 \approx f_\pi^2/27$ to 0.04%, tying lepton masses to the pion decay constant. The Seiberg Lagrange multiplier X has no kinetic term ($F_X = 0$), and all off-diagonal meson masses are positive; a block-diagonal obstruction in the Kähler sector prevents cross-flavour condensation at any perturbative order. The seesaw structure nevertheless embeds the Gatto–Sartori–Tonin relation $\sin\theta_C = \sqrt{m_d/m_s}$ algebraically, so the Cabibbo angle is naturally encoded in the mass hierarchy even though the mixing itself must originate from a mechanism beyond the confined-meson effective theory. After Koide and v_0-doubling constraints, only m_u, m_d, and m_s remain free; m_u and m_d are connected to the Cabibbo angle through the Gatto–Sartori–Tonin relation. The $SO(32)$ adjoint provides the UV completion, with the $E_8 \times E_8$ alternative blocked by the **15**-vs.-**10** obstruction.

The diquark sector, required to complete the SUSY multiplet count, fills the symmetric $\mathbf{15}+\overline{\mathbf{15}}$ of $SU(5)_f$—the representation produced naturally by the SUSY generator acting on quarks and by the $SO(32)$ decomposition. A kinematic theorem (§9.3) shows that *no* $J=0$ color-$\bar{3}$ diquark can have symmetric flavor, regardless of orbital angular momentum: the sBootstrap diquark in the **15** cannot be a qq composite. The Miyazawa supercharge $\mathcal{Q}_{[ij]} \in \overline{\mathbf{10}}$ (§14.5) connects mesons to baryons but generates diquark operators in $\overline{\mathbf{10}}$ (antisymmetric), not **15** (symmetric). The remaining candidate is a fundamental string-theoretic state from the $SO(32)$ adjoint.

The organizing principle is that each sector of the spectrum begins with a massless member (a seed), which is the necessary condition for spontaneous SUSY breaking. The breaking lifts the zero, generates the full triple, and the energy-balance condition $\langle z_k^2\rangle = z_0^2$ constrains the initial conditions. The bloom from seed to full triple is a phase rotation $\delta : 3\pi/4 \to \delta_{\text{phys}}$ in the Koide parametrization, at fixed Koide scale v_0; it is not a continuous deformation of the O'Raifeartaigh spectrum (which would break $Q=2/3$) but a nonperturbative rearrangement. The bloom mechanism (preceding section) identifies the dynamical origin of δ: the adiabatic bion potential is exactly flat on the Koide manifold, but with frozen topological signs it provides a restoring force opposing the bloom; the three-instanton potential $-\cos(9\delta)$ drives δ away from the seed, and their equilibrium lies within 2.8° of the physical quark bloom position. The unified superpotential has a layered structure: (i) the Seiberg effective superpotential governs the light sector, with SUSY breaking forced by the quantum constraint when $m_u = m_d = 0$; (ii) the O'Raifeartaigh condition $gv/m = \sqrt{3}$ (equivalently $\langle X\rangle = \sqrt{3}\,\mu$ in the magnetic dual) fixes the seed Koide ratio, predicting m_c from m_s; (iii) the bion Kähler correction $|\sum s_k\sqrt{\hat{m}_k}|^2$ predicts

$m_b = 4{,}177 \pm 40\,\mathrm{MeV}$ via v_0-doubling; (iv) the Yukawa sector governs the (c, b, t) triple as an overdetermination test. After these constraints, only two to three free parameters remain: m_u, m_d (connected to the Cabibbo angle through the GST relation), and the overall scale m_s. All quark Koide triples obligatorily mix up-type and down-type mass eigenstates. The energy-balance condition is a boundary condition on the superpotential, not an RG attractor; the Koide ratios are naturally protected against D-term perturbations because D-terms shift scalar masses but leave fermion masses untouched.

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