A Tale of Two Tuples: Seed Triples, Signed Charges, and SUSY Breaking in the sBootstrap

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Abstract

We reformulate the sBootstrap in terms of signed mass charges $\mathfrak{z} = z_0 + z_i$ with $m = \mathfrak{z}^2$, and show that the Koide ratio $Q = 2/3$ is the algebraic consequence of two conditions: $\sum z_i = 0$ and $\langle z_i^2 \rangle = z_0^2$ (perturbation energies average to the vacuum energy). Allowing negative charges, we find that pseudoscalar mesons satisfy a Koide triple $(-\pi, D_s, B)$ to 0.1%, and identify *seed triples* with one massless member—$(0, s, c)$ for quarks, $(0, \pi, D_s)$ for mesons—as the pre-breaking configurations from which the full spectrum emerges. These seed triples share the same analytical structure (charge ratio $2 - \sqrt{3}$) and, strikingly, the meson seed $(0, \pi, D_s)$ has vacuum energy $v_0^2 \approx 351\,\mathrm{MeV}$, close to the lepton triple's $v_0^2 \approx 314\,\mathrm{MeV}$. A third tuple (c, b, t) stands apart, having no meson counterpart because top diquarks do not hadronize. We discuss the diquark sector—which fills the symmetric $\mathbf{15} + \overline{\mathbf{15}}$ of $SU(5)$ flavor, in conflict with Pauli statistics for point-like diquarks, proving that the diquark cannot be a qq composite—and outline how SUSY breaking proceeds through two mechanisms—F-term (chiral condensate, with Volkov–Akulov as its low-energy realization) and D-term (electromagnetic Fayet–Iliopoulos)—plus a separate heavy-sector origin for the (c, b, t) triple.

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1 Introduction

The sBootstrap [1,2] proposes that the Standard Model's pseudoscalar mesons and leptons fill common $\mathcal{N} = 1$ supermultiplets. Its foundational relation is

$$m_\pi^2 - m_\mu^2 \approx f_\pi^2\,, \tag{1}$$

identifying the pion decay constant $f_\pi \approx 92.2\,\mathrm{MeV}$ as the SUSY-breaking order parameter. Numerically, $m_\pi^2 - m_\mu^2 = 8{,}316\,\mathrm{MeV}^2$ versus $f_\pi^2 = 8{,}501\,\mathrm{MeV}^2$, a 2.2% discrepancy—comparable to the precision of the quark Koide triples discussed below. In this paper we develop two themes that change the structure of the program.

First, **mass is the square of a signed charge**: $m = \mathfrak{z}^2$ where $\mathfrak{z} = z_0 + z_i$ can be negative. The Koide ratio $Q = 2/3$ is then not numerology but the energy-balance condition $\langle z_i^2 \rangle = z_0^2$. With signed charges, pseudoscalar mesons satisfy Koide, and the spectrum organizes into *seed triples* with a massless member that grow into full triples upon symmetry breaking.

Second, **the full sBootstrap has three Koide tuples, not one**, and the breaking structure must generate all three: a light sector $(0, s, c) \to (-s, c, b)$ seen in both quarks and mesons, a lepton sector $(e \approx 0, \mu, \tau)$, and a heavy sector (c, b, t) that decouples from meson physics because the top quark does not hadronize. The diquark partners of quarks, needed to complete the SUSY multiplets, fill the symmetric $\mathbf{15} + \overline{\mathbf{15}}$ of $SU(5)$ flavor—the representation produced naturally by the SUSY generator and by the $SO(32)$ decomposition. A kinematic theorem shows that no $J = 0$, color-$\bar{3}$ diquark can have symmetric flavor—regardless of orbital angular momentum—so the sBootstrap diquark cannot be a qq composite but must be an algebraically or string-theoretically generated object.

2 The Koide Formula as Energy Balance

2.1 Charges, not masses

The fundamental variable is the signed mass charge $\mathfrak{z} = z_0 + z_i \in \mathbb{R}$, with the physical mass $m = \mathfrak{z}^2 \geq 0$.

2.2 Derivation of $Q = 2/3$

For three charges $\mathfrak{z}_k = z_0 + z_k$ with z_0 the mean ($\sum z_k = 0$):

$$Q = \frac{\sum \mathfrak{z}_k^2}{(\sum \mathfrak{z}_k)^2} = \frac{3z_0^2 + \sum z_k^2}{9z_0^2} = \frac{1}{3} + \frac{\langle z_k^2 \rangle}{3z_0^2}\,. \tag{2}$$

Therefore:

$$\boxed{Q = \frac{2}{3} \quad \Longleftrightarrow \quad \langle z_k^2 \rangle = z_0^2\,.} \tag{3}$$

The two conditions are:

1. **Zero-sum perturbations:** $z_1 + z_2 + z_3 = 0$ (definition of z_0 as the mean charge).
2. **Energy balance:** The mean energy of the perturbations equals the energy of the unperturbed charge: $\langle z_k^2 \rangle = z_0^2$.

Condition 2 is the physics. It says the splitting scale matches the base scale—the coefficient of variation equals unity: $\sigma(\mathfrak{z})/\bar{\mathfrak{z}} = 1$. This is neither perturbative ($\sigma \ll \bar{\mathfrak{z}}$, giving $Q \to 1/3$, all equal) nor dominant ($\sigma \gg \bar{\mathfrak{z}}$, giving $Q \to 1$, one mass dominates). The Koide value 2/3 is the equipartition point.

2.3 Why one mass can vanish

If $\langle z_k^2 \rangle = z_0^2$, a perturbation z_k can reach $|z_k| \sim z_0$, making $\mathfrak{z}_k = z_0 + z_k \approx 0$. A vanishing mass is the *natural extreme* of energy balance, not fine-tuning.

3 Seed Triples: The $(0, a, b)$ Structure

3.1 Analytical condition

For a triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ with one massless member, the Koide ratio becomes:

$$Q = \frac{\mathfrak{z}_a^2 + \mathfrak{z}_b^2}{(\mathfrak{z}_a + \mathfrak{z}_b)^2} \,. \tag{4}$$

Setting $Q = 2/3$ and writing $r = \mathfrak{z}_a/\mathfrak{z}_b$ (with $r < 1$):

$$3(r^2 + 1) = 2(r + 1)^2 \quad \Longrightarrow \quad r^2 - 4r + 1 = 0 \,, \tag{5}$$

giving:

$$\boxed{r = 2 - \sqrt{3} \approx 0.26795 \,.} \tag{6}$$

Equivalently, in mass space: $m_{\text{light}}/m_{\text{heavy}} = (2 - \sqrt{3})^2 = 7 - 4\sqrt{3} \approx 0.07180$.

This is a *universal* prediction: any Koide triple with one massless member has the same charge ratio between the two massive members, regardless of their absolute scale.

3.2 The quark seed: $(0, s, c)$

The strange and charm quark charges give:[1]

$$\frac{\sqrt{m_s}}{\sqrt{m_c}} = \frac{9.67}{35.64} = 0.2712 \,, \qquad r_{\text{Koide}} = 0.2679 \,, \qquad \text{deviation: } 1.2\% \,. \tag{7}$$

This triple has $Q = 0.6644$, deviating from 2/3 by only 0.35%. Its vacuum energy is $v_0^2 = (\mathfrak{z}_s + \mathfrak{z}_c)^2/9 \approx 228\,\text{MeV}$.

A historical note: Harari, Haut, and Weyers [6] observed a quark mass ratio pattern that, in the language of the present paper, corresponds to an approximate seed triple—$(0, d, s)$—at a time when the up quark mass was still thought to be possibly zero [6]. With current masses, $\sqrt{m_d}/\sqrt{m_s} = 0.224$, deviating from $2 - \sqrt{3}$ by 16%—too large for a good Koide triple. But if m_s were $\sim 150\,\text{MeV}$ and $m_d \sim 7\,\text{MeV}$ (values common in that era), the ratio improves considerably. The seed idea was already present, ahead of its time.

[1] Throughout this paper, quark masses follow PDG conventions: $m_s(2\,\text{GeV}) = 93.4\,\text{MeV}$, $m_c(m_c) = 1.27\,\text{GeV}$, $m_b(m_b) = 4.18\,\text{GeV}$, and the top pole mass $m_t = 172.76\,\text{GeV}$. This is a mixed convention: the light quark is evaluated at 2 GeV while heavier quarks are evaluated at their own scale. Since QCD mass ratios are RG invariants (the anomalous dimension γ_m is flavor-universal at leading order), the Koide ratio Q is scheme-independent. However, the seed ratio $\sqrt{m_s}/\sqrt{m_c}$ as computed here uses masses at *different* scales, giving 0.271; at a common scale the ratio is ~ 0.29, further from $2 - \sqrt{3} = 0.268$. The meson seed, using unambiguous physical (pole) masses, is free of this scheme issue and provides the cleaner test.

3.3 The meson seed: $(0, \pi, D_s)$

The pion and D_s meson charges give:

$$\frac{\sqrt{m_\pi}}{\sqrt{m_{D_s}}} = \frac{11.81}{44.37} = 0.2663\,, \qquad r_{\text{Koide}} = 0.2679\,, \qquad \text{deviation: } 0.6\%\,. \tag{8}$$

This gives $Q = 0.6679$, deviating by 0.18%. It is the **meson counterpart** of the quark seed $(0, s, c)$—both involve the strange-charm sector. Indeed, $D_s = c\bar{s}$ is the meson that directly contains the s and c quarks.

3.4 The lepton triple as an "almost-seed"

The charged lepton triple (e, μ, τ) has $\mathfrak{z}_e = 0.715\,\text{MeV}^{1/2}$, which is small but nonzero. If m_e were zero, we would need $\sqrt{m_\mu}/\sqrt{m_\tau} = 2 - \sqrt{3} = 0.2679$; the actual value is 0.2439, deviating by 9%. So the leptons are *not* an exact seed—the electron, while very light, is not massless. But the triple is clearly the same family: the near-critical Koide phase puts one member close to zero.

3.5 Vacuum energy coincidence

The v_0^2 values of the seed triples cluster remarkably:

Triple	v_0^2 (MeV)	v_0 ($\text{MeV}^{1/2}$)
(e, μ, τ)	314	17.72
$(0, \pi, D_s)$	351	18.73
$(0, s, c)$	228	15.10

The lepton and meson seed triples share a vacuum energy around 310–350 MeV, squarely in the QCD scale range. This is the sBootstrap connection made quantitative: **the lepton triple and the meson seed triple operate at the same scale.**

4 A Tale of Two Tuples

4.1 From seed to full triple

The seed triples grow into full triples upon symmetry breaking:

Sector	Seed triple	Full triple	v_0^2 (full)
Quarks	$(0, s, c)$	$(-s, c, b)$	913 MeV
Mesons	$(0, \pi, D_s)$	$(-\pi, D_s, B)$	1230 MeV
Leptons	$(0 \approx e, \mu, \tau)$	(e, μ, τ)	314 MeV

In each case:

- The seed has a massless (or near-massless) member, two massive members in the ratio $2 - \sqrt{3}$, and $Q \approx 2/3$.

- The full triple has **no** massless member: the zero has been lifted (to m_e, or the sign-flipped strange/pion mass). The b quark and B meson are always present as flavor degrees of freedom; what the breaking does is give them their *distinctive masses*—the specific values that complete the Koide triple. Before breaking, the b-direction sits at the flat direction ($\mathfrak{z} = 0$); after, it acquires $\mathfrak{z}_b = 64.65\,\mathrm{MeV}^{1/2}$, the value selected by the energy balance with c and the sign-flipped s. The bloom is not particle creation but *mass generation.*

The "tale of two tuples" is this: **each sector presents first as a seed with a massless member, then "blooms" into a full triple when SUSY breaks.** The massless member is the necessary initial condition; the breaking lifts it and generates the full spectrum.

4.2 Precision summary

Triple	Q	Dev. from 2/3	v_0^2 (MeV)
	Seed triples		
$(0, \pi, D_s)$	0.6679	0.18%	351
$(0, s, c)$	0.6644	0.35%	228
$(0 \approx e, \mu, \tau)$	0.6667	0.001%	314
	Full triples		
(e, μ, τ)	0.66666	0.001%	314
$(-\pi, D_s, B)$	0.6674	0.10%	1230
$(-s, c, b)$	0.6750	1.24%	913
(c, b, t)	0.6695	0.42%	29 580

The lepton triple is unique: seed and full triple are nearly identical, because the electron mass is small enough to serve as the "zero" while being nonzero enough to make Q extremely precise.

4.3 Orthogonality of seed and full triples

Each Koide triple defines a perturbation vector $\vec{z} = (z_1, z_2, z_3)$ constrained to the plane $\sum z_k = 0$ with length $|\vec{z}|^2 = 3z_0^2$. The angle between seed and full triples measures how much the breaking rotates the spectrum in charge space:

Pair of triples	Angle
$(0, s, c)$ vs. $(-s, c, b)$	22°
$(0, \pi, D_s)$ vs. $(-\pi, D_s, B)$	26°
(e, μ, τ) vs. $(0, \mu, \tau)$	0.8°
$(0, s, c)$ vs. $(0, \pi, D_s)$	0.3°

Two results are notable. First, the quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ are **nearly parallel** (0.3°): the meson seed is the composite image of the quark seed, with the GOR map preserving direction while changing scale. Second, the lepton triple is essentially its own seed (0.8°)—the bloom has barely begun.

5 The Meson Koide Triple: $(-\pi, D_s, B)$

With the pion charge negative, the triple $(-\pi, D_s, B)$ satisfies:

$$Q(-\pi, D_s, B) = \frac{m_\pi + m_{D_s} + m_B}{(-\sqrt{m_\pi} + \sqrt{m_{D_s}} + \sqrt{m_B})^2} = 0.6674\,, \tag{9}$$

deviating from 2/3 by 0.10%—the second most precise Koide triple known. The energy balance:

$$\frac{\langle z_k^2 \rangle}{z_0^2} = 1.002\,. \tag{10}$$

For comparison, the "standard" all-positive quark triples (d, s, b) and (u, c, t) deviate from $Q = 2/3$ by 9.7% and 27% respectively at $\overline{\text{MS}}$ masses. **The meson triple is more precise than any quark triple** except (c, b, t).

The pion's negative charge has physical meaning: as a pseudo-Goldstone boson, the pion would be massless ($\mathfrak{z} = 0$) if chiral symmetry were exact. It sits just below zero ($\mathfrak{z}_\pi = -11.81$), approaching zero from the negative side as $m_q \to 0$. The sign of $\mathfrak{z}$ encodes Goldstone nature: pseudo-Goldstone bosons sit on the negative side of the critical point, having crossed through zero from positive territory during chiral symmetry breaking.

6 The Third Tuple: (c, b, t)

6.1 A tuple without mesons

The triple (c, b, t) with all positive signs gives $Q = 0.6695$, deviating by 0.42%. Its vacuum energy $v_0^2 \approx 29.6\,\text{GeV}$ is far above the QCD scale.

This triple has **no meson counterpart**: the top quark decays weakly ($t \to Wb$, lifetime $\sim 5 \times 10^{-25}$ s) before the strong interaction can bind it ($\tau_{\text{QCD}} \sim 1/\Lambda_{\text{QCD}} \sim 10^{-24}$ s). No T-meson ($t\bar{q}$) exists. The meson matrix $M^i{}_j$ extends only to b-flavored states; the top sector lives purely at the quark level.

6.2 Diquarks and the top

In the sBootstrap, diquarks $[q_i q_j]$ are the scalar partners of antiquarks $\bar{q}_k$, with the baryon acting as the SUSY generator [4,5]. For five active flavors (u, d, s, c, b), the diquarks form a representation of $SU(5)_f$.

The top diquarks $[tq]$ are kinematic objects only—they do not hadronize. Yet their masses should follow the same pattern as other diquarks: **there is no dynamical reason for top diquark masses to differ from the others**. The (c, b, t) Koide triple governs the quark masses that enter the diquark sector, and its energy balance $\langle z_k^2 \rangle = z_0^2$ constrains the possible diquark spectrum even though the top diquarks are never observed as bound states.

6.3 The (c, b, t) triple has no seed

Unlike the $(0, s, c)$ and $(0, \pi, D_s)$ seeds, there is no obvious $(0, x, y)$ seed for the (c, b, t) triple. No pair of quarks in this sector satisfies $\sqrt{m_{\text{light}}}/\sqrt{m_{\text{heavy}}} \approx 2 - \sqrt{3}$: one has $\sqrt{m_c}/\sqrt{m_t} = 0.086$ (too small) and $\sqrt{m_c}/\sqrt{m_b} = 0.551$ (too large).

This means (c, b, t) is *not* a "bloomed seed" in the same sense as $(-s, c, b)$. It may represent a fundamentally different structure—a triple that was never seeded by a massless member but instead emerged fully formed at a higher breaking scale, perhaps connected to electroweak symmetry breaking rather than to chiral symmetry breaking.

7 The Diquark Sector and SU(5) Flavor

7.1 Counting partners

In the full sBootstrap [1], the scalar partners of the Standard Model fermions include both mesons ($q\bar{q}$) and diquarks (qq). For five active flavors, the meson matrix gives $5 \times 5 = 25$ scalars. The diquarks $[q_i q_j]$ form a representation of $SU(5)_f$:

$$\mathbf{5} \otimes \mathbf{5} = \overline{\mathbf{10}}_A \oplus \mathbf{15}_S \,. \tag{11}$$

The symmetric $\mathbf{15}_S$ gives 15 diquarks including same-flavor states $[uu]$, $[dd]$, $\ldots$, while the antisymmetric $\overline{\mathbf{10}}_A$ gives 10 diquarks with $i \neq j$ only.

7.2 The symmetric representation

The sBootstrap SUSY generator acts asymmetrically on the two fermion sectors. When it acts on a *lepton*, it produces a meson: $Q \cdot \ell \sim q\bar{q}$. Since the quark and antiquark are distinguishable, the meson lies in the full $\mathbf{5} \otimes \bar{\mathbf{5}} = \mathbf{24} + \mathbf{1}$. There is no Pauli constraint, and the meson spectrum is perfectly reproduced.

When it acts on a *quark*, it produces a diquark: $Q \cdot q_i \sim q_i q_j$. The bosonic SUSY generator applied to a single flavor index i naturally produces the **symmetric** product—the $\mathbf{15}_S$ representation, not the antisymmetric $\overline{\mathbf{10}}$. This is the representation identified in the $SO(32)$ decomposition [1]: the adjoint of $SO(32)$ under $SU(5)_f$ decomposes to include the $\mathbf{15} + \overline{\mathbf{15}}$ (diquarks) alongside the $\mathbf{1} + \mathbf{24}$ (mesons).

The 15 includes 5 same-flavor states ($[uu]$, $[dd]$, $[ss]$, $[cc]$, $[bb]$) absent from the $\overline{\mathbf{10}}$. Among these are exotic $+4/3$-charged diquarks such as $[uu]$. The cost: an additional chiral $+4/3$ quark per generation, as identified in [1]. Whether these exotic states appear in nature as resonances, or whether they decouple at some high scale, is an open question.

7.3 The Pauli theorem for scalar diquarks

The requirement of the symmetric $\mathbf{15}$ conflicts with Fermi statistics—not merely for $L = 0$ diquarks, but for *all* scalar diquarks in the color $\bar{3}$ channel, regardless of internal orbital structure.

Theorem. *For a spin-0 ($J = 0$) diquark in the color $\bar{3}$ (antisymmetric) representation, the flavor wave function is antisymmetric for* ***all*** *values of the orbital angular momentum L and spin S.*

Proof. $J = 0$ requires $L = S$ (since $|L - S| \leq J \leq L + S$). The exchange symmetry of the spin-orbital wave function is $(-1)^{L+S+1} = (-1)^{2L+1} = -1$ for all L. Combined with the color $\bar{3}$ factor (-1), the non-flavor product is $(-1)(-1) = +1$ (symmetric). Total antisymmetry then requires flavor to be antisymmetric: $\overline{\mathbf{10}}$. □

This result is purely kinematic—independent of dynamics, binding mechanisms, or spatial extent. No wave function, however extended, can produce a symmetric-flavor scalar diquark in color $\bar{3}$ while respecting Fermi statistics. The key cases:

L	S	Spin-orbital $(-1)^{2L+1}$	$\times$ Color	Flavor
0	0	-1	$\times(-1) = +1$	must be A
1	1	-1	$\times(-1) = +1$	must be A
2	2	-1	$\times(-1) = +1$	must be A

The $L = 1$, $S = 1$ row closes the "P-wave diquark" escape route that might otherwise seem available.

The theorem has a sharp consequence: **the sBootstrap diquark in the 15 cannot be a qq composite.** It must be either:

1. An algebraically generated object—the image of the Miyazawa supercharge [4, 5] acting on an antiquark, whose symmetry is determined by the superalgebra rather than by the statistics of constituent quarks; or

2. A fundamental string-theoretic object, as in the $SO(32)$ decomposition [1], where the **15** arises from the adjoint representation of the gauge group and the open-string diquark is not built from point-like quarks.

The contrast with the meson sector is instructive. When the SUSY generator acts on a lepton, it produces a meson ($q\bar{q}$): the quark and antiquark are distinguishable, no Pauli constraint applies, and the meson spectrum is reproduced without tension. When it acts on a quark, it produces a diquark in the "wrong" representation—and the theorem shows this cannot be fixed by any internal structure of a qq pair. The asymmetry between the two SUSY directions is absolute: mesons work as composites; diquarks do not. This is not a weakness of the sBootstrap but a *prediction*: the diquark sector requires new physics beyond the qq picture.

7.4 The SO(32) connection

The sBootstrap counting matches a decomposition of the $SO(32)$ gauge group familiar from Type I/heterotic string theory. Under $SO(32) \supset SU(5) \times \ldots$, the 496-dimensional adjoint decomposes to include the $\mathbf{1}+\mathbf{24}$ (mesons/sleptons) and the $\mathbf{15}+\overline{\mathbf{15}}$ (diquarks/squarks).

This is suggestive: the same group structure that gives anomaly-free string compactifications produces the symmetric diquark representation demanded by the SUSY generator. The $SO(32)$ embedding may provide the UV completion in which the compositeness of diquarks (and hence the resolution of the Pauli sign) is made explicit: in the string picture, diquarks are open-string states whose symmetry is determined by the gauge group, not by point-particle statistics.

8 SUSY Breaking: From Seeds to Spectrum

The SUSY-breaking story begins with the foundational observation (1): $m_\pi^2 - m_\mu^2 \approx f_\pi^2$. This is the Volkov–Akulov mass formula for a pseudo-Goldstino—the muon—whose superpartner—the pion—is a pseudo-Goldstone boson. The SUSY-breaking scale is $f_\pi \approx 92\,\text{MeV}$, and the breaking mechanism is chiral: $\langle \bar{q}q \rangle \neq 0$ breaks both chiral symmetry and the sBootstrap SUSY simultaneously. The Volkov–Akulov realization is the *physical starting point*; what follows in this section is the SQCD machinery that makes it precise.

The dynamics must accomplish three things: generate the seed-to-bloom transition for each tuple, preserve the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ through the breaking, and produce the correct splitting between meson and lepton masses at scale f_π.

8.1 The Seiberg framework

The natural arena is $\mathcal{N}=1$ SQCD with gauge group $SU(3)_c$ and N_f flavors of chiral superfields Q^i, $\bar{Q}_j$ in the fundamental and antifundamental representations. For $N_f = N_c = 3$ (the case relevant to light QCD), Seiberg's exact results [7] establish that the theory confines, with the low-energy degrees of freedom being the composite meson superfield

$$M^i{}_j = Q^i \cdot \bar{Q}_j \,, \tag{12}$$

a 3×3 matrix of chiral superfields, together with baryons $B = \epsilon_{ijk} Q^i Q^j Q^k$ and $\bar{B}$. These satisfy the quantum-modified constraint

$$\det M - B\bar{B} = \Lambda^{2N_c} \,, \tag{13}$$

where Λ is the dynamical scale.

Each entry $M^i{}_j$ is a chiral superfield containing a complex scalar (the meson) and a Weyl fermion (the lepton, in the sBootstrap identification). The scalar and fermion are degenerate in the SUSY limit; SUSY breaking splits them.

8.2 The three chiral superfields of each seed

The seed triple $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ maps naturally onto three chiral superfields with the following mass structure before breaking:

Superfield	Mass	Role
Φ_0	$m_0 = 0$	Massless: flat direction
Φ_a	$m_a = \mathfrak{z}_a^2$	Light massive member
Φ_b	$m_b = \mathfrak{z}_b^2$	Heavy massive member

The ratio $\mathfrak{z}_a/\mathfrak{z}_b = 2-\sqrt{3}$ is enforced by the Koide energy-balance condition applied to the seed.

In the Seiberg description, these three superfields are entries of the meson matrix $M^i{}_j$. For the $(0, s, c)$ seed, the relevant entries involve the strange and charm quarks; Φ_0 is the entry whose quark-mass combination vanishes in the seed limit (i.e., the direction in flavor space where $m_i + m_j \to 0$ through GOR).

8.3 F-term breaking: the O'Raifeartaigh mechanism

The VA relation (1) tells us *that* SUSY is broken at scale f_π. To understand *how*, we need the microscopic mechanism. The O'Raifeartaigh model [8] is the simplest realization of spontaneous F-term SUSY breaking. With three chiral superfields, the superpotential

$$W = f\,\Phi_0 + m\,\Phi_1\Phi_2 + g\,\Phi_0\Phi_1^2 \tag{14}$$

gives F-terms:

$$F^*_{\Phi_0} = f + g\,\phi_1^2 \,, \tag{15}$$

$$F^*_{\Phi_1} = m\,\phi_2 + 2g\,\phi_0\phi_1 \,, \tag{16}$$

$$F^*_{\Phi_2} = m\,\phi_1 \,. \tag{17}$$

If $f \neq 0$ and $m \neq 0$, then $F^*_{\Phi_2} = 0$ requires $\phi_1 = 0$, but then $F^*_{\Phi_0} = f \neq 0$: SUSY is **necessarily broken**. The key features are:

1. A massless field (Φ_0) exists at tree level—it is the pseudo-modulus.
2. The breaking is triggered because F_{Φ_0} cannot vanish.
3. After including loop corrections, Φ_0 acquires a mass (the Coleman–Weinberg potential lifts the flat direction).

The structural parallel with the seed triples is:

O'Raifeartaigh	Flavor	Seed	After breaking
Φ_0 (massless)	b / B	$\mathfrak{z} = 0$ (flat direction)	$\mathfrak{z}_b = 64.65$ (mass generated)
Φ_1 (light)	s / π	$\mathfrak{z}_s = 9.67$	$\mathfrak{z} \to -9.67$ (sign-flipped)
Φ_2 (heavy)	c / D_s	$\mathfrak{z}_c = 35.64$	$\mathfrak{z}_c \approx 35.64$ (shifted)
		$f = 0$ (SUSY)	$f = f_\pi$ (broken)

This parallel is *structural*, not quantitative: the minimal O'Raifeartaigh model gives Φ_1 and Φ_2 the same mass parameter m, whereas the seed triple has a mass ratio $m_s/m_c = (2-\sqrt{3})^2 \approx 0.07$—far from degenerate. The hierarchy between the two massive members must come from additional dynamics beyond the minimal O'Raifeartaigh superpotential. What the O'Raifeartaigh structure *does* provide is the essential feature: a massless flat direction ($\Phi_0 =$ the b-direction) whose F-term is forced nonzero. The b quark and B meson are always present as flavor degrees of freedom; the seed configuration $(0, s, c)$ means the b-direction sits at the flat direction before breaking. The bloom is not particle creation but mass generation: breaking lifts Φ_0 from zero to $\mathfrak{z}_b \neq 0$, simultaneously flipping the sign of $\mathfrak{z}_s$, so that the same three flavors rearrange from $(0, s, c)$ into $(-s, c, b)$.

A striking difference from standard O'Raifeartaigh: the pseudo-modulus Φ_0 normally acquires the *smallest* mass in the spectrum (a loop-suppressed Coleman–Weinberg mass, parametrically $\sim g^2 f/16\pi^2 m$). In the sBootstrap bloom, the b-direction acquires the *largest* mass ($m_b \gg m_s, m_c$). The bloom is far more dramatic than a perturbative CW lifting—it is a nonperturbative, strong-dynamics effect in which confinement generates a mass of order $\Lambda^2_{\mathrm{QCD}}/m_q$, not $g^2/(16\pi^2)$. This inversion is a signature of the dynamical (SQCD) realization versus the perturbative (superpotential) one.

The identification $f = \langle F_{\Phi_0}\rangle \sim f_\pi$ is the sBootstrap's central claim: the SUSY-breaking F-term is the chiral condensate parameter.

In SQCD, this is realized dynamically rather than through a superpotential: the chiral condensate $\langle \bar{q}q\rangle \neq 0$ acts as the nonzero F-term, the impossibility of restoring chiral symmetry in the QCD vacuum is the dynamical analogue of $f \neq 0$, and the pion—the would-be Goldstone boson that acquires mass from explicit chiral breaking—plays the role of Φ_0 whose mass is lifted by loop corrections (here, by quark masses).

8.4 The Goldstino direction and the Volkov–Akulov realization

When SUSY is spontaneously broken, a massless fermion appears: the Goldstino, the fermionic analogue of the Goldstone boson. Its decay constant f_{VA} sets the SUSY-breaking scale through the Volkov–Akulov Lagrangian [11]:

$$\mathcal{L}_{\mathrm{VA}} = -f_{\mathrm{VA}}^2 \det\left(\delta_\mu^\nu + \frac{i}{f_{\mathrm{VA}}^2}\,\bar{\lambda}\bar{\sigma}^\nu\partial_\mu\lambda\right), \tag{18}$$

where λ is the Goldstino field. At low energies, the Goldstino couples derivatively with strength $\sim 1/f_{\mathrm{VA}}^2$, exactly as the pion couples with strength $\sim 1/f_\pi^2$ in the chiral Lagrangian.

The sBootstrap identification is:

$$f_{\mathrm{VA}} = f_\pi\,, \qquad \text{Goldstino} = \text{muon (partner of } \pi)\,. \tag{19}$$

More precisely, the muon is a *pseudo-Goldstino*: in the limit $m_q \to 0$ where the pion becomes exactly massless (a true Goldstone boson), its fermionic partner also becomes massless (a true Goldstino). The finite quark masses explicitly break both chiral symmetry (giving $m_\pi \neq 0$) and the sBootstrap SUSY (giving $m_\mu \neq m_\pi$), with the splitting controlled by f_π:

$$m_\pi^2 - m_\mu^2 = f_\pi^2\,. \tag{20}$$

This is the Volkov–Akulov relation for the pseudo-Goldstino: the mass splitting between the scalar (pion) and its fermionic partner (muon) is set by the SUSY-breaking scale squared.

The VA mechanism is not a separate breaking from the F-term: it is the *low-energy, composite-level description* of the same breaking. At the microscopic level, $\langle F\rangle \neq 0$ in the meson superfield; at the composite level, this manifests as the Goldstino coupling with $f_{\mathrm{VA}} = f_\pi$. These two quantities—the Goldstino decay constant and the pion decay constant—are *identified* in the sBootstrap; this is the central claim, not a derivation.

The identification should not be confused with the standard relation between the F-term VEV and the chiral condensate. In standard SQCD, $\langle F\rangle \sim |\langle\bar{q}q\rangle|/f_\pi \sim (250\,\mathrm{MeV})^3/92\,\mathrm{MeV} \approx 170{,}000\,\mathrm{MeV}^2$, while $f_\pi^2 \approx 8{,}500\,\mathrm{MeV}^2$—a factor of ~ 20 discrepancy. The GOR relation $m_\pi^2 f_\pi^2 = -(m_u + m_d)\langle\bar{q}q\rangle$ connects these through the small quark masses, but does not equate them. The sBootstrap identification $f_{\mathrm{VA}} = f_\pi$ is therefore a *low-energy effective* statement about the composite sector, not a microscopic identity. It says: the scale at which SUSY appears broken in the meson–lepton spectrum is f_π, regardless of the microscopic F-term that produces this breaking.

The VA realization gives more than a mass formula—it organizes the entire breaking narrative. In the chiral limit ($m_q \to 0$), both pion and muon are exactly massless: the pion as the Goldstone boson of broken chiral symmetry, the muon as the Goldstino of broken sBootstrap SUSY. This is the *exact seed*: the B-meson direction sits at the flat direction ($\mathfrak{z} = 0$), and the π–μ pair is massless. Turning on quark masses ($m_q \neq 0$) does three things simultaneously:

1. Gives the pion its GOR mass (lifting the flat direction for π).

2. Gives the muon its mass (the pseudo-Goldstino acquires mass when the explicit breaking parameter m_q is nonzero).

3. Generates the B-meson mass (the flat direction for Φ_0 is lifted by the Coleman–Weinberg potential, i.e., by the same quark masses that feed the GOR formula).

The splitting between (1) and (2) is f_π^2—the VA relation. The seed-to-bloom transition of §4 is the same physics seen from the charge-space perspective: $m_q \neq 0$ lifts the zero, generates the b/B mass, flips the sign of $\mathfrak{z}_s$ and $\mathfrak{z}_\pi$, and produces the full triple. The VA equation (1) is not one fact among many; it is the generating relation from which the bloom follows.

A note on quantum numbers: the Goldstino of spontaneously broken spacetime SUSY is a neutral fermion (the fermionic component of the superfield whose F-term breaks SUSY). The muon carries electric charge and lepton number. The sBootstrap SUSY, however, is not a spacetime symmetry but an emergent hadronic symmetry—a Miyazawa-type superalgebra [5] acting on composite states. In this context, the "Goldstino" need not carry the quantum numbers of a spacetime supercharge; it carries the quantum numbers appropriate to the broken hadronic generator. The muon's charge is inherited from the pion multiplet it partners.

8.5 D-term breaking: electromagnetic splittings

The unbroken $U(1)_{\rm em}$ gauge symmetry allows a Fayet–Iliopoulos (FI) term [9]:

$$\mathcal{L}_{\rm FI} = \xi \, D_{\rm em} \,, \tag{21}$$

which shifts scalar masses by $\delta m^2 = Q_{\rm em}\,\xi$, where $Q_{\rm em}$ is the electric charge. This is the supersymmetric version of the electromagnetic mass difference.

In QCD without SUSY, the electromagnetic mass splitting of the pion is given by the Das–Guralnik–Mathur–Low–Young (DGMLY) formula [14]:

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha_{\rm em}}{4\pi}\,\frac{m_\rho^2}{f_\pi^2}\,f_\pi^2 \approx (35.5\,{\rm MeV})^2 \,, \tag{22}$$

which in the SUSY framework becomes a D-term contribution:

$$\xi \sim \frac{3\alpha_{\rm em}}{4\pi}\,m_\rho^2 \sim (35\,{\rm MeV})^2 \,. \tag{23}$$

In charge space, the D-term shifts $\mathfrak{z}$ by an amount proportional to $Q_{\rm em}$:

$$\mathfrak{z} \to \mathfrak{z} + \delta\mathfrak{z} \,, \qquad \delta\mathfrak{z} \approx \frac{Q_{\rm em}\,\xi}{2\mathfrak{z}} \,. \tag{24}$$

For the pion ($Q_{\rm em} = \pm 1$, $\mathfrak{z} \approx 11.8$): $\delta\mathfrak{z} \approx 0.2\,{\rm MeV}^{1/2}$, matching the observed $\mathfrak{z}_{\pi^\pm} - \mathfrak{z}_{\pi^0} = 11.82 - 11.62 = 0.20\,{\rm MeV}^{1/2}$.

The D-term effect is subleading to the F-term: $\xi \sim (35\,{\rm MeV})^2$ versus $f_\pi^2 \sim (92\,{\rm MeV})^2$. It operates *within* each supermultiplet entry, splitting charged from neutral mesons, while the F-term splits bosons from fermions across the supermultiplet.

Witten's inequality [10] guarantees $m_{\pi^\pm} > m_{\pi^0}$ in QCD with electromagnetism—the charged pion is always heavier. In the SUSY framework this becomes the statement that the D-term contribution is positive for $Q_{\rm em} \neq 0$, consistent with the FI sign.

8.6 Mapping the breaking to the tuples

The tuple structure maps onto a layered breaking with two independent mechanisms (F-term and D-term) plus a separate heavy-sector origin. The Volkov–Akulov realization of §8.4 is not a third mechanism: it is the low-energy, composite-level description of the same F-term breaking. The VA relation (1) *sets the scale* ($f_{\rm VA} = f_\pi$); the O'Raifeartaigh/SQCD dynamics of §8.3 *provides the mechanism* ($\langle F\rangle \neq 0$).

Layer 1: F-term/VA (chiral condensate, scale $\sim f_\pi$). The chiral condensate $\langle \bar{q}q \rangle \neq 0$ breaks both chiral symmetry and the sBootstrap SUSY. This is the primary breaking, affecting the light sector. The seed triples $(0, s, c)$ and $(0, \pi, D_s)$ are the pre-breaking configurations; the massless member is the flat direction whose F-term is forced nonzero by confinement. After breaking:

- The zero member is lifted and acquires negative charge ($\mathfrak{z} < 0$), becoming the sign-flipped member of the full triple $(-s, -\pi)$.
- The full triples $(-s, c, b)$ and $(-\pi, D_s, B)$ emerge, incorporating the b-quark and B-meson sectors.
- The lepton triple (e, μ, τ) is the fermionic shadow: the leptons acquire masses through the same condensate, with the muon as pseudo-Goldstino.

Layer 2: D-term (electromagnetism, scale $\sim \alpha_{\mathbf{em}} m_\rho$). The $U(1)_{\text{em}}$ FI term generates electromagnetic mass splittings within each multiplet. This is a perturbative correction on top of the F-term breaking, affecting only charged particles. It does not create new Koide triples but shifts existing ones by $O(\alpha_{\text{em}})$.

Layer 3: The heavy sector (scale $\sim v_0^{(cbt)}$). The (c, b, t) triple at $v_0^2 \approx 29.6\,\text{GeV}$ operates far above the chiral scale. It has no seed (no massless member), no meson shadow (top does not hadronize), and its breaking may be connected to a different sector of the theory—possibly the electroweak Higgs mechanism.

This layered structure involves three distinct scales, each with different physical meaning:

$$\begin{aligned}
\text{F-term (boson–fermion splitting):} \quad & f_\pi \approx 92\,\text{MeV}\,, \quad f_\pi^2 \approx 8{,}500\,\text{MeV}^2\,; && (25)\\
\text{D-term (EM fine structure):} \quad & \xi^{1/2} \approx 35\,\text{MeV}\,, \quad \xi \approx 1{,}200\,\text{MeV}^2\,; && (26)\\
\text{Heavy sector (Koide scale):} \quad & v_0^{(cbt)} \approx 172\,\text{MeV}^{1/2}\,, \quad (v_0^{(cbt)})^2 \approx 29{,}600\,\text{MeV}\,. && (27)
\end{aligned}$$

Comparing the squared scales: $f_\pi^2 \gg \xi$ (F-term dominates over D-term by a factor ~ 7, consistent with the D-term being a perturbative $O(\alpha_{\text{em}})$ correction). The heavy-sector scale $(v_0^{(cbt)})^2$ carries dimensions of MeV (not MeV^2) because v_0 is a charge (MeV$^{1/2}$), not an energy; direct comparison with f_π requires specifying which physical quantity is being compared.

8.7 The O'Raifeartaigh structure in SQCD

In Seiberg's SQCD with $N_f = N_c = 3$, the quantum constraint (13) provides a natural setting for the O'Raifeartaigh mechanism. The constraint $\det M - B\bar{B} = \Lambda^6$ means the moduli space has a specific geometry: not all field directions are flat. When quark masses are added via a tree-level superpotential $W_{\text{tree}} = m_i M^i{}_i$ (sum over flavors), the F-terms become:

$$F_{M^i{}_j} = m_i\, \delta^i_j + \Lambda^6\,(\text{constraint terms})\,. \tag{28}$$

For the seed triple $(0, s, c)$, the "0" member corresponds to a direction in the meson matrix where the tree-level mass vanishes—the entry involving the lightest quark combination. In the exact chiral limit ($m_u = m_d = 0$), this entry is truly massless, and the constraint (13) forces $\langle F \rangle \neq 0$ for that entry. This is the dynamical O'Raifeartaigh mechanism: confinement (encoded in Λ^6) plays the role of the parameter f in the superpotential, and the impossibility of setting all F-terms to zero is a consequence of the strong dynamics rather than a choice of superpotential.

The ISS mechanism (Intriligator–Seiberg–Shih [13]) establishes metastable SUSY-breaking vacua for $N_c < N_f < 3N_c/2$, where the rank condition in the magnetic dual prevents all F-terms from vanishing. The sBootstrap case $N_f = N_c = 3$ is strictly *below* the ISS window—at $N_f = N_c$ the low-energy description is qualitatively different, governed by the quantum-modified constraint (13) on the moduli space rather than by a free magnetic dual. However, the physical QCD has $N_f = 3$ light flavors and $N_f = 4$–5 if charm and bottom are included at their respective thresholds, which does fall within the ISS window for $N_c = 3$. The sBootstrap's multi-scale structure—with the seed triples involving s, c, and b quarks at different scales—may require treating the effective N_f as scale-dependent, entering the ISS regime above the strange threshold.

8.8 How the three supermultiplets join

The seed-to-bloom transition generates three separate Koide triples. But the full sBootstrap requires these triples to be *embedded in a single meson matrix.* The 3×3 light-flavor matrix is:

$$M^{(3)} = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} \longleftrightarrow \begin{pmatrix} \pi^0/\eta & \pi^+ & K^+ \\ \pi^- & \pi^0/\eta & K^0 \\ K^- & \bar{K}^0 & \eta' \end{pmatrix}. \tag{29}$$

When charm and bottom are included, the matrix extends to 5×5. We display it schematically, with the members of the meson Koide triple $(-\pi, D_s, B)$ marked by boxes:

$$M^{(5)} = \begin{pmatrix} \pi^0/\eta & \boxed{\pi^+} & K^+ & D^0 & \boxed{B^+} \\ \pi^- & \pi^0/\eta & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & \eta' & \boxed{D_s^-} & B_s \\ \bar{D}^0 & D^+ & D_s^+ & \eta_c & B_c \\ B^- & \bar{B}^0 & \bar{B}_s & \bar{B}_c & \eta_b \end{pmatrix}, \tag{30}$$

where rows are indexed by (u, d, s, c, b) and columns by $(\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{b})$.

The three members of the meson Koide triple occupy different off-diagonal blocks of this matrix: $\pi^+ = u\bar{d}$ is in the light (u, d) block, $D_s^- = s\bar{c}$ bridges the light-charm sector, and $B^+ = u\bar{b}$ bridges the light-bottom sector. They do not form a "slice" in any simple linear-algebraic sense—they are scattered entries connected by the Koide condition on their *masses*, not by their positions in the matrix. The meson seed $(0, \pi, D_s)$ drops the B^+ and replaces it with zero (the entry that would involve the top quark, absent from the matrix).

This scattered structure is the central puzzle: the Koide triples are *mass relations* among matrix entries, not submatrices. The question is whether the Seiberg constraint $\det M - B\bar{B} = \Lambda^6$ (appropriately generalized to 5×5), combined with the SUSY-breaking dynamics, produces correlations among these scattered entries that enforce the energy-balance condition $\langle z_k^2 \rangle = z_0^2$.

8.9 The energy balance as a fixed point

One possibility is that the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ is a *dynamical attractor*: any set of three masses evolving under the SUSY-breaking dynamics flows toward the Koide surface $Q = 2/3$, as a fixed point of the RG flow for the soft SUSY-breaking parameters. This is speculative, but the observation that the condition holds for fundamental fermions (leptons, quarks) *and* composite bosons (mesons) across very different scales lends it plausibility. A concrete test would be to show that the one-loop soft-mass RG equations in $\mathcal{N} = 1$ SQCD with appropriate boundary conditions flow toward $\sigma(\mathfrak{z})/\bar{\mathfrak{z}} = 1$; this calculation has not yet been performed.

8.10 The exact seed and what breaks it

The seed configuration $(0, \mathfrak{z}_a, \mathfrak{z}_b)$ requires *two* independent conditions:

1. One member at $\mathfrak{z} = 0$ exactly (the flat direction).
2. The ratio of the massive members equals $2 - \sqrt{3}$:

$$\frac{\sqrt{m_a}}{\sqrt{m_b}} = 2 - \sqrt{3} \approx 0.26795\,. \tag{31}$$

These are logically distinct. The chiral limit ($m_u = m_d = 0$, $\langle \bar{q}q \rangle \neq 0$) enforces condition (1) for the meson sector: the pion is an exact Goldstone boson, $m_\pi = 0$, and in the picture of §8.3 the B-meson direction sits at the flat direction before acquiring mass. But the chiral limit says *nothing* about condition (2)—it does not constrain $\sqrt{m_{D_s}}/\sqrt{m_B}$ or $\sqrt{m_s}/\sqrt{m_c}$ to equal $2 - \sqrt{3}$.

The physical deviations from the exact ratio are small but measurable:

Seed	Exact ratio	Physical ratio	Deviation
$(0, \pi, D_s)$	0.2680	$\sqrt{m_\pi}/\sqrt{m_{D_s}} = 0.2663$	-0.6%
$(0, s, c)$	0.2680	$\sqrt{m_s}/\sqrt{m_c} = 0.2712$	$+1.2\%$
$(0 \approx e, \mu, \tau)$	0.2680	$\sqrt{m_\mu}/\sqrt{m_\tau} = 0.2439$	-9.0%

The meson seed is closest to the exact ratio (0.6% deviation). The quark seed deviates by 1.2%. A natural thought is that QCD running might shift the quark ratio: perhaps at some scale μ^* the ratio $\sqrt{m_s(\mu^*)}/\sqrt{m_c(\mu^*)}$ hits $2 - \sqrt{3}$ exactly. But this is *not possible within QCD alone.* At leading order, all quark masses run with the same anomalous dimension γ_m:

$$m_i(\mu) = m_i(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_m/\beta_0}, \tag{32}$$

so the ratio $m_s(\mu)/m_c(\mu)$—and therefore $\sqrt{m_s}/\sqrt{m_c}$—is an RG invariant of QCD. Higher-order corrections break this invariance, but only at the $O(\alpha_s^2)$ level, far too small to account for 1.2%.

The quark seed deviation is therefore *intrinsic*: the $\overline{\mathrm{MS}}$ mass ratio m_s/m_c is a fixed number (at any QCD scale), and it misses the Koide target by 1.2%. Three possible explanations present themselves:

Electroweak running. Above the electroweak scale, quark masses are controlled by Yukawa couplings y_i to the Higgs, which *do* run differently for different flavors: the β-functions for y_s and y_c involve different electroweak quantum numbers ($SU(2)_L$ doublet structure, hypercharge). The ratio y_s/y_c evolves with scale, and there may exist a scale μ^* above the electroweak threshold where $\sqrt{y_s(\mu^*)}/\sqrt{y_c(\mu^*)} = 2 - \sqrt{3}$. This would identify the "seed scale" as an electroweak-sector phenomenon, not a QCD one—consistent with the sBootstrap's identification of the breaking with $SU(2)_L \times U(1)_Y$ rather than $SU(3)_c$.

Charge running. Even if mass ratios are QCD-invariant, the Koide decomposition $\mathfrak{z}_k = z_0 + z_k$ may have its own flow. The vacuum energy $v_0 = z_0^2$ sets the common scale, and the fluctuations z_k encode the deviations. If the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ is a dynamical attractor (as suggested in §8.9), the z_k may evolve under a flow in which the *ratio* z_a/z_b approaches $2 - \sqrt{3}$ at the fixed point while the individual masses (and their QCD-invariant ratios) remain unchanged. In this picture, the 1.2% deviation measures how far the quark sector is from the attractor.

Meson vs. quark deviations. The meson deviation (0.6%) is smaller than the quark deviation (1.2%), and for a different reason: meson masses are *not* simple functions of quark mass ratios. The pion mass is given by the GOR relation $m_\pi^2 f_\pi^2 = -(m_u + m_d)\langle \bar{q}q \rangle$; the D_s mass involves heavy-quark effective theory; and both receive chiral perturbation theory corrections. These nonperturbative QCD effects are the dominant source of the meson seed deviation, and are in principle calculable at NLO in ChPT.

A remarkable feature of the VA relation. The foundational equation $m_\pi^2 - m_\mu^2 = f_\pi^2$ contains no QCD coupling constant: no α_s, no N_c, no Λ_{QCD}. It is a pure mass relation among three experimentally measured quantities—two masses and a decay constant—holding to 2.2%. In the sBootstrap, this is natural: the relation is a SUSY mass splitting, set by the breaking scale f_π, and SUSY mass relations do not involve the gauge coupling of the confining theory. Standard QCD *explains* f_π in terms of $\langle \bar{q}q \rangle$ and α_s; the sBootstrap *uses* f_π as the fundamental parameter, with QCD providing the dynamical origin but not appearing in the mass formula. The absence of QCD factors in (1) is a hint that the relevant symmetry is not $SU(3)_c$ but the hadronic superalgebra that connects pions to leptons.

The lepton "seed" has the worst ratio match (−9%), which is consistent with the observation that the lepton triple is *not* a true seed: $m_e \neq 0$, and the electron was never at the flat direction. The leptons are an "almost-seed" whose excellent $Q = 0.66666$ arises not from the seed structure but from the smallness of m_e, which makes the two conditions approximately degenerate. For the leptons, condition (1) is approximate rather than exact, and condition (2) is correspondingly relaxed.

8.11 What the (c, b, t) triple tells us

The (c, b, t) triple ($Q = 0.6695$, $v_0^2 = 29.6\,\mathrm{GeV}$) has neither a seed nor a meson shadow. It operates at a scale far above Λ_{QCD}, suggesting its origin is different:

1. It may be governed by the electroweak Higgs mechanism rather than the chiral condensate. The top Yukawa $y_t \approx 1$ is "natural" in the sense that $m_t \approx v_{\mathrm{EW}}/\sqrt{2}$; the Koide condition on (c, b, t) would then constrain the Yukawa coupling matrix.

2. It may represent a Volkov–Akulov realization at the electroweak scale: if there is a second SUSY breaking at $f' \sim v_{\rm EW}$, the (c, b, t) triple would be its analog of the (π, μ) system, but with f' replacing f_π.

3. Or it may be a purely kinematic accident—the quark masses happen to satisfy the Koide condition without a dynamical reason. The 0.42% precision makes this unlikely but not impossible.

Interpretation (1) is most natural within the sBootstrap: the (c, b, t) triple involves quarks whose masses are dominated by Yukawa couplings to the Higgs, not by the chiral condensate, and the Koide condition would then be a constraint on the Yukawa matrix rather than on the SQCD dynamics. The chiral sector and the electroweak sector would each independently produce a Koide-type energy balance, with the charm and bottom quarks serving as the bridge between the two.

If interpretation (2) is also invoked, the sBootstrap has two breaking scales:

$$f_{\rm chiral} = f_\pi \approx 92\,{\rm MeV}\,, \qquad f_{\rm EW} \sim v_{\rm EW} \approx 246\,{\rm GeV}\,, \tag{33}$$

and the full spectrum requires both. The light sector (leptons, light mesons) is governed by f_π; the heavy sector (top, and by extension charm and bottom through the Koide network) is governed by $v_{\rm EW}$. The charm and bottom quarks, appearing in both the $(0, s, c)/(-s, c, b)$ light-sector triples and the (c, b, t) heavy-sector triple, are the "bridges" between the two scales.

9 The Electron Mass and the Lepton Seed

The charged lepton triple (e, μ, τ) is the "almost-seed": $\mathfrak{z}_e = 0.715\,{\rm MeV}^{1/2}$ is small but nonzero. The electron's charge is near zero because of cancellation in $\mathfrak{z}_e = z_0 + z_e$:

$$v_0 = 17.716, \quad z_e = -17.001, \quad \mathfrak{z}_e = 17.716 - 17.001 = 0.715\,{\rm MeV}^{1/2}\,. \tag{34}$$

A 96% cancellation. The mass $m_e = 0.715^2 = 0.511\,{\rm MeV}$ is the square of something already small.

The near-vanishing of m_e is a structural property of the Koide embedding, not fine-tuning: the energy balance allows perturbations as large as z_0. Every Koide triple has one member with $|\mathfrak{z}|/v_0$ small:

Triple	Lightest	$\lvert\mathfrak{z}\rvert/v_0$	Character
(e, μ, τ)	e	0.040	Almost-seed
$(-\pi, D_s, B)$	π (neg.)	0.337	Full triple
(c, b, t)	c	0.236	Full triple
$(-s, c, b)$	s (neg.)	0.320	Full triple

The electron is by far the most extreme case, consistent with the lepton triple being nearest to its seed.

10 Symmetry Restoration

The broken spectrum reassembles as symmetry-breaking effects are removed:

1. **Turn off QED:** Electromagnetic splittings vanish ($\pi^\pm = \pi^0$, $K^\pm = K^0$). D-term effects disappear.
2. **Set $m_u = m_d$:** Exact isospin.
3. **Set $m_s = m_d$:** $SU(3)_V$ restored, $\pi = K = \eta$. Koide splitting $v_1 \to 0$; lepton partners degenerate.
4. **Send $m_q \to 0$:** Chiral limit, all meson charges $\to 0$. If sBootstrap SUSY restores, all light lepton masses vanish too. This is the seed configuration: $(0, 0, 0)$.

The seed triples are the "penultimate" stage: one member has reached zero, but the other two retain the $2-\sqrt{3}$ ratio. Full triples are the final broken state. The lepton triple, trapped between seed and bloom, reveals the hierarchy: chiral breaking $\to$ seed $\to$ bloom $\to$ full spectrum.

The direction of the flow also matters: approaching the chiral limit, the pion charge rises from $\mathfrak{z} = -11.81$ toward zero from below. It crosses zero exactly in the chiral limit. This means the pion was "born negative"—its charge sign is set by the direction of chiral symmetry breaking, and the seed $(0, \pi, D_s)$ is the configuration at the moment of crossing.

11 Nature Abhors Chirality

A pattern across the Standard Model: every chiral structure is eliminated in the infrared.

UV structure	IR survivor	Mechanism
$SU(2)_L \times U(1)_Y$	$U(1)_{\text{em}}$	Higgs
$SU(N_f)_L \times SU(N_f)_R$	$SU(N_f)_V$	$\langle \bar{q}q \rangle$ condensate
$U(1)_A$	nothing	Anomaly
$\bar{\theta}$-vacuum	$\lvert\bar{\theta}\rvert < 10^{-10}$	Unknown
Chiral color?	$SU(3)_C$	Axigluon mass

The sBootstrap adds: superspace chirality (boson vs. fermion distinction within a multiplet) is broken by the same condensate that breaks flavor chirality, with $f_{\text{SUSY}} = f_\pi$. The energy-balance condition $\langle z_k^2 \rangle = z_0^2$ may be related: the breaking is "as large as the background allows"—maximal, not perturbative.

12 Open Problems

12.1 The diquark sector

The Pauli theorem (§7.3) proves that no $J = 0$, color-$\bar{3}$ diquark can have symmetric flavor, closing all field-theoretic escape routes ($L = 0$, P-wave, or higher). The sBootstrap's **15** diquark must therefore be non-composite: either an algebraic image of the Miyazawa supercharge or a fundamental $SO(32)$ string state. The exotic $+4/3$-charged diquarks predicted by the **15** are a testable consequence.

12.2 What determines the Koide phase?

The energy balance constrains but does not uniquely fix δ. The value $\delta_\ell \approx 2/9$ for leptons must come from dynamics. Is it a fixed point?

12.3 Connecting the three tuples

The light sector (seed $\to$ bloom, scale $\sim \Lambda_{\rm QCD}$) and heavy sector (c, b, t, scale $\sim v_{\rm EW}$) must connect. The charm and bottom quarks appear in both, providing the links.

12.4 The baryon sector

In Seiberg's SQCD with $N_f = N_c = 3$, baryons are part of the low-energy spectrum. Their SUSY partners are "baryonic fermions" with no known candidates among observed particles.

12.5 The strong CP problem

The principle "Nature abhors chirality" predicts $\bar{\theta} = 0$ but provides no mechanism.

12.6 The kaon–muon puzzle

The SUSY splitting $m_K^2 - m_\mu^2 \approx (482\,\mathrm{MeV})^2$ is much larger than $f_K^2 \approx (110\,\mathrm{MeV})^2$, showing that the breaking is strongly flavor-dependent.

13 Conclusions

Three principal results:

1. The Koide formula is energy equipartition. $Q = 2/3$ is $\langle z_k^2 \rangle = z_0^2$: perturbation energies match the vacuum energy. The signed-charge framework resolves the electron mass puzzle (near-cancellation, not fine-tuning) and reveals that mesons satisfy Koide with $(-\pi, D_s, B)$ to 0.1%.

2. Seed triples set the stage for SUSY breaking. The quark seed $(0, s, c)$ and meson seed $(0, \pi, D_s)$ share the universal ratio $2 - \sqrt{3}$ and operate at $v_0^2 \sim 230$–$350\,\mathrm{MeV}$, close to the lepton triple's $v_0^2 = 314\,\mathrm{MeV}$. The massless member is the necessary condition for F-term (O'Raifeartaigh) SUSY breaking. Upon breaking, seeds bloom into full triples $(-s, c, b)$ and $(-\pi, D_s, B)$. The lepton triple is the "almost-seed" that has barely bloomed.

3. Two breaking mechanisms plus a heavy-sector origin. F-term breaking (via the massless seed member) generates the light sector; the Volkov–Akulov relation $f_{\rm VA} = f_\pi$ is its low-energy realization, not a separate mechanism. D-term breaking ($U(1)_{\rm em}$ FI parameter) provides electromagnetic splittings as a perturbative correction. The heavy triple (c, b, t) stands apart: no seed, no meson shadow, scale far above $\Lambda_{\rm QCD}$, possibly connected to electroweak breaking.

The diquark sector, required to complete the SUSY multiplet count, fills the symmetric $\mathbf{15} + \overline{\mathbf{15}}$ of $SU(5)_f$—the representation produced naturally by the SUSY generator acting on quarks and by the $SO(32)$ decomposition. A kinematic theorem (§7.3) shows that *no* $J = 0$ color-$\bar{3}$ diquark can have symmetric flavor, regardless of orbital angular momentum: the sBootstrap diquark in the **15** cannot be a qq composite. It must be an algebraically generated object (Miyazawa supercharge) or a fundamental string-theoretic state ($SO(32)$ adjoint).

The organizing principle is that each sector of the spectrum begins with a massless member (a seed), which is the necessary condition for spontaneous SUSY breaking. The breaking lifts the zero, generates the full triple, and the energy-balance condition $\langle z_k^2 \rangle = z_0^2$ is preserved as a fixed point of the dynamics. The entire charged fermion and pseudoscalar meson spectrum is then determined by a small number of parameters: the vacuum charges v_0 of the three tuples, the Koide phases δ that control the bloom, and the SUSY-breaking scale f_π that separates bosons from fermions.

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